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## THE SETS OF JULIA AND MANDELBROT FOR MULTI-DIMENSIONAL CASE OF LOGISTIC MAPPING

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**The sets of Julia and Mandelbrot for multi-dimensional case of logistic mapping**

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***Abstract** - The present paper is devoted to investigation of the multidimensional case of the logistic mapping on the plane to itself. In this paper we learnt the properties of the sets of Julia and Mandelbrot for some two-dimensional logistic mappings. The sets of Julia and Mandelbrot help to define asymptotical behavior of the trajectories of certain mappings. The analytical solutions of the equations for finding fixed and periodic points and the computational simulations for describing the sets of Julia and Mandelbrot are the main results of this paper.*

***Аннотация** - Настоящая работа посвящена исследованию многомерного случая логистического отображения плоскости в себя. В этой статье мы изучили свойства множеств Жюлиа и Мандельброта для некоторых двумерных логистических отображений. Множества Жюлиа и*

*Мандельброта* помогают определить асимптотическое поведение траекторий некоторых отображений. Аналитические решения уравнений для нахождения неподвижных и периодических точек и вычислительное моделирование для описания множеств Жюлиа и Мандельброта являются основными результатами этой статьи.

**Annotatsiya** - Bu maqolada logistik akslantirishning tekislikni o'ziga akslantiruvchi ko'p o'lchovli holati tatbiq qilingan. Maqolada ikki o'lchovli logistik akslantirish uchun Julia va Mandelbrot to'plamlarining xossalari o'rganildi. Julia va Mandelbrot to'plamlari berilgan akslantirish uchun traektoriyalarning asimptotikalarini aniqlash uchun asosiy vosita hisoblanadi. Berilgan akslantirishning qo'zgalmas va davriy nuqtalarini topish uchun tuzilgan tenglamalarning analitik yechimlari va Julia va Mandelbrot to'plamlarining kompyuterda olingan tasvirlari maqolaning asosiy natijalari hisoblanadi.

**Key words:** Julia set, Mandelbrot set, Graphical analysis, Chaos.

**Ключевые слова:** множество Жюлиа, множество Мандельброта, Графический анализ, Хаос

**Kalit so'zlar:** Julia to'plami, Mandelbrot to'plami, Grafik tahlil, Xaos.

**MSC (2010):** 34C28, 37-XX, 39Axx.

## **I.Introduction**

The logistic map is a polynomial mapping of degree 2, often cited as an archetypal example of how complex, chaotic behavior can arise from very simple non-linear dynamical equations [3]. The map was popularized in a 1976 paper by the biologist Robert May,[3] in part as a discrete-time demographic model analogous to the logistic equation first created by Pierre Francois Verhulst [6] Mathematically, the logistic map is written

$$x_{n+1} = rx_n(1 - x_n)$$

where  $x_n$  is a number between zero and one that represents the ratio of existing population to the maximum possible population. The values of interest for the

parameter  $r$  are those in the interval  $[0,4]$ . This nonlinear difference equation is intended to capture two effects:

**Reproduction** where the population will increase at a rate proportional to the current population when the population size is small.

**Starvation** where the growth rate will decrease at a rate proportional to the value obtained by taking the theoretical "carrying capacity" of the environment less the current population. However, as a demographic model the logistic map has the pathological problem that some initial conditions and parameter values lead to negative population sizes. This problem does not appear in the older Ricker model, which also exhibits chaotic dynamics.

## II. Preliminaries

Let  $x = (x_1, x_2, \dots, x_n) \in R^n$ ,  $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_n) \in R^n$ ,  $I = 1, 2, \dots, n$  and  $\pi : I \rightarrow I$  some permutations. We call this mapping

$$x'_k = \lambda_{\pi(k)} x_{\pi(k)} (1 - x_{\pi(k)}), k = \overline{1, n} \quad (2.1)$$

on  $R^n$  to itself is **multi - dimensional case** of logistic mapping. If the permutation  $\pi$  expansions in product of the several cycles,  $R^n$  also expansions Cartesian product of sub spaces every from invariant at (2.1) mapping. Therefore dynamical properties also defined by Cartesian product of dynamical properties of invariant sub spaces. Hence it is enough to learn when  $\pi$  - cyclical permutation has maximal length. First we learn when  $n = 2$ . In this case mapping (2.1) is

$$F_{\lambda\mu} : \begin{cases} x' = \lambda y(1 - y) \\ y' = \mu x(1 - x) \end{cases} \quad (2.2)$$

where  $(x, y) \in R^2$  and  $(\lambda, \mu) \in R^2$ .

**Definition 2.1.** The filled Julia [2], [4] set  $K(F_{\lambda\mu})$  of a mapping (2.2) is defined as the set of all points  $(x, y)$ , that have bounded orbit with respect to mapping (2.2).

$$K(F_{\lambda\mu}) = \{(x, y) : F_{\lambda\mu}^n(x, y) \rightarrow \infty \text{ as } n \rightarrow \infty\}$$

**Definition 2.2.** The Julia set is the common boundary of the filled Julia set

$$J(F_{\lambda\mu}) = \partial K(F_{\lambda\mu}).$$

**Definition 2.3.** The critical points of the mapping (2.2) are all points  $(x_c, y_c)$  which determinant of Jacobian matrix at these points is equal to zero

$$\Delta(J(F_{\lambda\mu}(x_c, y_c))) = 0.$$

**Definition 2.4.** The Mandelbrot set [14], [15], [16]  $M_{F_{\lambda\mu}}$  for the mapping (2.2) is the set of all points  $(\lambda, \mu)$  on the parameter plane, which the orbits of the all critical points are bounded.

### III.Fixed point of the mapping (2.2).

For finding fixed points of the mapping (2.2) necessary to solve the following equation [6], [7], [8], [10], [13],

$$x = \lambda\mu x(1-x)(1-\mu x(1-x)) = -\lambda\mu^2 x^4 + 2\lambda\mu^2 x^3 - \lambda\mu(1+\mu)x^2 + \lambda\mu x.$$

Let  $f(x) = -\lambda\mu^2 x^4 + 2\lambda\mu^2 x^3 - \lambda\mu(1+\mu)x^2 + (\lambda\mu - 1)x$  the polynomial of fourth degree and has two parameters  $\lambda$  and  $\mu$ .

Let  $f(x) = a_0 x^n + a_1 x^{n-1} + \dots + a_n$  and  $g(x) = b_0 x^m + b_1 x^{m-1} + \dots + b_m$  are the polynomials.

We know from [23] the following determinant

$$\begin{vmatrix} a_0 & a_1 & a_2 & \dots & a_n & 0 & \dots & 0 \\ 0 & a_0 & a_1 & a_2 & \dots & a_n & \dots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \dots & a_0 & a_1 & a_2 & \dots & a_n \\ b_0 & b_1 & b_2 & \dots & b_m & 0 & \dots & 0 \\ 0 & b_0 & b_1 & b_2 & \dots & b_m & \dots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \dots & b_0 & b_1 & b_2 & \dots & b_m \end{vmatrix}$$

is called **the resultant** of the polynomials of  $f(x)$  and  $g(x)$ . And from

[23] discriminant of polynomial  $f(x)$  is equal

$$D(f) = \prod_{i>j}^4 (z_i - z_j)^2$$

$D(f(x)) = 0$  and  $R(f, f') = 0$  equations are equivalent.

We calculate the resultant  $R(f, f') =$

$$= \begin{vmatrix} -\lambda\mu^2 & 2\lambda\mu^2 & -\lambda\mu(1+\mu) & \lambda\mu-1 & 0 & 0 & 0 \\ 0 & -\lambda\mu^2 & 2\lambda\mu^2 & -\lambda\mu(1+\mu) & \lambda\mu-1 & 0 & 0 \\ 0 & 0 & -\lambda\mu^2 & 2\lambda\mu^2 & -\lambda\mu(1+\mu) & \lambda\mu-1 & 0 \\ -4\lambda\mu^2 & 6\lambda\mu^2 & -2\lambda\mu(1+\mu) & \lambda\mu-1 & 0 & 0 & 0 \\ 0 & -4\lambda\mu^2 & 6\lambda\mu^2 & -2\lambda\mu(1+\mu) & \lambda\mu-1 & 0 & 0 \\ 0 & 0 & -4\lambda\mu^2 & 6\lambda\mu^2 & -2\lambda\mu(1+\mu) & \lambda\mu-1 & 0 \\ 0 & 0 & 0 & -4\lambda\mu^2 & 6\lambda\mu^2 & -2\lambda\mu(1+\mu) & \lambda\mu-1 \end{vmatrix} =$$

$$= -\lambda^3\mu^6(-1+\lambda\mu)^2(-27+18\lambda\mu-4\lambda^2\mu-4\lambda\mu^2+\lambda^2\mu^2).$$

The equation  $R(f, f') = D(f) = 0$  defines the multiple real roots of the polynomial  $f(x)$ . Hence, if  $R(f, f') = D(f) = 0$ , then parabolas

$$\begin{aligned} x &= \lambda y(1-y), \\ y &= \mu x(1-x). \end{aligned}$$

have a common tangent point, i.e. multiple root. The equation  $D(f) = 0$  is equivalent to

$$-\lambda^3\mu^6(-1+\lambda\mu)^2(-27+18\lambda\mu-4\lambda^2\mu-4\lambda\mu^2+\lambda^2\mu^2) = 0 \quad (3.1)$$

We investigate the case  $\lambda > 0, \mu > 0$ . Then we must investigate the following equations of  $\lambda$  and  $\mu$   $(-1+\lambda\mu)^2 = 0$  and

$$-27+18\lambda\mu-4\lambda^2\mu-4\lambda\mu^2+\lambda^2\mu^2 = 0 \quad (3.2)$$

which will be considered as a function  $\mu(\lambda)$  given implicitly. How many ordinary functions are defined by implicit functions (3.1)? To answer for this question, we calculate the discriminant of a polynomial (3.2) with respect to the variable  $\mu$ .

We find

$$D = 16\lambda(\lambda-3)^3$$

Since the quadratic equation for  $D < 0$  has no real root, and for  $D > 0$ , two real roots, we get the following statement.

**Theorem 3.1.** *If  $\lambda > 3$  then (3.1) defines three functions, and for get  $\lambda < 3$  only one function.*

**Theorem 3.2.** *The algebraic curve (3.1) splits the parameters plane  $(\lambda, \mu)$  into three areas.*

**Proof.** The discriminant of  $f(x)$

$$D(f) = -\lambda^3 \mu^6 (-1 + \lambda \mu)^2 D_1(f)$$

where

$$D_1(f) = -27 + 18\lambda\mu - 4\lambda^2\mu - 4\lambda\mu^2 + \lambda^2\mu^2$$

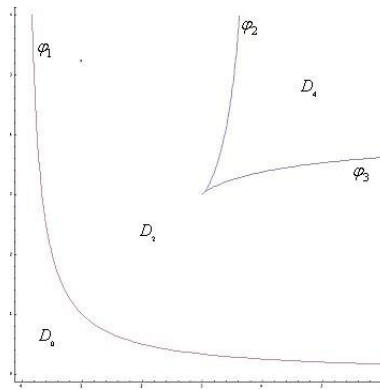
by the elementary functions

$$D_1(f) = \left(\mu - \frac{2\lambda^2 - 9\lambda - 2\sqrt{\lambda(\lambda-3)^3}}{\lambda(\lambda-4)}\right) \left(\mu - \frac{2\lambda^2 - 9\lambda + 2\sqrt{\lambda(\lambda-3)^3}}{\lambda(\lambda-4)}\right)$$

Hence  $D(f) = 0$  equation equivalent to

$$\left(\mu - \frac{1}{\lambda}\right) \left(\mu - \frac{2\lambda^2 - 9\lambda - 2\sqrt{\lambda(\lambda-3)^3}}{\lambda(\lambda-4)}\right) \left(\mu - \frac{2\lambda^2 - 9\lambda + 2\sqrt{\lambda(\lambda-3)^3}}{\lambda(\lambda-4)}\right) = 0$$

It means that If  $\lambda > 3$  then (3.1) defines three functions, and for get  $\lambda < 3$  only one function. We depict the curve of  $D(f) = 0$  in picture 1.

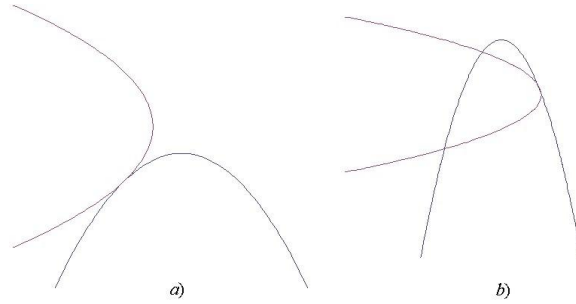


**Picture. 1.** The curves of  $D(f) = 0$ .

So, the graphs of these functions split the  $(\lambda, \mu)$  parameter plane into three open areas  $D_0, D_2$  and  $D_4$ . If  $(\lambda, \mu) \in D_2$  then has two and If  $(\lambda, \mu) \in D_4$  then has four real roots. If  $(\lambda, \mu) \in \varphi_1$  then parabolas

$$\begin{cases} x' = \lambda y(1 - y), \\ y' = \mu x(1 - x). \end{cases} \quad (3.3)$$

tangent externally (Pic.2a.), Finally if  $(\lambda, \mu) \in \varphi_2$  or  $(\lambda, \mu) \in \varphi_3$  then parabolas tangent externally (Pic.2b.)



**Picture. 2.** The graphs of parabolas  $x = \lambda y(1 - y)$  and  $y = \mu x(1 - x)$ .

**Lemma 3.3.** *If  $\lambda = \mu = 3$  then the parabolas at tangents with third order.*

For an arbitrary initial point the orbits is determined by the following formula

$$\begin{cases} x_{n+1} = \lambda y_n(1 - y_n), \\ y_{n+1} = \mu x_n(1 - x_n), \end{cases} n = 0, 1, 2, \dots$$

#### IV. Graphical analysis.

There exist in [1],[5],[6],[7], [9],[10],[17],[12],[18], [21],[22] method graphical analysis for one dimensional dynamical systems. In this part of the paper, we introduce a geometric procedure that will help us understand the dynamics of some two-dimensional mappings. This procedure enables us to use the graphs of functions to determine the behavior of orbits in many cases. Suppose we have the two-dimensional mapping. In this part of the paper we introduce a geometric procedure that will help us understand the dynamics of some two-dimensional mappings. This procedure, called **graphical analysis**, enables us to use the graphs of functions to determine the behavior of orbits in many cases. Suppose we have the two-dimensional mapping

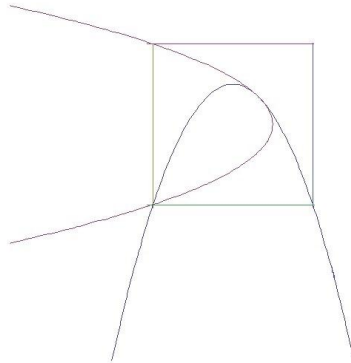
$$F_{\lambda\mu} : \left\{ \begin{array}{l} x' = \lambda y(1 - y), \\ y' = \mu x(1 - x), \end{array} \quad n = 0, 1, 2, \dots \right\}$$

and wish to display the orbit of a given point  $(x_0, y_0)$ . We begin by superimposing the graph of  $x = f(y, \lambda)$  on the graph of  $y = g(x, \mu)$ . The points of intersection of the graph  $x = f(y, \lambda)$  with the graph of  $y = g(x, \mu)$  give us the **fixed points** of  $F_{\lambda\mu}$ . To find the orbit of  $(x_0, y_0)$ , we begin at the point  $(x_0, y_0)$  on the  $XOY$  plane. We



first draw a horizontal line to the graph of  $x = f(y, \lambda)$ . When this line meets the graph of  $x = f(y, \lambda)$ , we have reached the point  $(f(y_0, \lambda), y_0)$  then draw a vertical line and denote it by  $V_1$ . We again begin at the point  $(x_0, y_0)$  on the  $XOY$  plane we draw a vertical line to the graph of  $y = g(x, \mu)$ . When this line meets the graph of  $y = g(x, \mu)$ , we have reached the point  $(x_0, g(x_0, \mu))$  then draw a horizontal line and denote it by  $H_1$ . The intersection point of  $V_1$  and  $H_1$  is  $(f(y_0, \lambda), g(x_0, \mu)) = (x_1, y_1)$  the next point of the orbit of given point  $(x_0, y_0)$ . To display the orbit of  $(x_0, y_0)$  geometrically, we thus continue this procedure over and over, in the next step we denote  $V_{i+1}$  instead of  $V_i$  and  $H_{i+1}$  instead of  $H_i$ . The intersection point of  $V_i$  and  $H_i$  is the  $i$  th point of the orbit of  $(x_0, y_0)$  by the mapping of  $F_{\lambda\mu}$ . In the Picture 3. we depicted graphical analysis of  $(x_0, y_0)$  for

$$\begin{cases} x_{n+1} = \lambda y(1 - y_n), \\ y_{n+1} = \mu x_n(1 - x_n), n = 0, 1, 2, \dots \end{cases}$$



**Picture. 3.** The graphical analysis.

Let  $J(f)$  be the set of all points  $(x_0, y_0) \in \mathbb{R}^2$  that the orbit of them bounded. By the method of the graphical analysis we obtain following theorems.

**Theorem 3.4.** *If  $(\lambda, \mu) \notin D_0$  then, the orbit of an arbitrary initial point  $(x_0, y_0) \notin \{[0, 1] \times [0, 1]\}$  tends to infinity, i. e.  $x_n \rightarrow +\infty, y_n \rightarrow +\infty$  with  $n \rightarrow +\infty$ .*

**Theorem 3.5.** *If  $(\lambda, \mu) \in D_0$  then  $J(f) \subset [0, 1] \times [0, 1]$*

**Theorem 3.6.** *Let the points  $(p_1, q_1), (p_2, q_2), \dots, (p_n, q_n)$  are the fixed points for the mapping (2.2), if  $i \neq j$  the points  $(p_i, q_i) \neq (p_j, q_j)$  are arbitrary two points of them, the points  $(p_i, q_j)$  and  $(p_j, q_i)$  are periodic points with prime period two.*

**Proof.** The points are  $(p_i, q_i) \neq (p_j, q_j)$  fixed, let

$$\begin{aligned}x_0 &= p_i \\y_0 &= q_j\end{aligned}$$

next point of the orbit of  $(x_0, y_0)$

$$\begin{aligned}x_1 &= \lambda q_j(1 - q_j) = p_j \\y_1 &= \mu p_i(1 - p_i) = q_i\end{aligned}$$

hence

$$\begin{aligned}x_2 &= \lambda q_i(1 - q_i) = p_i, \\y_2 &= \mu p_j(1 - p_j) = q_j\end{aligned}$$

We see  $(p_i, q_j) \rightarrow (p_j, q_i) \rightarrow (p_i, q_j)$  by (2.2). The theorem is proved.

**Statement:** The equations for finding fixed points and for finding periodic points with period two are the same, therefore this theorem is true. In first section we learnt the properties of fixed points, many of them are true for the periodic points with period two.

### **V.Periodic orbits of the mapping (2.2) with prime period four.**

For finding fixed points of the mapping (2.2) necessary to solve the following equation [3]

$$\begin{aligned}
& x = (\lambda\mu)^2 x(1-x)(1-\mu x(1-x))(1-\lambda\mu x(1-x)(1-\mu x(1-x))) \times \\
& \times (1-\mu\lambda\mu x(1-x)(1-\mu x(1-x))(1-\lambda\mu x(1-x)(1-\mu x(1-x)))) = \\
& = -x(1-\lambda\mu + \lambda\mu x + \lambda\mu^2 x - 2\lambda\mu^2 x^2 + \lambda\mu^2 x^3) \times \\
& \times (1 + \lambda\mu - \lambda\mu x - \lambda\mu^2 x - \lambda^2\mu^2 x - \lambda^2\mu^3 x + 2\lambda\mu^2 x^2 + \lambda^2\mu^2 x^2 + 4\lambda^2\mu^3 x^2 + \lambda^2\mu^4 x^2 + \\
& + 2\lambda^3\mu^4 x^2 - \lambda\mu^2 x^3 - 5\lambda^2\mu^3 x^3 - 4\lambda^2\mu^4 x^3 - 5\lambda^3\mu^4 x^3 - 4\lambda^3\mu^5 x^3 - \lambda^4\mu^5 x^3 + \\
& + 2\lambda^2\mu^3 x^4 + 6\lambda^2\mu^4 x^4 + 4\lambda^3\mu^4 x^4 + 14\lambda^3\mu^5 x^4 + 3\lambda^4\mu^5 x^4 + 2\lambda^3\mu^6 x^4 + 3\lambda^4\mu^6 x^4 - \\
& - 4\lambda^2\mu^4 x^5 - \lambda^3\mu^4 x^5 - 18\lambda^3\mu^5 x^5 - 3\lambda^4\mu^5 x^5 - 9\lambda^3\mu^6 x^5 - 12\lambda^4\mu^6 x^5 - 3\lambda^4\mu^7 x^5 + \lambda^2\mu^4 x^6 + \\
& + 10\lambda^3\mu^5 x^6 + \lambda^4\mu^5 x^6 + 16\lambda^3\mu^6 x^6 + 18\lambda^4\mu^6 x^6 + 15\lambda^4\mu^7 x^6 + \lambda^4\mu^8 x^6 - 2\lambda^3\mu^5 x^7 - \\
& - 14\lambda^3\mu^6 x^7 - 12\lambda^4\mu^6 x^7 - 30\lambda^4\mu^7 x^7 - 6\lambda^4\mu^8 x^7 + 6\lambda^3\mu^6 x^8 + 3\lambda^4\mu^6 x^8 + 30\lambda^4\mu^7 x^8 + \\
& + 15\lambda^4\mu^8 x^8 - \lambda^3\mu^6 x^9 - 15\lambda^4\mu^7 x^9 - 20\lambda^4\mu^8 x^9 + 3\lambda^4\mu^7 x^{10} + 15\lambda^4\mu^8 x^{10} - 6\lambda^4\mu^8 x^{11} + \\
& + \lambda^4\mu^8 x^{12})
\end{aligned}$$

Let

$$\begin{aligned}
g(x) = & 1 + \lambda\mu - \lambda\mu x - \lambda\mu^2 x - \lambda^2\mu^2 x - \lambda^2\mu^3 x + 2\lambda\mu^2 x^2 + \lambda^2\mu^2 x^2 + 4\lambda^2\mu^3 x^2 + \lambda^2\mu^4 x^2 + \\
& + 2\lambda^3\mu^4 x^2 - \lambda\mu^2 x^3 - 5\lambda^2\mu^3 x^3 - 4\lambda^2\mu^4 x^3 - 5\lambda^3\mu^4 x^3 - 4\lambda^3\mu^5 x^3 - \lambda^4\mu^5 x^3 + 2\lambda^2\mu^3 x^4 + \\
& + 6\lambda^2\mu^4 x^4 + 4\lambda^3\mu^4 x^4 + 14\lambda^3\mu^5 x^4 + 3\lambda^4\mu^5 x^4 + 2\lambda^3\mu^6 x^4 + 3\lambda^4\mu^6 x^4 - 4\lambda^2\mu^4 x^5 - \\
& - \lambda^3\mu^4 x^5 - 18\lambda^3\mu^5 x^5 - 3\lambda^4\mu^5 x^5 - 9\lambda^3\mu^6 x^5 - 12\lambda^4\mu^6 x^5 - 3\lambda^4\mu^7 x^5 + \lambda^2\mu^4 x^6 + 10\lambda^3\mu^5 x^6 + \\
& + \lambda^4\mu^5 x^6 + 16\lambda^3\mu^6 x^6 + 18\lambda^4\mu^6 x^6 + 15\lambda^4\mu^7 x^6 + \lambda^4\mu^8 x^6 - 2\lambda^3\mu^5 x^7 - 14\lambda^3\mu^6 x^7 - \\
& - 12\lambda^4\mu^6 x^7 - 30\lambda^4\mu^7 x^7 - 6\lambda^4\mu^8 x^7 + 6\lambda^3\mu^6 x^8 + 3\lambda^4\mu^6 x^8 + 30\lambda^4\mu^7 x^8 + 15\lambda^4\mu^8 x^8 - \\
& - \lambda^3\mu^6 x^9 - 15\lambda^4\mu^7 x^9 - 20\lambda^4\mu^8 x^9 + 3\lambda^4\mu^7 x^{10} + 15\lambda^4\mu^8 x^{10} - 6\lambda^4\mu^8 x^{11} + \lambda^4\mu^8 x^{12}
\end{aligned}$$

the polynomial of twenties degree and has two parameters  $\lambda$  and  $\mu$ .

$$D(g(x)) = 0 \text{ and } R(g, g') = 0 \text{ equations are equivalent.}$$

We calculate the discriminant

$$\begin{aligned}
R(g, g') = & \lambda^{70} \mu^{140} (1 + \lambda\mu)(-27 + 18\lambda\mu - 4\lambda^2\mu - 4\lambda\mu^2 + \lambda^2\mu^2) \times \\
& \times (125 - 85\lambda\mu + 12\lambda^2\mu + 12\lambda\mu^2 + 15\lambda^2\mu^2 - 4\lambda^3\mu^2 - 4\lambda^2\mu^3 + \lambda^3\mu^3)^3 \times \\
& \times (91125 + 77760\lambda\mu - 19872\lambda^2\mu - 19872\lambda\mu^2 - 432\lambda^2\mu^2 - 13680\lambda^3\mu^2 + \\
& + 3328\lambda^4\mu^2 - 13680\lambda^2\mu^3 + 4896\lambda^3\mu^3 + 4792\lambda^4\mu^3 - 2176\lambda^5\mu^3 + 256\lambda^6\mu^3 \\
& + 3328\lambda^2\mu^4 + 4792\lambda^3\mu^4 + 1682\lambda^4\mu^4 - 832\lambda^5\mu^4 - 2176\lambda^3\mu^5 - 832\lambda^4\mu^5 - 1504\lambda^5\mu^5 + \\
& + 720\lambda^6\mu^5 - 64\lambda^7\mu^5 + 256\lambda^3\mu^6 + 720\lambda^5\mu^6 + 8\lambda^6\mu^6 - 112\lambda^7\mu^6 + \\
& + 16\lambda^8\mu^6 - 64\lambda^5\mu^7 - 112\lambda^6\mu^7 + 64\lambda^7\mu^7 - 8\lambda^8\mu^7 + 16\lambda^6\mu^8 - 8\lambda^7\mu^8 + \lambda^8\mu^8)^2.
\end{aligned}$$

For any  $\lambda$  and  $\mu$  the expression

$$\begin{aligned}
& (91125 + 77760\lambda\mu - 19872\lambda^2\mu - 19872\lambda\mu^2 - 432\lambda^2\mu^2 - 13680\lambda^3\mu^2 + 3328\lambda^4\mu^2 - \\
& - 13680\lambda^2\mu^3 + 4896\lambda^3\mu^3 + 4792\lambda^4\mu^3 - 2176\lambda^5\mu^3 + 256\lambda^6\mu^3 + 3328\lambda^2\mu^4 + \\
& + 4792\lambda^3\mu^4 + 1682\lambda^4\mu^4 - 832\lambda^5\mu^4 - 2176\lambda^3\mu^5 - 832\lambda^4\mu^5 - 1504\lambda^5\mu^5 + 720\lambda^6\mu^5 - \\
& - 64\lambda^7\mu^5 + 256\lambda^3\mu^6 + 720\lambda^5\mu^6 + 8\lambda^6\mu^6 - 112\lambda^7\mu^6 + 16\lambda^8\mu^6 - 64\lambda^5\mu^7 - 112\lambda^6\mu^7 + \\
& + 64\lambda^7\mu^7 - 8\lambda^8\mu^7 + 16\lambda^6\mu^8 - 8\lambda^7\mu^8 + \lambda^8\mu^8)^2
\end{aligned}$$

is not equal to zero.

At the above we considered curves (3.1) which define the fixed points of (2.2). Hence, the equation  $D(g) = 0$  is equivalent to

$$125 - 85\lambda\mu + 12\lambda^2\mu + 12\lambda\mu^2 + 15\lambda^2\mu^2 - 4\lambda^3\mu^2 + \lambda^3\mu^3 = 0 \quad (3.4)$$

define the periodic points with prime period four which will be considered as a function  $\mu(\lambda)$  given implicitly. How many ordinary functions are defined by implicit functions (3.4)? To answer for this question, we calculate the discriminant of a polynomial (3.4) with respect to the variable  $\mu$ . By the known [23] formulas we find

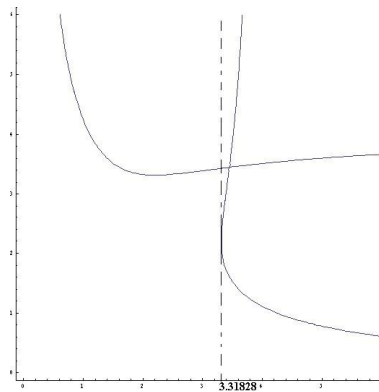
$$D = 256\lambda^3(-5 - 2\lambda + \lambda^2)^2(-135 + 144\lambda - 61\lambda^2 + 9\lambda^3)$$

From  $D = 0$  when  $\lambda \approx 3.31828\dots$

Since the cubic equation for  $D < 0$  has one real, and for  $D > 0$  has three real roots, we get the following statement.

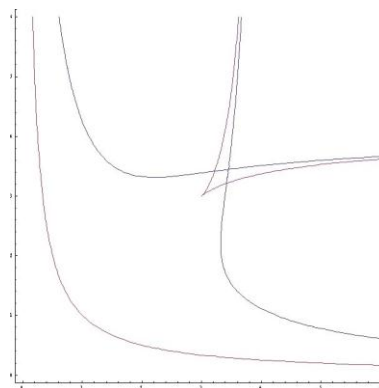
**Lemma 3.7.** *If  $\lambda > 3.31828\dots$  then (3.4) defines three functions and for  $\lambda < 3.31828\dots$  get only one function.*

In the pic. 4. we depicted the curves defined by (3.4).



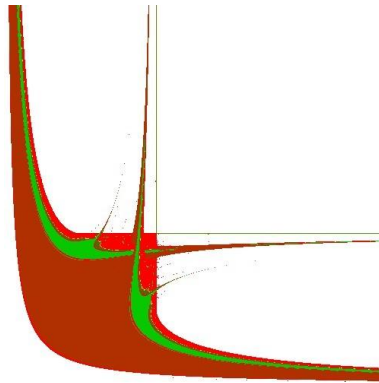
**Picture. 4.** The curves defined by (3.4).

In the pic. 5. we depict the curves defined by (3.1) and (3.4) together.



**Picture. 5.** The curves defined by (3.1) and (3.4).

Pictures 4 and 5 are the analytical approaches to the Mandelbrot set. For improving our approaching we developed the computer program to get the picture of Mandelbrot set. In Pic. 6. we depict our programs result.



**Picture. 6.** Mandelbrot set on  $(\lambda, \mu)$  plane.

**Definition.** If the orbits have following three properties then it is chaotic [6],[7],[19]:

- i.* Dense periodic points.
- ii.* Transitivity.
- iii.* Sensitive dependence of initial condition.

### **VI. Conclusion.**

When  $\lambda = \mu = 4$  there exist so many periodic and chaotic orbits on the filled Julia set [20],[21],[22]. All periodic points are repeller. It means sensitive dependence of initial condition. The red area of the pic 6. on the parameter plane is such parameters dynamics of the mapping (2.2) on them are chaotic.

We have investigated in this paper two dimensional case of logistic mappings. It is learned fixed points, periodic points and their some properties of the mapping (2.2). It is appeared that there are chaotic dynamics for two dimensional logistic mappings.

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