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Erratum

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ЎЗБЕКИСТОН РЕСПУБЛИКАСИ ОЛИЙ ВА ЎРТА МАХСУС ТАЪЛИМ ВАЗИРЛИГИ

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TRAJECTORIES OF QUADRATIC OPERATORS WHICH MAP I_2 TO ITSELF

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Abstract: *In this paper described some quadratic operators which map the $(n-1)$–dimensional simplex of idempotent measures to itself. Such operators are divided to two classes: the first class contains all $n \times n \times n$ - cubic matrices with nonpositive entries which in each $n \times n$ dimensional $k-th$ matrix contains exactly one non-zero row and exactly one non-zero column; the second class contains all $n \times n \times n$ - cubic matrices with non-positive entries which has at least one quadratic zero-matrix. These matrices play a role of the stochastic matrices in the case of idempotent measures. For both classes of quadratic maps we find fixed points and their characters. And also, we find trajectories of quadratic maps which map I_2 to itself.*

Key words: *quadratic operator, simplex, idempotent measure, fixed point, attracting fixed point, repelling fixed point, trajectories.*

ТРАЕКТОРИИ КВАДРАТИЧНЫЕ ОПЕРАТОРЫ КОТОРЫЕ ОТОБРАЖАЕТ I_2 НА СЕБЕ

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Аннотация: *В статье изучено квадратичные операторы которые отображает $(n-1)$–мерное симплекс идемпотентных мер на себе. Такие операторы разделяется на два класса: первый класс содержит все $n \times n \times n$ - кубические матрицы с неотрицательными элементами которые в каждой из $n \times n$ мерное $k-$матрица содержит ровно один не равным нулю строка и ровно один не равным нулю столбца; второй класс содержит все $n \times n \times n$ - кубические матрицы с неотрицательными элементами которые имеют по крайней мере один квадратная нулевая матрица. Эти матрицы играют роль стохастических матриц в случае идемпотентных мер. Для обоих классов квадратичных операторов мы находили неподвижные точки и их характеристика. И также, мы находили траекториям квадратичных операторов которые отображает I_2 на себе.*

Ключевые слова: *Квадратичный оператор, симплекс, идемпотентная мера, неподвижная точка, привлекающая неподвижная точка, отталкивающая неподвижная точка, траектория.*

I_2 NI O'ZINI O'ZIGA AKSLANTIRUVCHI KVADRATIK OPERATORLARNING TRAEKTORIYALARI

Jo'rayev Ilxomjon Tursunmuxammedovich
Namangan davlat universiteti

Annotatsiya: *Ushbu maqolada $(n-1)$–o'lchovli idempotent ehtimolliklar simpleksini o'zini o'ziga akslantiruvchi kvadratik operatorlar o'rganilgan. Bunday operatorlar ikki sinfga bo'linadi: birinchi sinf nomusbat qiymatlarni qabul qiluvchi va har bir $n \times n$ o'lchovli $k-$matritsa*

aynan bitta nol bo'lmagan ustun va aynan bitta nol bo'lmagan satrdan iborat barcha kubik matritsalar; ikkinchi sinf esa, kamida bitta $n \times n$ o'lchovli matritsasi nol matritsa bo'lgan, nomusbat elementli $n \times n \times n$ - o'lchovli barcha kubik matritsalardir. Ushbu matritsalar idempotent ehtimolliklar o'lchovi uchun stoxastik matritsa rolini o'ynaydi. Mazkur ishda har ikkala sinf uchun ham qo'zg'almas nuqtalar va ularning xarakterlari topilgan. Shuningdek, I_2 ni o'zini o'ziga akslantiruvchi kvadratik operatorlarning traektoriyalari ham topilgan.

Kalit so'zlar: *kvadratik operator, simplex, idempotent o'lchov, qo'zg'almas nuqta, tortuvchi qo'zg'almas nuqta, itaruvchi qo'zg'almas nuqta, traektoriyalar.*

1. Introduction It is known that the set of all probability measures on the set $E = \{1, 2, \dots, m\}$ is $(m-1)$-dimensional simplex of the form

$$S^{m-1} = \left\{ x = (x_1, x_2, \dots, x_m) \in R^m : x_i \geq 0, \sum_{i=1}^{m} x_i = 1 \right\}$$

Any element of S^{m-1} can be considered as a state of a system with m elements. Then one of well-known dynamics of the system can be given by iterations of a stochastic matrix Φ, i.e. it describes a Markov chain [1]. Moreover if on S^{m-1} one considers a quadratic stochastic operator V (see [2]) then the dynamical system

Describes evolution of a population.

In this paper we consider quadratic operators which map a finite-dimensional simplex of idempotent measures to itself. In idempotent mathematics the usual arithmetic operations are replaced with a new set of basic associative operations (a new addition $\oplus$ and a new multiplication $\otimes$) so that all the semifield or semiring axioms hold; moreover, the new addition is idempotent, i.e., $x \oplus x = x$ for every element x of the corresponding semiring, see, e.g., [3–7].

A typical example is the semifield $R_{\max} = R \cup \{-\infty\}$ known as the Max-Plus algebra. This semifield consists of all real numbers and an additional element $\mathbf{0} = -\infty$. This element $\mathbf{0}$ is the zero element in $R_{\max}$, and the basic operations are defined by the formulas $x \oplus y = \max\{x, y\}$ and $x \otimes x = x + y$; the identity (or unit) element $\mathbf{1}$ coincides with the usual zero 0.

In this paper, we consider the simplex I_n of idempotent measures on $\{1, 2, \dots, \mathrm{n}\}$, where

$$I_n = \{(x_1, x_2, \dots, x_n) \in R^n_{\max} : \max_{1 \leq i \leq n} x_i = 0\} =$$

$$= \{(x_1, x_2, \dots, x_n) \in R^n_{\max} : x_1 \oplus x_2 \oplus \dots \oplus x_n = 1\}$$

and consider quadratic maps of I_n to itself. The term idempotent measure in our case is used in the sense of idempotent analysis ([3, 6, 7]) rather than in the sense of abstract harmonic analysis as in [1].

In [8] linear maps of the set of idempotent measures are constructed and their dynamical systems are studied. The quadratic operators defined on the set of idempotent measures which we shall consider in this paper are "idempotent" analogies of quadratic stochastic operators of [2].

The paper is organized as follows. In Section 2 we describe all quadratic operators which map the $(n-1)$-dimensional simplex of idempotent measures to itself. Such operators are divided to two classes. Section 3 contains a description of fixed points for both classes of quadratic maps. In Section 4 we will see the trajectories of quadratic maps of I_2

2. Quadratic maps of I_n to itself

It is known (see, e.g., [1]) that the properties of homogenous Markov chains with the space of states $E=\{1,2,\dots,m\}$ can by completely determined by the initial distribution $x\in S^{m-1}$ and a stochastic matrix P, i.e., P maps S^{m-1} to itself if P is a stochastic matrix. The theory of dynamical systems (Markov chains) generated by P is well known (see for example [1]).

In this section we shall answer the question: which quadratic map maps I_n to itself? The following theorem describes such quadratic maps.

For a cubic matrix $P=\left(p_{ij,k}\right)_{i,j,k=1}^{n}$, a fixed $k\in\{1,2,\dots,\mathrm{n}\}$, the matrix $\left(p_{ij,k}\right)_{i,j=1}^{n}$ is called the k-th matrix. We will find the result with formula:

$$y_k=\sum_{i,j=1}^{n}p_{ij,k}x_ix_j \qquad (1)$$

Theorem1. [9] A quadratic operator $P=\left(p_{ij,k}\right)_{i,j,k=1}^{n}$ with $p_{ij,k}\le 0$ maps I_n to itself, if and only if it satisfies one of following conditions:

i) Each k-th matrix contains exactly one non-zero row and exactly one non-zero column: In each k-th matrix all of elements except i_k-th row and i_k-th column are zeroes. Where $i_j\ne i_m$ if $j\ne m$ ($i=\overline{1,n}$, $j=\overline{1,n}$, $m=\overline{1,n}$ $k=\overline{1,n}$);

ii) P has at least one zero matrix ($\exists m\le n$ such that all of elements of m-th $P_m=\left(p_{ij,m}\right)_{i,j=1}^{n}$ matrix consists of only zeroes).

3. Fixed points of P (cases $n=2$ and $n=3$)

Recall that fixed points of $P:I_n\to I_n$ are solutions to $P(x)=x$. Denote by $Fix(P)$ the set of all fixed point of P. Let's find fixed points of quadratic operator $P:I_2\to I_2$.

Assume, P is given as a cubic matrix: $P=\left(p_{ij,k}\right)_{i,j,k=1}^{2}$. Then by the formula (1) the equation $P(x)=x$ has the form

$$\begin{cases} x_1=p_{11,1}x_1^2+\left(p_{12,1}+p_{21,1}\right)x_1x_2+p_{22,1}x_2^2\\ x_2=p_{11,2}x_1^2+\left(p_{12,2}+p_{21,2}\right)x_1x_2+p_{22,2}x_2^2\end{cases} \qquad (2)$$

Let P satisfies condition (i) of theorem 1 and $i_k=k$ e.g., $p_{22,1}=0$, $p_{11,2}=0$ and then by (2) we'll find following system:

$$\begin{cases} x_1=p_{11,1}x_1^2+\left(p_{12,1}+p_{21,1}\right)x_1x_2\\ x_2=p_{22,2}x_2^2+\left(p_{12,2}+p_{21,2}\right)x_1x_2\end{cases}$$

Following three cases are possible:

1a) Let $x_1 < 0$, $x_2 = 0$ then we get $x_1 = p_{11,1}x_1^2$. The solution is $x_1 = \frac{1}{p_{11,1}}$, $x_2 = 0$ $(p_{11,1} \neq 0)$, hence the fixed point is $\left(\frac{1}{p_{11,1}}, 0\right)$.

1b) Assume $x_1 = 0$, $x_2 < 0$ then we get $x_2 = p_{22,2}x_2^2$. The solution is $x_1 = 0$, $x_1 = \frac{1}{p_{22,2}}$ $(p_{22,2} \neq 0)$, hence the fixed point is $\left(0, \frac{1}{p_{22,2}}\right)$

1c) Let $x_1 = x_2 = 0$ then we get fixed point $(0,0)$.

Now, assume P satisfies condition (i) of theorem 1 and $i_k \neq k$, e.g., $p_{11,1} = 0$ and $p_{22,2} = 0$ then by (2) we'll find the following system

$$\begin{cases} x_1 = p_{22,1}x_2^2 + (p_{12,1} + p_{21,1})x_1x_2 \\ x_2 = p_{11,2}x_1^2 + (p_{12,2} + p_{21,2})x_1x_2 \end{cases}$$

Three cases are possible:

2a) Let $x_1 < 0$, $x_2 = 0$. Then from equation $x_1 = p_{22,1}x_2^2 + (p_{12,1} + p_{21,1})x_1x_2$ we get $x_1 = 0$. Then solution is $x_1 = x_2 = 0$ and fixed point is $(0,0)$.

2b) Let $x_1 = 0$, $x_2 < 0$. Then from equation $x_2 = p_{11,2}x_1^2 + (p_{12,2} + p_{21,2})x_1x_2$ we'll find $x_2 = 0$. Then solution is $x_1 = x_2 = 0$ and fixed point is $(0,0)$.

2c) Assume $x_1 = x_2 = 0$. Then fixed point is $(0,0)$.

Consequently, it is not difficult to see that if P satisfies the condition (ii) of the theorem 1, then exists only one fixed point as: $(0,0)$. [see 9].

Now, let's find fixed points of quadratic operator $P: I_3 \to I_3$. First of all we denote

$$\begin{aligned} \Delta_{ij} &= p_{ii,i} \cdot p_{jj,j} - (p_{ij,i} + p_{ji,i})(p_{ij,j} + p_{ji,j}) \\ \alpha_{ij} &= \frac{p_{jj,j} - (p_{ij,i} + p_{ji,i})}{\Delta_{ij}} \\ \beta_{ij} &= \frac{p_{ii,i} - (p_{ij,j} + p_{ji,j})}{\Delta_{ij}} \end{aligned} \tag{3}$$

We will get following preposition after some elementary analysis:

Proposition 2. **[9]** If $n = 3$ and condition (i) of the theorem 1 is satisfied, then

$$Fix(P)=\begin{cases}\left(\frac{1}{p_{11,1}},\ 0,\ 0\right) \text{ if } i_1=1 \text{ and } p_{11,1}\neq 0;\\ \left(0,\ \frac{1}{p_{22,2}},\ 0\right) \text{ if } i_2=2 \text{ and } p_{22,2}\neq 0;\\ \left(0,\ 0,\ \frac{1}{p_{33,3}}\right) \text{ if } i_3=3 \text{ and } p_{33,3}\neq 0;\\ (\alpha_{12},\ \beta_{12},\ 0) \text{ if } \Delta_{12}\neq 0,\ i_k=k,\ p_{11,1}\neq 0 \text{ and } p_{22,2}\neq 0;\\ (\alpha_{13},\ 0,\ \beta_{13}) \text{ if } \Delta_{13}\neq 0,\ i_k=k,\ p_{11,1}\neq 0 \text{ and } p_{33,3}\neq 0;\\ (0,\ \alpha_{23},\ \beta_{23}) \text{ if } \Delta_{23}\neq 0,\ i_k=k,\ p_{22,2}\neq 0 \text{ and } p_{33,3}\neq 0;\\ (0,\ 0,\ 0) \text{ if } i_k\neq k;\\ (0,\ 0,\ 0) \text{ for all other cases.}\end{cases} \qquad (4)$$

4. Trajectories of quadratic maps of I_2 to itself

When we map the non-fixed points by using quadratic maps they will change their coordinates. In this paper our aim is to find the trajectory of non-fixed points if we use quadratic maps to them so many times. In other words, we should find the solution of the following limit:

$$\lim_{n\to\infty} P^n(x)=\lim_{n\to\infty} P(P(P(\ldots(P(x))\ldots))) \quad (5)$$

For getting the trajectories of quadratic maps of I_2 to itself let's use the results 1a (fixed point is $\left(\frac{1}{p_{11,1}},0\right)$) and 1b (fixed point is $\left(0,\frac{1}{p_{22,2}}\right)$) in Section 3.

In case 1a we will check the points which are situated by the two sides of fixed point $\left(\frac{1}{p_{11,1}},0\right)$. These points we can denote as $\left(\frac{k}{p_{11,1}},0\right)$. In this case, if $k>1$ we will get point which situated between points $(-\infty,0)$ and $\left(\frac{1}{p_{11,1}},0\right)$; if $0<k<1$ we will get point which situated between points $\left(\frac{1}{p_{11,1}},0\right)$ and $(0,0)$. Then we solve the expression (5) step by step:

Case 1a. Let $x_1<0$, $x_2=0$ then for $\left(\frac{k}{p_{11,1}},0\right)$ and $n=1$ we will get following:

$$P(x)=P\begin{pmatrix}\frac{k}{p_{11,1}}\\0\end{pmatrix}=\left(\begin{array}{cc|cc}p_{11,1} & p_{12,1} & 0 & p_{12,2}\\ p_{21,1} & 0 & p_{21,2} & p_{22,2}\end{array}\right)\cdot\begin{pmatrix}\frac{k}{p_{11,1}}\\0\end{pmatrix}=\begin{pmatrix}x_1^{(1)}\\x_2^{(1)}\end{pmatrix}$$

$$\begin{cases} x_1^{(1)} = p_{11,1}\cdot\left(\dfrac{k}{p_{11,1}}\right)^2 + \left(p_{12,1}+p_{21,1}\right)\cdot\dfrac{k}{p_{11,1}}\cdot 0 = \dfrac{k^2}{p_{11,1}} \\ x_2^{(1)} = p_{22,2}\cdot 0^2 + \left(p_{12,2}+p_{21,2}\right)\cdot\dfrac{k}{p_{11,1}}\cdot 0 = 0 \end{cases}$$

$$P(x)=\begin{pmatrix} \dfrac{k^2}{p_{11,1}} \\ 0 \end{pmatrix}$$

After this we should find the solution of expression (5) for case $n=2$.

$$P^2(x)=P(P(x))=P\begin{pmatrix} \dfrac{k^2}{p_{11,1}} \\ 0 \end{pmatrix}=\left(\begin{array}{cc|cc} p_{11,1} & p_{12,1} & 0 & p_{12,2} \\ p_{21,1} & 0 & p_{21,2} & p_{22,2} \end{array}\right)\cdot\begin{pmatrix} \dfrac{k^2}{p_{11,1}} \\ 0 \end{pmatrix}=\begin{pmatrix} x_1^{(2)} \\ x_2^{(2)} \end{pmatrix}$$

$$\begin{cases} x_1^{(2)} = p_{11,1}\cdot\left(\dfrac{k^2}{p_{11,1}}\right)^2 + \left(p_{12,1}+p_{21,1}\right)\cdot\dfrac{k^2}{p_{11,1}}\cdot 0 = \dfrac{k^4}{p_{11,1}} \\ x_2^{(2)} = p_{22,2}\cdot 0^2 + \left(p_{12,2}+p_{21,2}\right)\cdot\dfrac{k^2}{p_{11,1}}\cdot 0 = 0 \end{cases}$$

$$P^2(x)=P(P(x))=\begin{pmatrix} \dfrac{k^{2^2}}{p_{11,1}} \\ 0 \end{pmatrix}$$

Then, we will find the solution of expression (5) for case $n=3$

$$P^3(x)=P(P(P(x)))=P\begin{pmatrix} \dfrac{k^4}{p_{11,1}} \\ 0 \end{pmatrix}=\left(\begin{array}{cc|cc} p_{11,1} & p_{12,1} & 0 & p_{12,2} \\ p_{21,1} & 0 & p_{21,2} & p_{22,2} \end{array}\right)\cdot\begin{pmatrix} \dfrac{k^4}{p_{11,1}} \\ 0 \end{pmatrix}=\begin{pmatrix} x_1^{(3)} \\ x_2^{(3)} \end{pmatrix}$$

$$\begin{cases} x_1^{(3)} = p_{11,1}\cdot\left(\dfrac{k^4}{p_{11,1}}\right)^2 + \left(p_{12,1}+p_{21,1}\right)\cdot\dfrac{k^4}{p_{11,1}}\cdot 0 = \dfrac{k^8}{p_{11,1}} \\ x_2^{(3)} = p_{22,2}\cdot 0^2 + \left(p_{12,2}+p_{21,2}\right)\cdot\dfrac{k^4}{p_{11,1}}\cdot 0 = 0 \end{cases}$$

$$P^3(x)=P(P(P(x)))=\begin{pmatrix} \dfrac{k^{2^3}}{p_{11,1}} \\ 0 \end{pmatrix}$$

Continuously doing these operations we will get the solution of expression (5) for all natural values of n as:

$$\lim_{n\to\infty} P^n(x)=\lim_{n\to\infty} P(P(P(\ldots(P(x))\ldots)))=\left(\dfrac{k^{2^n}}{p_{11,1}}, 0\right) \qquad (6)$$

From (6) we get conclusion, for $k>1$, i.e. if the point is on the left from the fixed point in case 1a it will be move to the fixed point $(-\infty, 0)$; for $k<1$, i.e. if the point is on the right from the fixed point in case 1a it will be move to the fixed point $(0,0)$.

There are 3 cases possible:

1) for $k>1$ we will get the point as: $\lim\limits_{n\to\infty} P^n(x) = \lim\limits_{n\to\infty}\left(\frac{k^{2^n}}{p_{11,1}}, 0\right) = (-\infty, 0)$;

2) for $k<1$ we will get the point as: $\lim\limits_{n\to\infty} P^n(x) = \lim\limits_{n\to\infty}\left(\frac{k^{2^n}}{p_{11,1}}, 0\right) = (0, 0)$;

3) for $k=1$ we will get the point as: $\lim\limits_{n\to\infty} P^n(x) = \lim\limits_{n\to\infty}\left(\frac{k^{2^n}}{p_{11,1}}, 0\right) = \left(\frac{1}{p_{11,1}}, 0\right)$.

Case 1b. Let $x_1 = 0$, $x_2 < 0$ then for $\left(0, \frac{k}{p_{22,2}}\right)$ and $n=1$ we will get following:

$$P(x) = P\begin{pmatrix} 0 \\ \frac{k}{p_{22,2}} \end{pmatrix} = \left(\begin{array}{cc|cc} p_{11,1} & p_{12,1} & 0 & p_{12,2} \\ p_{21,1} & 0 & p_{21,2} & p_{22,2} \end{array}\right) \cdot \begin{pmatrix} 0 \\ \frac{k}{p_{22,2}} \end{pmatrix} = \begin{pmatrix} x_1^{(1)} \\ x_2^{(1)} \end{pmatrix}$$

$$\begin{cases} x_1^{(1)} = p_{11,1} \cdot 0^2 + (p_{12,1} + p_{21,1}) \cdot 0 \cdot \frac{k}{p_{22,2}} = 0 \\ x_2^{(1)} = p_{22,2} \cdot \left(\frac{k}{p_{22,2}}\right)^2 + (p_{12,2} + p_{21,2}) \cdot \frac{k}{p_{22,2}} \cdot 0 = \frac{k^2}{p_{22,2}} \end{cases}$$

$$P(x) = \begin{pmatrix} 0 \\ \frac{k^2}{p_{22,2}} \end{pmatrix}$$

As analogue, for cases $n=2$ and $n=3$ we will get:

$$P^2(x) = P(P(x)) = \begin{pmatrix} 0 \\ \frac{k^{2^2}}{p_{22,2}} \end{pmatrix}$$

$$P^3(x) = P(P(P(x))) = \begin{pmatrix} 0 \\ \frac{k^{2^3}}{p_{22,2}} \end{pmatrix}$$

For for all natural values of n expression (5) will be defined as:

$$\lim_{n\to\infty} P^n(x) = \lim_{n\to\infty} P(P(P(\ldots(P(x))\ldots))) = \left(0, \frac{k^{2^n}}{p_{22,2}}\right) \qquad (7)$$

From (7) we do conclusion, for $k>1$, i.e. if the point is at the bottom of the fixed point in case 1b it will be move to the fixed point $(0, -\infty)$; for $k<1$, i.e. if the point is at the top of the fixed point in case 1b it will be move to the fixed point $(0, 0)$.

There are 3 cases possible:

1) for $k>1$ we will get the point as: $\lim\limits_{n\to\infty} P^n(x) = \lim\limits_{n\to\infty}\left(0, \frac{k^{2^n}}{p_{22,2}}\right) = (0, -\infty)$;

2) for $k<1$ we will get the point as: $\lim_{n\to\infty} P^n(x) = \lim_{n\to\infty}\left(0, \frac{k^{2^n}}{p_{22,2}}\right) = (0,0)$;

3) for $k=1$ we will get the point as: $\lim_{n\to\infty} P^n(x) = \lim_{n\to\infty}\left(0, \frac{k^{2^n}}{p_{22,2}}\right) = \left(0, \frac{1}{p_{22,2}}\right)$.

In other cases (1c, 2a, 2b, 2c) if we map the quadratic operator one time all of the point in I_2 become fixed point as $(0,0)$. That's why, we don't need to research them.

5. Conclusion.

There are two markedly different types of fixed points, attracting and repelling fixed points. In figure 1 we can see, any $x=(x_1,0)$ which coordinates between points $\left(\frac{1}{p_{11,1}},0\right)$ and $(0,0)$, no matter how close to $\left(\frac{1}{p_{11,1}},0\right)$ lead to an orbit that tends "far" from $\left(\frac{1}{p_{11,1}},0\right)$ and close $(0,0)$; all point as $x=(0,x_2)$ which coordinates between points $\left(0,\frac{1}{p_{22,2}}\right)$ and $(0,0)$, lead to an orbit that tends "far" from $\left(0,\frac{1}{p_{22,2}}\right)$ and close $(0,0)$. Other point which coordinates between $\left(\frac{1}{p_{11,1}},0\right)$ and $(-\infty,0)$ or $\left(0,\frac{1}{p_{22,2}}\right)$ and $(0,-\infty)$ are lead to an orbit that tends "far" from $\left(\frac{1}{p_{11,1}},0\right)$ or $\left(0,\frac{1}{p_{22,2}}\right)$. So, fixed points $\left(\frac{1}{p_{11,1}},0\right)$ and $\left(0,\frac{1}{p_{22,2}}\right)$ are repelling fixed points. Finally, fixed points $(0,0)$, $(-\infty,0)$ and $(0,-\infty)$ are attracting fixed points. (Figure 1)

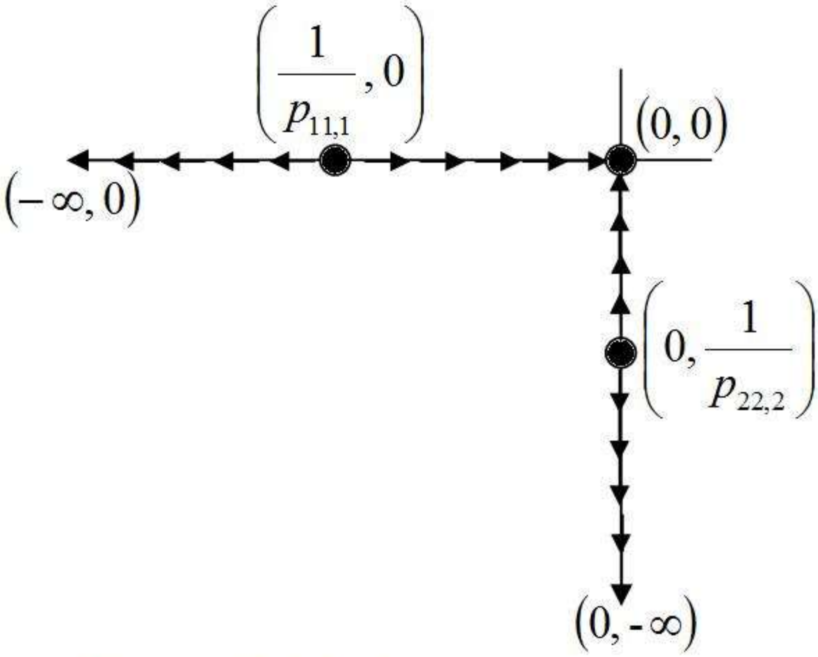

Figure 1. Trajectories of quadratic maps

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МУНДАРИЖА

01.00.00 ФИЗИКА-МАТЕМАТИКА ФАНЛАРИ
ФИЗИКО-МАТЕМАТИЧЕСКИЕ НАУКИ
PHYSICAL AND MATHEMATICAL SCIENCES

02.00.00 КИМЁ ФАНЛАРИ
ХИМИЧЕСКИЕ НАУКИ
CHEMICAL SCIENCES

03.00.00 БИОЛОГИЯ ФАНЛАРИ
БИОЛОГИЧЕСКИЕ НАУКИ
BIOLOGICAL SCIENCES

05.00.00 ТEXNIKA ФАНЛАРИ / ТЕХНИИЧЕСКИЕ НАУКИ / TECHNICAL SCIENCES

07.00.00 ТАРИХ ФАНЛАРИ / ИСТОРИЧЕСКИЕ НАУКИ / HISTORICAL SCIENCES

09.00.00 ФАЛСАФА ФАНЛАРИ / ФИЛОСОФКИЕ НАУКИ / PHILOSOPHICAL SCIENCES

10.00.00 ФИЛОЛОГИЯ ФАНЛАРИ / ФИЛОЛОГИЧЕСКИЕ НАУКИ / PHILOLOGICAL SCIENCES

13.00.00 ПЕДАГОГИКА ФАНЛАРИ
ПЕДАГОГИЧЕСКИЕ НАУКИ
PEDAGOGICAL SCIENCES