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АННОТАЦИЯ. Назарий ва амалий жиҳатдан муҳим аҳамиятга эга бўлган физика, механика масалалари интеграл тенгламалар усули билан ечилади. Математик физика масалаларни ечишда интеграл тенгламаларидан фойдаланилади. Шунинг учун интеграл тенгламаларни ечимларини топишнинг аҳамяти каттадир, лекин ҳар доим ҳам интеграл тенгламаларнинг аналитик ечимларини топишиниг имконни йўқ. Шу сабабли интерал тенгламаларнинг сонли усулларини ўрганамиз. Ушбу мақолада МАТЛАБ дастури ёрдамида Волтерранинг иккинчи тур интеграл тенгламасининг сонли ечимлари топиш кўрсатилган. Натижалар аниқ ечим билан таққосланади.

Калит сўзлар: сонли усуллар, интеграл тенгламалар, Волтерра интеграл тенгламаси.

АННОТАЦИЯ. Задачи физики, механики теоретического и практического значения решаются методом интегральных уравнений. Математическая физика использует интегральные уравнения для решения задач. Поэтому важно найти решения интегральных уравнений. Но не всегда можно найти аналитические решения интегральных уравнений. Поэтому мы изучаем численные методы интегральных уравнений. В этой статье показано, как найти численное решение интегрального уравнения Вольтерры 2-го рода с помощью программы MATLAB. Результаты сравниваются с точным решением с помощью компьютерного моделирования.

Ключевые слова: численные методы, интегральные уравнения, интегральные уравнения Вольтерра.

ABSTRACT. Problems of physics, mechanics of theoretical and practical importance are solved by the method of integral equations. Mathematical physics uses integral equations to solve problems. Therefore, it is important to find solutions to integral equations. But it is not always possible to find analytical solutions of integral equations. Thus, we study the numerical methods of integral equations. This paper shows how to find numerical solutions to Volterra equation of the second kind using the MATLAB program. The results are compared with the exact solution by using computer simulations.

Keywords: numerical methods, integral equations, Volterra equation.

Introduction

The linear Volterra equation of the second kind has the following form:

$$u(x)-\int_a^x K(x,s)u(s)ds = f(x) \tag{1}$$

where $u(x)$ is an unknown function, $f(x)$ is a known function, and $K(x,s)$ is another known function of two variables, often called the kernel of the integral equation. The homogeneous equation (for $f\equiv 0$) has only a trivial solution, and the conditions for the existence of a solution of an inhomogeneous equation (1) are related to various restrictions on the kernel $K(x,s)$ and $f(x)$ ([1],[2]). In particular ([3]), a solution exists and is unique in the class of functions continuous on the interval $[a,b]$ if the kernel is continuous inside and on the sides of a triangle bounded by straight lines $s=a$, $x=b$, $x=s$ and the function f (x) is continuous on [a, b].

Equation (1) contains the integral operator

$$A\varphi(x) = \int_a^x K(x,s)\varphi(s)ds. \tag{2}$$

It is clear that the values of the function $\psi(x) = A\varphi(x)$ for any x are determined by the values of function $\varphi(s)$ only for $s<x$. The integral operators characterized by this

property are called Volterra operators and are widely used in the description of processes with aftereffect and feedback [1].

Research Methodology.

In the numerical solution of integral equations, the integrals entering into them are usually replaced by finite sums [4], [5], [6]. We calculate the integrals using a quadrature formula based on

$$X = \left\{ x_k = a + kh,\ h = \frac{b-a}{N},\ k = 0,1,\dots,N \right\}$$

grid. According to the quadrature method, the integral operators are replaced by the sums obtained using various quadrature formulas [7], [8], [9],[10]:

$$\int_a^b g(x)dx = \sum_{i=1}^{n} d_i g(x_i) + R. \tag{3}$$

Here $a \le x_1 < x_2 < \dots < x_n \le b$ — nodes, d_i, $i = 1, 2, \dots, n$ are weights and R is the approximation error of the quadrature formula.

To apply the quadrature method to the solution of equation (1), it is necessary to use the following equalities:

$$u(x_i) - \int_a^{x_i} K(x_i, s)u(s)ds = f(x_i),\ i = 1, 2, \dots, n. \tag{4}$$

They are obtained from the original equation for fixed values x_i of independent variable x. Grid nodes x_i can be selected in a special way or predefined if, for example, the right part $f(x)$ is specified by a table.

We take the values x_i as the nodes of the quadrature formula and use it to replace the integral in (4) with a finite sum. We get following system

$$u(x_i) - \sum_{j=1}^{i} d_j K(x_i, x_j)u(x_j) = f(x_i) + R_i,\ i = 1, 2, \dots, n, \tag{5}$$

where d_j are weights of the quadrature formula, R_j are approximation errors.

We set errors R_j small and discard them. We get a system of linear algebraic equations:

$$u(x_i) - \sum_{j=1}^{i} d_j K_{ij} u_j = f(x_i), \ i = 1, 2, ..., n. \tag{6}$$

Here $u_i = u(x_i)$, $u_i = u(x_i), u_i = u(x_i)$. The solution of the system of equations (6) gives approximate values of the desired function at nodes x_i. We bring the system (6) to the following form:

$$-\sum_{j=1}^{i-1} d_j K_{ij} u_j + (1 - d_i K_{ii}) u_i = f_i, \ i = 1, 2, ..., n, \tag{7}$$

more details,

$$\begin{pmatrix} 1-d_1K_{11} & & & \\ -d_1K_{21} & 1-d_2K_{22} & & \\ \vdots & \vdots & \ddots & \\ -d_1K_{n1} & -d_2K_{n2} & \dots & 1-d_nK_{nn} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{pmatrix}. \tag{8}$$

We rewrite the system of linear algebraic equations (8) in the form

$$DU = F, \tag{9}$$

where

$$D = \begin{pmatrix} 1-d_1K(x_1,x_1) & & & \\ -d_1K(x_2,x_1) & 1-d_2K(x_2,x_2) & & \\ \vdots & \vdots & \ddots & \\ -d_1K(x_n,x_1) & -d_2K(x_n,x_2) & \dots & 1-d_nK(x_n,x_n) \end{pmatrix},$$

$$U = \left(u(x_1), u(x_2), ..., u(x_n)\right)^T,$$

$$F = \left(f(x_1), f(x_2), ..., f(x_n)\right)^T.$$

According to (9), the equality

$$U = D^{-1}F$$

The fulfillment of conditions $(1-d_i K_{ii}) \neq 0, i=1,2,\ldots,n$ can be achieved by choosing the nodes of the quadrature formula and ensuring that the coefficients d_j are sufficiently small.

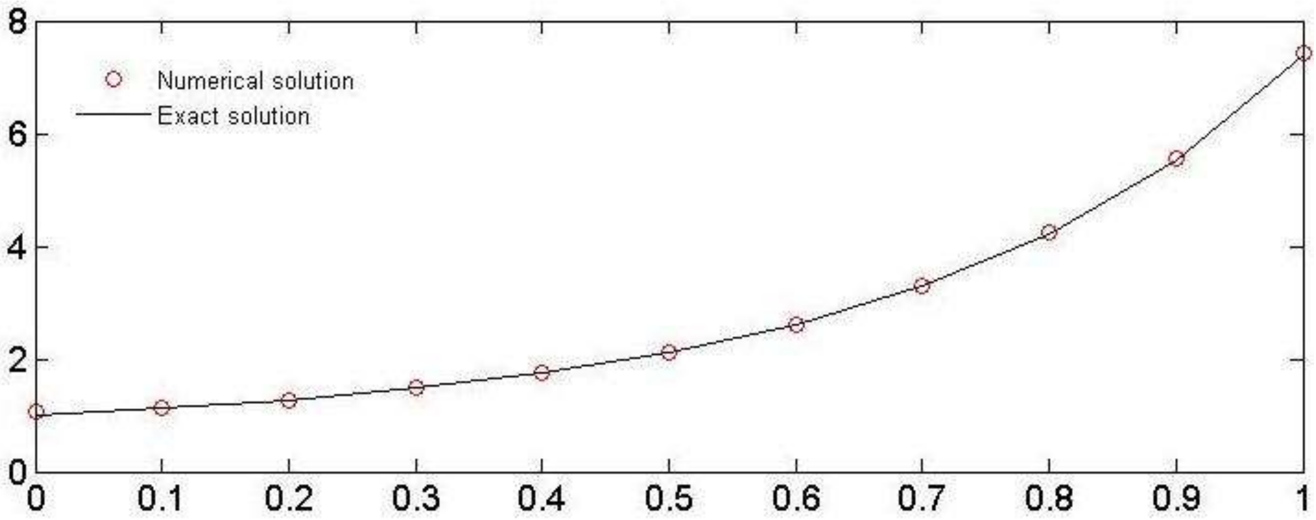
Analysis and results.

The values of all parameters are selected equal to $\{h=0.1,\ n=11\}$ for area $[0,1]$. Let $K(x,s)=e^{x^2-s^2}, f(x)=e^{x^2}$. In this case, according to the linear Volterra equation of the second kind (1), we obtain the exact solution in the form

$$u(x)=e^{x^2+x}.$$

Numerical and exact solutions of Volterra integral equation of the second kind is shown at n=11 as following.

x_i	**Numerical solution**	**Exact solution**
0.0	1.0526315789473684	1.0
0.1	1.1191691602040643	1.1162780704588713
0.2	1.2746479812240938	1.2712491503214047
0.3	1.4810532734217747	1.4769807938826427
0.4	1.7556461476306326	1.7506725002961012
0.5	2.1231915887628814	2.117000016612675
0.6	2.619553495173338	2.611696473423118
0.7	3.2972452268817323	3.2870812073831184
0.8	4.234100033501857	4.220695816996553
0.9	5.546983448652364	5.528961477624004
1.0	7.413759872283178	7.38905609893065



Conclusion.

In this work, we have used a numerical method to get approximate solution of the linear Volterra equation of the second kind. The analyzed examples illustrated the ability and reliability of the present method. The obtained solutions, in comparison with exact solutions admit a remarkable accuracy.

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