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CONSERVATIVE SCHEMES OF THE NON-STATIONARY PROBLEM FOR THE OPTIMAL SELECTION OF THE LOCATION OF HEAT SOURCES IN THE ROD

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Cover Page Footnote

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Erratum

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CONSERVATIVE SCHEMES OF THE NON-STATIONARY PROBLEM FOR THE OPTIMAL SELECTION OF THE LOCATION OF HEAT SOURCES IN THE ROD

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Abstract: *In this paper, we develop a method and algorithm for solving the problem of the optimal selection of the density of heat sources on the rod in such a way that the temperature inside the considered region is within the given limits. At the same time, heat sources must provide a given temperature regime of the minimum total power and temperature in a given corridor. Conservative approximations of the original problem are constructed in the form of a linear programming problem. A method for constructing conservative schemes for solving the heat equation with variable coefficients, a brief description of the developed software application for constructing computational grids and solving equations is given. A new method is proposed and justified for the numerical solution of non-stationary problems of the optimal selection of heat sources in the rod. A software application for conducting numerical experiments to solve the problem has been created. A description of the based algorithm and the results of numerical experiments is provided.*

Keywords*: non-stationary problems, optimal selection, heat sources, heat equation, balance equation, conservation law, integro-interpolation method, implicit schemes, conservative schemes, simplex method.*

КОНСЕРВАТИВНЫЕ СХЕМЫ НЕСТАЦИОНАРНОЙ ЗАДАЧИ ПО ОПТИМАЛЬНОМУ ВЫБОРУ МЕСТОПОЛОЖЕНИЯ ИСТОЧНИКОВ ТЕПЛА В СТЕРЖНЕ

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Аннотация: *В данной работе разработаны метод и алгоритм решения задачи об оптимальном выборе плотности источников тепла на стержне таким образом, чтобы температура внутри рассматриваемой области находилась в заданных пределах. При этом источники тепла должны обеспечить заданный температурный режим минимальной суммарной мощности и температуру в заданном температурном коридоре. Строятся консервативными конечномерные аппроксимации исходной задачи в виде задачи линейного программирования. Приводится метод построения консервативные* *разностные схемы для решения уравнения теплопроводности с переменными коэффициентами, краткое описание разработанного программного приложения для построения расчётных сеток и решения уравнений. Предлагается и обосновывается новый метод численного решения нестационарных задач оптимального выбора источников тепла в стержне. Создано программное приложение для проведения численных экспериментов решения поставленной задачи. Приводятся описание основанного алгоритма и результатов численных экспериментов.*

Ключевые слова: *нестационарные задачи, оптимальный выбор, источники тепла, уравнение теплопроводности, уравнение баланса, закон сохранения, интегро-интерполяционный метод, неявные схемы, консервативные схемы, симплекс-метод.*

СТЕРЖЕНДАГИ ИССИҚЛИК МАНБАЛАРИНИНГ ЖОЙЛАШУВИНИ ОПТИМАЛ ТАНЛАШ БЎЙИЧА НОСТАЦИОНАР МАСАЛАЛАРНИНГ КОНСЕРВАТИВ СХЕМАЛАРИ

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Аннотация: *Ушбу ишда ўрганилаётган соҳа ичидаги ҳарорат берилган чегаралар ичида бўлиши учун стержендаги иссиқлик манбаларининг жойлашувини оптимал танлаш масаласини ечиш усули ва алгоритми ишлаб чиқилган. Бундай ҳолда, иссиқлик манбалари берилган ҳарорат режимида қувватларнинг йиғиндиси минимал бўлишини ва ҳарорат берилган ҳарорат чегаралари оралиғида бўлишини таъминлаши керак. Масалани чизиқли программалаш кўринишида консерватив чекли ўлчовли аппроксимацияси қурилган. Ўзгарувчи коэффицентли иссиқлик тарқалиш тенгламасини ечиш учун консерватив айирмали схемаларни қуриш усули, ҳисоблаш тўрларини қуриш ва тенгламаларни сонли ечиш учун яратилган дастурий таъминотнинг қисқача тавсифи келтирилган. Стерженда иссиқлик манбаларини оптимал танлашнинг ностационар масалаларини сонли ечиш учун янги усул таклиф қилинган ва асосланган. Масалани ечишда сонли экспериментларни ўтказиш учун дастурий таъминот яратилди. Асосланган алгоритм тавсифи ва сонли экспериментнинг натижалари берилган.*

Калит сўзлар: *ностационар масалалар, оптимал танлаш, иссиқлик манбалари, иссиқлик тарқалиш тенгламаси, баланс тенгламаси, сақланиш қонуни, интеграл-интерполяцион метод, ошкормас схемалар, консерватив схемалар, симплекс методи.*

1. Introduction

One of the types of objects that are widespread in various areas of human activity are heat sources at the border, providing heat in a state of non-stationary heat balance with the environment. It is clear that the temperature inside the body depends on the temperature of the heating medium located at the border of the region. In a typical formulation, the problem of the optimal selection of the power of the heating medium is that the temperature field generated by them inside the body is in a given corridor. Similar problems arise in the organization of heating residential and industrial premises, greenhouses and, if necessary, to maintain a given temperature regime in homogeneous and inhomogeneous solids [1]. They allow a number of settings that are not equivalent due to differences in optimization criteria. Here we consider the problem of finding the density of heat sources of minimum power, which provides a given temperature regime in a certain body under the conditions of its unsteady heat balance with the environment. Possible formulations and ways of solving the stationary problem are discussed in [2]. In the paper [3], a solution to the problem of optimal placement of sources in inhomogeneous media is proposed, in which scalar stationary fields are described by elliptic equations. The algorithms for solving the problem are based on effective methods for evaluating the values of the functional on the set of possible locations of sources, which makes it possible to select the optimal option by implementing the branch and bound method in each specific case. In the work [4], on the basis of the Pontryagin maximum principle, the problems of optimal heating of a room are considered. The methodology for calculating the optimal control of transient modes during heating of the room is presented. The work [5] is devoted to the explicit formulation of the mathematical problem of optimization of heat supply in terms of its energy efficiency and the search for its solutions. In the paper [6], difference schemes are proposed for the case of an incompressible viscous fluid, reflecting the properties of the original equations: the grid approximation of nonlinear transfer terms does not contribute to the energy and entropy balance. The schemes are built on a non-uniform grid in a rectangle using the integro-interpolation method. In the work [7], a conservative algorithm for the numerical solution is constructed for the kinetic Boltzmann equation on the basis of the method of splitting by physical factors. The formulation of the corresponding discrete boundary and initial conditions is given. A number of examples show the effectiveness of the method, which allows to significantly increase the accuracy of calculations. In the paper [8] studied differential-differential problem of control of the diffusion process, an analogue of the maximum principle, allows to determine such moments on and off the maximum source power, at which the inside of a parallelepiped set allowable level concentration at the observed level of concentration of the substance at the interface of the parallelepiped. In the paper [9], the third boundary value problem of parabolic type is considered. The distribution of heat in the body under consideration is controlled by a function that is located on the boundary of the body, the problem is solved, in the event of a conflict, about the possibility of transferring the initial position of the body to the desired state. From a mathematical point of view, this problem belongs to the optimal control problems [10, 11] for elliptic boundary value problems. The existence of a solution and general properties of similar problems for quadratic objective functionals, as well as approximate methods for their solution, have been studied by a number of authors [12-14]. Our problem can also be attributed to inverse problems of heat conduction, methods of an approximate solution of which are considered in [15]. In the works [16, 17], a method and an algorithm for solving the non-stationary problem of the optimal selection of the density of heat sources on simple geometric regions is developed so that the temperature inside the considered region is within the specified limits. In this case, the heat sources must provide a given temperature regime of the minimum total power and temperature in a given corridor filled with a homogeneous or inhomogeneous medium. In these works, to solve the problem, finite-dimensional approximations of the original problem are constructed in the form of a linear programming problem and the results of numerical experiments are presented. In the numerical solution of this problem, a number of difficulties arise, and so far have practically not been considered. An exact solution to this problem may not exist. In this work, the statement of the problem is clarified and the so-called conservative is introduced, which is quite acceptable from an applied point of view. Here, a method is proposed for conservative approximation of the problem, on the basis of which the main algorithms are developed, and a method for the approximate finding of generalized solutions is created.

In this paper, we consider the problem of finding the density distribution of heat sources, which provides a given temperature regime at the minimum total power of these sources. A method and an algorithm for solving non-stationary problems with the optimal selection of the density of heat sources on the rod in such a way that the temperature is within the specified limits are proposed. A software application has been created for carrying out computational experiments using this algorithm.

2. Statement of the problem and its finite-dimensional approximation

In the rectangle $D=\{a \le x \le b,\ 0 \le t \le T\}$ is required to define the function $f(x,t) \ge 0$, which delivers for each $t \in [0,T]$ is the minimum of the linear functional

$$J\{f\} = \int_a^b f(x,t)dx \to \min, \tag{1}$$

under the following conditions:

$$\begin{gathered} \frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(\chi(x,t)\frac{\partial u}{\partial x}\right) + f(x,t), \quad a<x<b,\ 0<t\le T, \\ u(x,0)=u_0(x), \quad a\le x\le b, \\ u(a,t)=\mu_1(t), \quad 0<t\le T, \\ u(b,t)=\mu_2(t), \quad 0<t\le T, \end{gathered} \tag{2}$$

$$m(x,t) \le u(x,t) \le M(x,t), \quad a\le x\le b,\ 0\le t\le T, \tag{3}$$

Here $u=u(x,t)$ – is the temperature of the rod at point x at time t; $\chi(x,t)>0$ – is the thermal conductivity coefficient; $u_0(x)$, $\mu_1(t)$, $\mu_2(t)$, $m(x,t)$, $M(x,t)$ – are given functions. The functions $m(x,t)$, $M(x,t)$ have the meaning of the functions of the minimum and maximum temperature profile in the region D respectively. The density of heat sources is described by the square-integrable functions $f(x,t)$ in the space $L_2(D)$. The solution to this boundary value problem can be obtained in an analytical form using the Fourier method [18].

Let's build a grid in D. Let be

$$\bar{\omega}_h = \{x_i = ih,\ i=0,1,\dots,N_1,\ N_1h=b-a\}$$

uniform mesh with step h on the segment $a\le x\le b$,

$$\bar{\omega}_\tau = \{t_j = j\tau,\ j=0,1,\dots,N_2,\ N_2\tau=T\}$$

grid with step τ on the segment $0\le t\le T$,

$$\bar{\omega}_{h\tau} = \bar{\omega}_h \times \bar{\omega}_\tau = \{(x_i, t_j),\ x_i \in \bar{\omega}_h,\ t_j \in \bar{\omega}_\tau\}$$

grid in D,

$$\omega_{h\tau} = \omega_h \times \omega_\tau = \{(x_i, t_j),\ x_i = ih,\ 0 < i < N_1,\ t_j = j\tau,\ 0 < j \le N_2\}.$$

To obtain homogeneous conservative difference schemes, we use the integro-interpolation method.

Earlier, when constructing difference schemes, we approximated differential operators, determined the stability and convergence of the schemes, assuming that the differential problem has a sufficiently smooth solution. However, not all physical processes are described using differentiable functions. For example, on a shock wave, the gas velocity, its density, pressure, and temperature are discontinuous functions. The corresponding differential problems have no smooth solutions. There is no smooth solution for the heat equation with variable heat conductivity coefficient (2). In the general case, when the differential equation does not make sense, to obtain the difference equations, we will use the recording of physical conservation laws not in differential, but in integral form. Integral conservation laws also make sense for nonsmooth functions that cannot be differentiated, but can be integrated.

The differential equation of heat conduction is based on the integral law of heat conservation:

$$\int_x^{x+\Delta x} u(\xi, t+\Delta t)d\xi - \int_x^{x+\Delta x} u(\xi, t)d\xi = \int_t^{t+\Delta t} W(x,\tau)d\tau - \int_t^{t+\Delta t} W(x+\Delta x,\tau)d\tau + \int_t^{t+\Delta t}\int_x^{x+\Delta x} f(\xi,\tau)d\xi d\tau, \quad (4)$$

where Δt and Δx are arbitrary numbers, $u(x,t)$ – is the temperature, $W(x,t)$ – is the heat flux (the amount of heat that has flowed per unit time through single site)

$$W(x,t) = -\chi(x,t)\frac{\partial u(x,t)}{\partial x}.$$

The integral law (4) is obtained by calculating the heat balance on an arbitrary section of the rod for an arbitrary period of time and therefore is also called the heat balance equation. The above law establishes that the change in the amount of heat in the rod on the segment $[x, x+\Delta x]$ during the time Δt is determined by: the difference in the amount of heat that flowed in and out through the cross-section of the rod x and $x+\Delta x$ for the time Δt; the amount of heat released in the segment $[x, x+\Delta x]$ during the time Δt due to the heat sources distributed on it with the density $f(x,t)$.

To obtain the difference equation, consider the integral heat balance equation on an elementary grid cell on the segment $x_{i-1/2} \le x \le x_{i+1/2}$ for the time interval $t_j \le t \le t_{j+1}$:

$$\int_{x_{i-1/2}}^{x_{i+1/2}} u(x,t_{j+1})dx - \int_{x_{i-1/2}}^{x_{i+1/2}} u(x,t_j)dx = \int_{t_j}^{t_{j+1}} W(x_{i-1/2},t)dt - \int_{t_j}^{t_{j+1}} W(x_{i+1/2},t)dt + \int_{t_j}^{t_{j+1}}\int_{x_{i-1/2}}^{x_{i+1/2}} f(x,t)dxdt.$$

We approximate the integrals included in the balance equation by approximate formulas [19]

$$\int_{x_{i-1/2}}^{x_{i+1/2}} u(x,t_{j+1})dx \approx hu_i^{j+1}; \quad \int_{x_{i-1/2}}^{x_{i+1/2}} u(x,t_j)dx \approx hu_i^j;$$

$$\int_{t_j}^{t_{j+1}} W(x_{i-1/2},t)dt \approx \tau W_{i-1/2}^{j+1}; \quad \int_{t_j}^{t_{j+1}} W(x_{i+1/2},t)dt \approx \tau W_{i+1/2}^{j+1};$$

$$\int_{t_j}^{t_{j+1}} \int_{x_{i-1/2}}^{x_{i+1/2}} f(x,t)dxdt \approx \tau h f_i^{j+1},$$

$$W_{i+1/2}^{j+1} = -\chi_{i+1/2}^{j+1} \frac{u_{i+1}^{j+1} - u_i^{j+1}}{h}; \quad W_{i-1/2}^{j+1} = -\chi_{i-1/2}^{j+1} \frac{u_i^{j+1} - u_{i-1}^{j+1}}{h}.$$

Moreover $\chi_{i\pm 1/2}^{j+1}$ and f_i^{j+1} is defined by the equalities

$$\chi_{i+1/2}^{j+1} = \chi\left(\frac{x_i + x_{i+1}}{2}, t_{j+1}\right), \quad \chi_{i-1/2}^{j+1} = \chi\left(\frac{x_i + x_{i-1}}{2}, t_{j+1}\right), \quad \chi_i^{j+1} = \chi(x_i, t_{j+1}), \quad f_i^{j+1} = f(x_i, t_{j+1}).$$

After such a replacement, the integral heat balance equation turns into a discrete heat balance equation for a unit cell

$$hu_i^{j+1} - hu_i^j = \tau W_{i-1/2}^{j+1} - \tau W_{i+1/2}^{j+1} + \tau h f_i^{j+1}$$

which, after dividing by τh gives the difference equation

$$\frac{u_i^{j+1} - u_i^j}{\tau} = \frac{1}{h}\left[\chi_{i+1/2}^{j+1} \frac{u_{i+1}^{j+1} - u_i^{j+1}}{h} - \chi_{i-1/2}^{j+1} \frac{u_i^{j+1} - u_{i-1}^{j+1}}{h}\right] + f_i^{j+1} \tag{5}$$

The method of obtaining difference equations, based on the approximation of integral conservation laws for elementary cells, is called the integro-interpolation method. Using this method, we have obtained a discrete heat balance equation (5) for a unit cell only. This problem in a complete mathematical setting will be solved by the integro-interpolation method on a uniform grid.

The implicit homogeneous conservative scheme for the problem (2) has the form [18]:

$$\begin{cases} \dfrac{u_i^{j+1} - u_i^j}{\tau} = \left[\chi_{i+1/2}^{j+1} \dfrac{u_{i+1}^{j+1} - u_i^{j+1}}{h^2} - \chi_{i-1/2}^{j+1} \dfrac{u_i^{j+1} - u_{i-1}^{j+1}}{h^2}\right] + f_i^{j+1}, \; i = 1, 2, \ldots, N_1 - 1, \; j = 0, 1, \ldots, N_2 - 1, \\ u_i^0 = u_0(x_i), \quad i = 0, 1, \ldots, N_1, \\ u_0^{j+1} = \mu_1(t_{j+1}), \quad j = 0, 1, \ldots, N_2 - 1, \\ u_{N_1}^{j+1} = \mu_2(t_{j+1}), \quad j = 0, 1, \ldots, N_2 - 1. \end{cases} \tag{6}$$

The operator $Lu = \dfrac{\partial u}{\partial t} - \dfrac{\partial}{\partial x}\left(\chi(x,t)\dfrac{\partial u}{\partial x}\right)$ with the initial and boundary condition will be self-adjoint, positive definite in $L_2(D)$, which means that it has a bounded inverse operator $G = L^{-1}$. It can be used to reformulate the problem (1)-(3) as a problem on the minimum of the functional (1) under the following conditions on the density of sources:

$$f(\cdot,\cdot) \in L_2(D), \; f(x,t) \geq 0, \quad m(x,t) \leq (Gf)(x,t) \leq M(x,t), \tag{7}$$

this can be done, since the matrix corresponding to this operator has the form:

$$G=\begin{bmatrix} \left(\frac{1}{\tau}+\frac{\chi_{1\pm1/2}^{j+1}}{h^2}\right) & -\frac{\chi_{1+1/2}^{j+1}}{h^2} & 0 & \dots & 0 \\ -\frac{\chi_{1-1/2}^{j+1}}{h^2} & \left(\frac{1}{\tau}+\frac{\chi_{1\pm1/2}^{j+1}}{h^2}\right) & -\frac{\chi_{1+1/2}^{j+1}}{h^2} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & -\frac{\chi_{1-1/2}^{j+1}}{h^2} & \left(\frac{1}{\tau}+\frac{\chi_{1\pm1/2}^{j+1}}{h^2}\right) & -\frac{\chi_{1+1/2}^{j+1}}{h^2} \\ 0 & \dots & 0 & -\frac{\chi_{(N_1-1)-1/2}^{j+1}}{h^2} & \left(\frac{1}{\tau}+\frac{\chi_{(N_1-1)\pm1/2}^{j+1}}{h^2}\right) \end{bmatrix}.$$

Let us construct a conservative approximation (1)-(7) as a linear programming problem. Divide the area D by x into N_1 and t into N_2 equal parts: $D=\bigcup_{j=1}^{N_2}\bigcup_{i=1}^{N_1}D_i^j$, where $D_i^j=\{(x,t),\ x_{i-1}\le x\le x_i,\ t_{j-1}\le t\le t_j\}$, $i=\overline{1,N_1}$, $j=\overline{1,N_2}$. Denote by $S_{N_1}^{N_2}(D)$ the subspace of the space $L_2(D)$, in which piecewise constants with functions of the form $f(x,t)=f_i^j$, $(x,t)\in D_i^j$ $(i=1,2,\dots,N_1-1,\ \ j=1,2,\dots,N_2)$. Introduce in $S_{N_1}^{N_2}(D)$ a basis consisting of functions $e_i^j(x,t)=1$, for $(x,t)\in D_i^j$ and $e_i^j(x,t)=0$, for $(x,t)\notin D_i^j$. Then $f(x,t)\approx\sum_{i=1}^{N_1-1}f_i^je_i^j(x,t)$. Let $g_{ik}^j=(Ge_k^j,e_i^j)$, $(m(x,t),e_i^j(x,t))=m_i^j$, $(M(x,t),e_i^j(x,t))=M_i^j$, $i=\overline{1,N_1-1}$, $k=\overline{1,N_1-1}$, $j=\overline{1,N_2}$, where $(\cdot,\cdot)$ – is the dot product in $L_2(D)$. Substitute the expression for $f(x,t)$ in (1) and scalarly multiply the inequalities (7) by $e_i^j(x,t)$ in $L_2(D)$. We get a linear programming problem

$$\begin{gathered} J_j\{f\}=\sum_{i=1}^{N_1-1}(mesD_i^j)f_i^j\to\min,\quad j=1,2,\dots,N_2, \\ m_i^j\le\sum_{k=1}^{N_1-1}g_{ik}^jf_k^j\le M_i^j,\quad i=1,2,\dots,N_1-1,\ j=1,2,\dots,N_2, \\ f_k^j\ge0,\ k=1,2,\dots,N_1-1,\ j=1,2,\dots,N_2. \end{gathered}\tag{8}$$

By solving the problem (8) numerically, the function $u_i^j=\sum_{k=1}^{N_1-1}g_{ik}^jf_k^j$, which is a solution to the boundary value problem (2) with f_k^j. In this case, the problem (8) is solved by the simplex method [20].

3. Description of algorithms and results of numerical experiments

For an approximate solution of the problem (1)-(8) a software application has been developed written in the C# language and does not use third-party mathematical libraries. This software application allows you to accept all the necessary input data: constants, coefficients, mesh parameters, including initial and boundary conditions, temperature functions in the form of scripts in the C# language. Graphic modules have been developed to present the results. The software application uses the reduction of the task to the task (8) with a uniform partition of the area along each coordinate axis. The flowchart (Fig. 1) shows the general algorithm for solving the problem using the numerical method to calculate the function f_i^j.

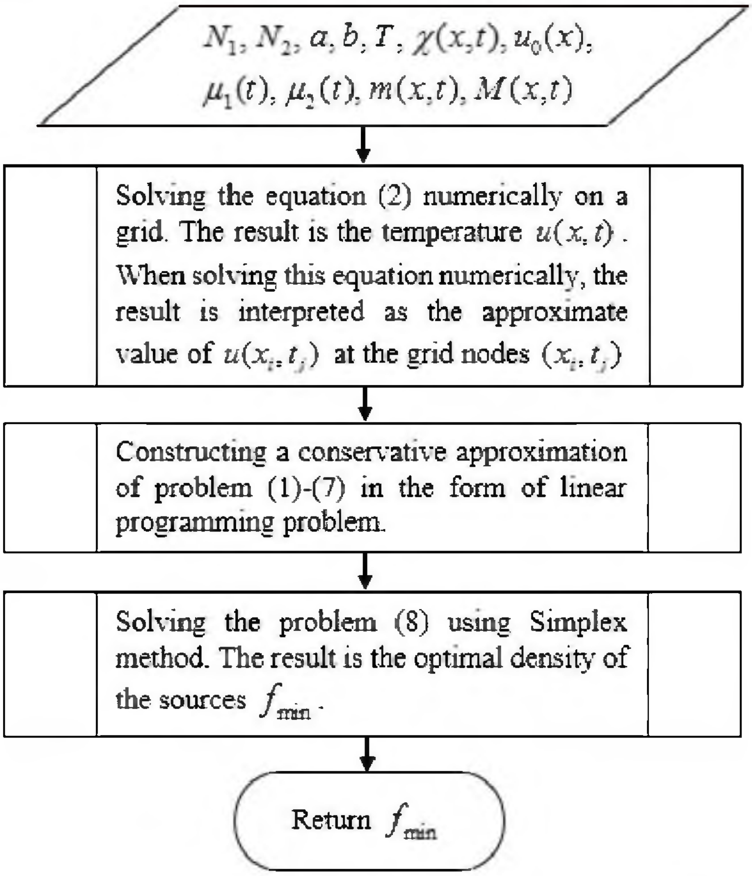

Figure 1. Flowchart of the general algorithm for solving the problem

Example 1. Let us find the optimal distribution density of the sources on the segment. As the computational domain, we take the segment $[0,1]$ with the thermal conductivity function $\chi(x,t)=e^{x-t}$ m^2/s. To determine the initial and boundary conditions, we define the function $u_0(x)=3+x^2$ m/s, $\mu_1(t)=3+t^2$ m/s, $\mu_2(t)=4+t^2$ m/s. The limiting temperature curves are defined by the functions $m(x,t)=1+x^2+t^2$ K, $M(x,t)=5+x^2+t^2$ K, end time $T=1$. Computational grid with the number of sources $N_2\times(N_1-1)=50\times49$. In Fig. 2 shows the result of the numerical solution of the problem (8). The minimum for the numerical solution of the value of the functional is $J_{\min}=3374.39$ K$\cdot$m/s. In Fig. 2 shows the results of the minimum (boundaries with blue), maximum (boundaries with red) and approximate (green) temperatures. To illustrate the effectiveness of the developed method, Fig. 3, the optimal distribution of sources is shown as a surface of different colors.

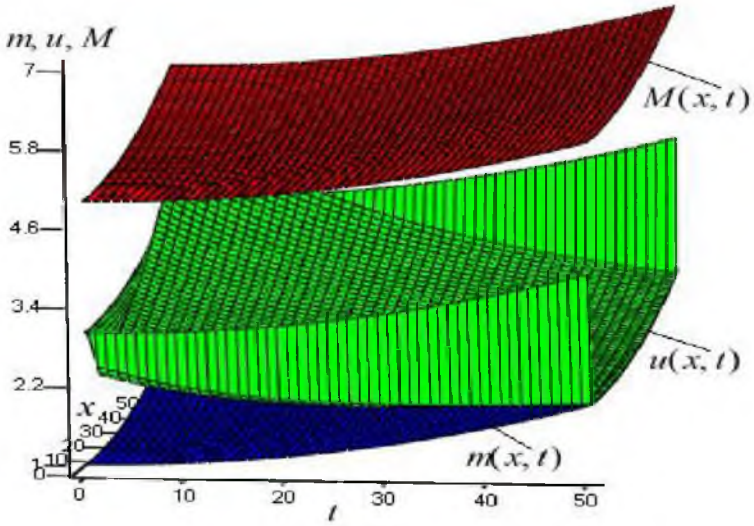

Figure 2. Figure for solving the problem (8)

Example 2. With the same input parameters of the region, we will carry out calculations with the heat conduction function $\chi(x,t)=\begin{cases}1, & 0<x<0.5,\\ 2, & 0.5<x<1\end{cases}$ m^2/s. Computational grid with the number of sources $N_2\times(N_1-1)=50\times49$. In Fig. 4 shows the result of the numerical solution of the problem (8). The minimum for the numerical solution of the value of the functional is $J_{\min}=4279.71$ K·m/s. In Fig. 4 shows the results of minimum (boundaries with blue), maximum (boundaries with red) and approximate (green) temperatures. To illustrate the effectiveness of the developed method, Fig. 5 shows the optimal distribution of sources as a surface of different colors.

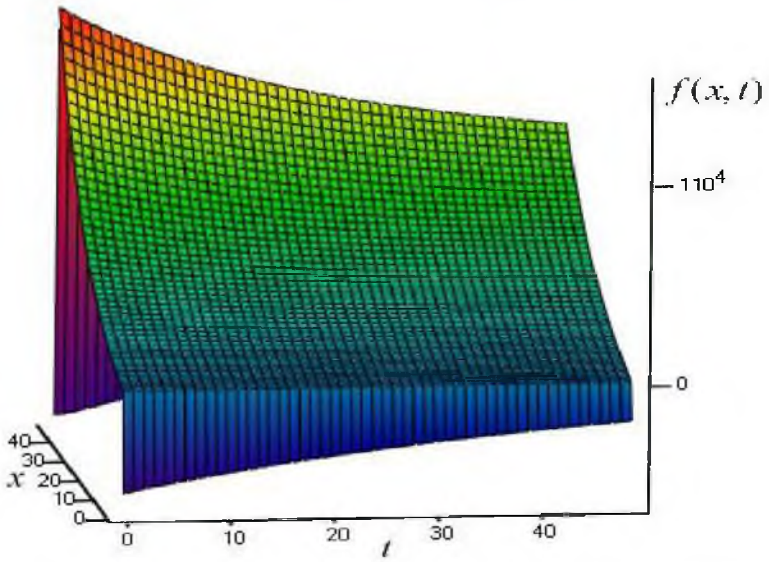

Figure 3. Distribution of the optimal density of heat sources $f(x,t)$

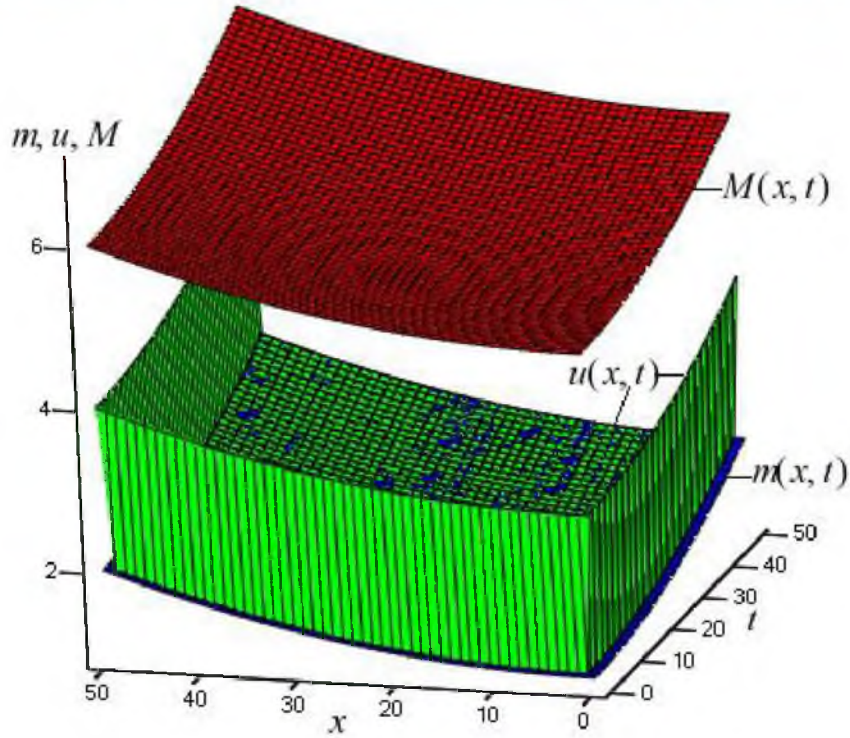

Figure 4. Figure for solving the problem (8)

4. Conclusions

The paper investigates the question of the fundamental possibility of numerically solving the problem of finding the density of heat sources of the minimum total power, which provides a given temperature regime in a region filled with an inhomogeneous stationary medium. Algorithms for the numerical solution of this problem have been developed, based on its reduction to the solution of a conservative finite-dimensional linear programming problem (8). As a result of the computational experiments for a stationary inhomogeneous medium, the effectiveness of the developed algorithms and the entire technique as a whole has been confirmed.

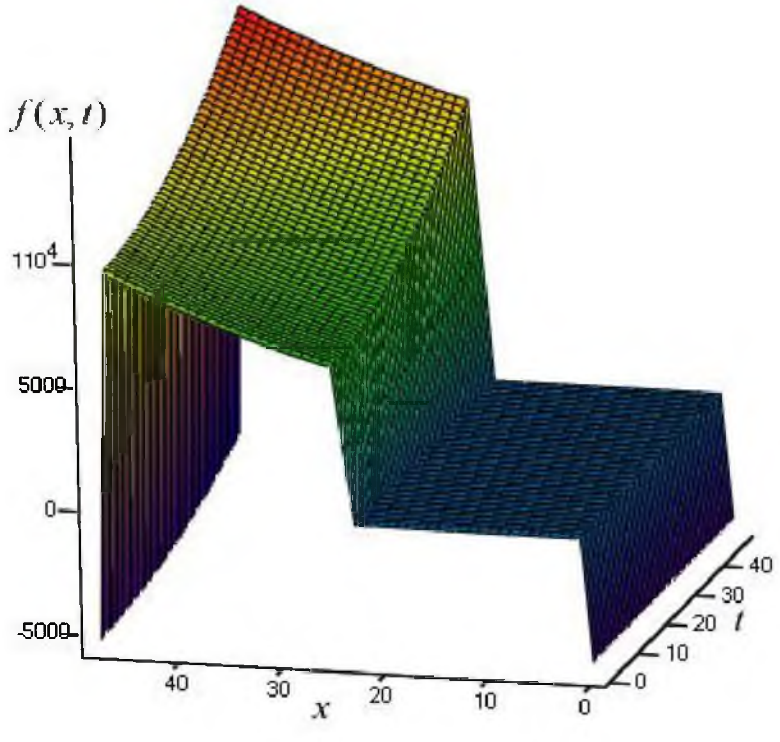

Figure 5. Distribution of the optimal density of heat sources $f(x,t)$

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МУНДАРИЖА