TOPOLOGICAL STATES IN RESONANT PHOTONIC CRYSTALS

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1. Introduction. Topology in geometry and band theory of solids

Topology is a branch of mathematics concerned with geometrical properties of objects that are insensitive to smooth deformations. For example, a sphere can be continuously deformed into many different shapes, such as the surface of a disk or a pialat, Fig. 1. Another example of geometrical topological equivalence is a torus (doughnut) which can be transformed into a cup.
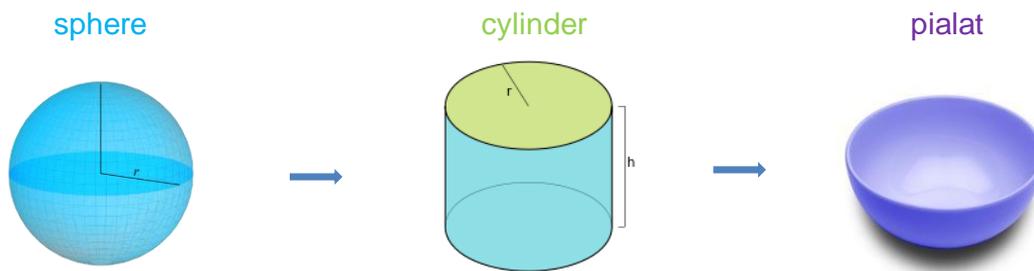

Fig.1. Three topologically equivalent surfaces.

Thus, the topological equivalence in geometry means a possibility to transform one surface into another by a smooth deformation. An insulator is a material that has an energy gap for electronic excitations which separates the ground state from all excited states. This allows for a notion of topological equivalence based on the principle of adiabatic continuity of the Hamiltonians: Insulators are equivalent if they can be transformed into one another by a smooth transformation of one Hamiltonian into the other without the gap closure [1-3].

In geometry, there exists a topological invariant, the same for all equivalent surfaces. It is the Euler characteristic

$$\chi = \frac{1}{2\pi} \int_S K \, dS \, ,$$

where K is the Gaussian curvature, S is the surface and dS is the surface area element. For a sphere, one has $\chi = 2$ and, for a torus, $\chi = 0$. In the topological band theory there also exist topological invariants. Thus, in a particular case the role of the Gaussian curvature is played by the Berry curvature and the role of the Euler characteristic is played by the Chern number. The Berry curvature $F(\boldsymbol{k})$ is expressed via the Berry connection $\boldsymbol{A}(\boldsymbol{k})$ as

$$F(\boldsymbol{k}) = \frac{\partial A_y(\boldsymbol{k})}{\partial k_x} - \frac{\partial A_x(\boldsymbol{k})}{\partial k_y} = [\boldsymbol{\nabla}_{\boldsymbol{k}} \times \boldsymbol{A}(\boldsymbol{k})]_z \, .$$

In its turn the Berry connection is expressed in terms of the electron Bloch function $\psi_k(\boldsymbol{r}) = \mathrm{e}^{\mathrm{i}kr} u_k(\boldsymbol{r})$ by

$$\boldsymbol{A}(\boldsymbol{k}) = -\mathrm{i} \int u_{\boldsymbol{k}}^*(\boldsymbol{r}) \frac{\partial u_{\boldsymbol{k}}(\boldsymbol{r})}{\partial \boldsymbol{k}} d\boldsymbol{r} \, .$$

For the 2D system, the Chern number is defined by

$$\mathcal{C} = \frac{1}{2\pi} \int_{\mathrm{BZ}} F(\boldsymbol{k}) d^2\boldsymbol{k} \, .$$

Here the integration is performed over the whole 2D Brillouin zone. The Chern number of the free space equals zero.

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Let us consider a spatial interface between two topologically distinct insulators as shown in Fig. 2. If the sums of the Chern numbers C_1+C_2 and $C_{1'}+C_{2'}$ are different, then somewhere along the
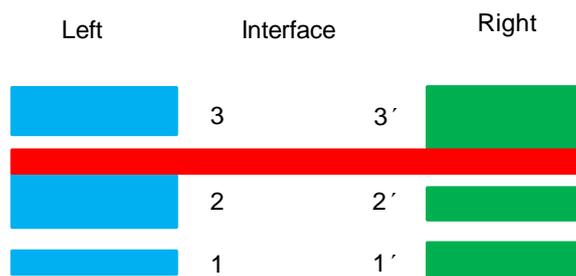
Fig. 2. The schematic presentation of the interface between two materials, left- and right-hand-side. The shaded rectangles show the allowed bands, and the white stripes between them show the band gaps. The long stripe represents the intersection of two gaps between the allowed bands 2 and 3 of the left material and the allowed bands 2′ and 3′ of the right material.

normal to the interface the energy gap has to go to zero, because otherwise the two phases would be equivalent. The gapless states are edge states, see Fig. 3, that are topologically protected and exhibit conducting properties.
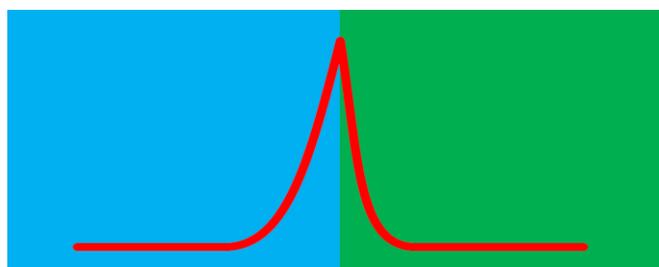
Fig. 3. The schematic illustration of an edge state on the boundary between two topologically nonequivalent insulators.

It follows then that, in addition to metals and conventional insulators (and semiconductors), there is a new class of solids, topological insulators, which have electron band gaps in their interior like ordinary insulators but possess conducting states on their edges or surfaces, Fig. 4.
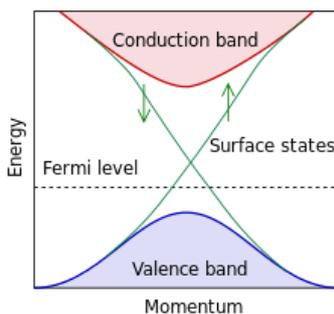
Fig. 4. Idealized band structure of a topological insulator.

The photonic analogues of the electronic topological insulators are defined in the same way [4,5] although the photons are bosons and carry no electric charge. This talk concerns the studies of the photonic topological systems.

2. Two-dimensional lattice of ring resonators

Figure 5 borrowed from Ref. [5] illustrates the system designed to model the photonic topolo-gical insulator.

The Hamiltonian describing the nearest-neighbor coupling between the site resonators is given by

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$$\mathcal{H} = -J \sum_{n,m} \left(\hat{a}^{\dagger}_{n,m} \hat{a}_{n-1,m} + \hat{a}^{\dagger}_{n,m} \hat{a}_{n,m-1} e^{2\pi i b n} \right) + \text{h.c.}$$
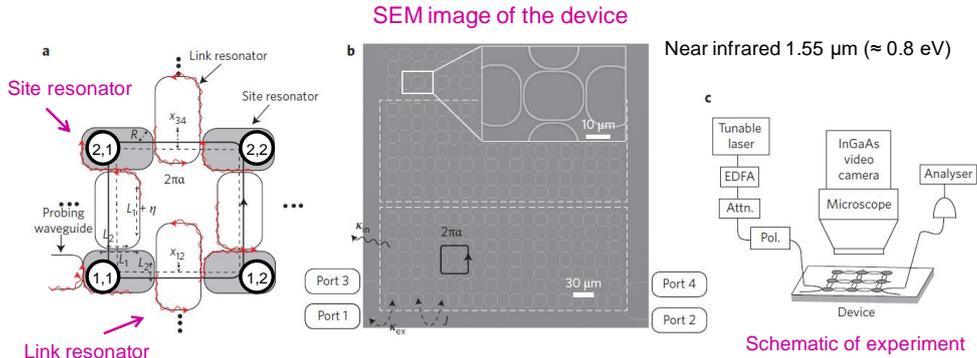

Fig. 5. (a) Four site ring resonators coupled by the link ring resonators. (b) Scanning electron microscope image of the system of 10×10 array of site ring resonators. (c) Schematic of the experimental set-up. Each site is characterized by two integers (n,m). [5]

where b is dependent on the positioning of the link resonators. For $b=1/3$, the system is periodic with the period 1 along the vertical direction y and the period 3 along the horizontal direction x. The band structure and the Chern numbers of the allowed bands are shown in Fig. 6. The edge states have been observed experimentally [5].
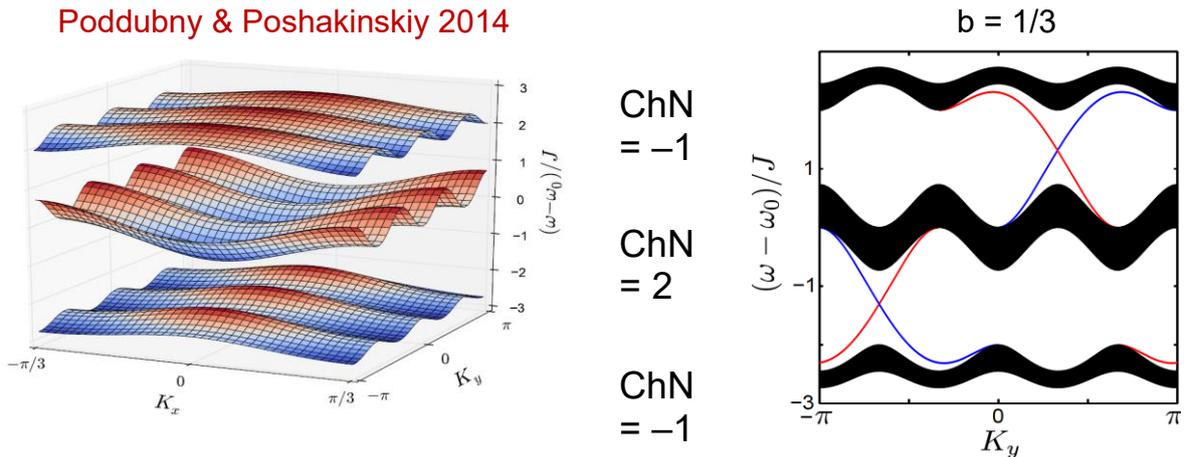

Fig. 6. (Left panel) The dispersion of the modes in the photonic topological insulator [5] for $b=1/3$. ω_0 is the resonance frequency of the ring resonator, J is the modulus of the coupling parameter, K_x and K_y are the components of the 2D wave vector. The Brillouin zone is defined by the conditions $-\pi/3 \leq K_x \leq \pi/3$, $-\pi \leq K_y \leq \pi$. (Right panel) The allowed and forbidden bands. The curves show the dispersion of the edge states. The symbol ChN means the Chern number of the corresponding allowed band. [6]

3. Radiative topological states in 1D resonant photonic crystals

In Ref. [7] we have shown how the ideas of 2D topological systems can be used to analyze the edge states in one-dimensional (1D) structures. For definiteness, we consider a multi-quantum- well structure with identical wells centered according to the bichromatic sequence

$$z_n = d\left[n + \eta \cos\left(\frac{2\pi n}{3} + \varkappa + \frac{\pi}{6} \right) \right],$$

where n is an integer, η is the dimensionless parameter smaller than unity, the phase $\varkappa$ is defined between $-\pi$ and π, the additional phase $\pi/6$ is introduced for convenience.

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The structure is shown schematically in Fig. 7, it is periodic with the period $D = 3d$. Let us replace the index n by the double index (s,l), where $s=0, \pm1, \pm2…, l = 1, 2, 3$. In the infinite structure the normal modes are
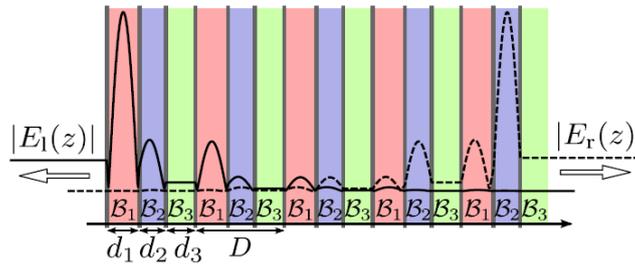

Fig. 7. Illustration of the periodic structure with three quantum wells in the unit cell. Vertical lines indicate the wells, the labels B_1, B_2 and B_3 mark the barriers of different thicknesses. The solid line shows the electric field distribution for the state localized on the left edge, the dashed line corresponds to the right-edge state. [7]

described by the Bloch waves $P(z_{sl}) \equiv P_l(s) = e^{iksD}P_l$, where the wave vector k lies between $-\pi/D$ and π/D. Here $P(z_{sl})$ is the exciton polarization in the quantum well (s,l), the three values P_l satisfy the equations

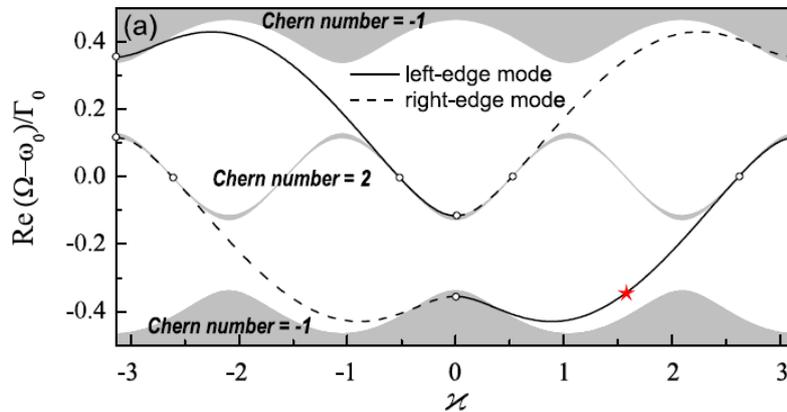$$\hbar(\omega - \omega_0)P_l(k, \varkappa) = \sum_{l'=1,2,3} \mathcal{H}_{ll'}(k, \varkappa)P_{l'}(k, \varkappa).$$

Fig. 8. The band structure as a function of the "ancestor" lattice wave vector $\varkappa$ characterizing the distribution of three quantum wells in the unit cell. The gray regions are the allowed exciton-polariton bands, while the white regions are the stop-bands. The lines show the dependence of the polariton frequency of the left-edge (solid) and the right-edge (dashed) mode. Circles indicate points where the edge modes are absent. The star shows the value of $\varkappa$ in the analysis made in Ref. [7].

We explicitly take into account the dependence of the modes on $\varkappa$ and consider this parameter as the second component of the effective 2D wave vector $(k, \varkappa)$. Then the Hamiltonian $H_{ll'}(k, \varkappa)$ effectively becomes that of the "ancestor" 2D lattice and one can analyze the topological properties of such the lattice. Figure shows the dependence of frequency of the edge modes on the parameter $\varkappa$. The possible methods of experimental observation of the calculated edge modes is discussed in Ref. [7].

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