Scientific Bulletin of Namangan State University

Volume 1 | Issue 5 Article 1

5-10-2019

THE STRATEGY OF PARALLEL PURSUIT FOR DIFFERENTIAL GAME OF THE SECOND ORDER

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Erratum

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Abstract. *In this work is considered a differtial game of the second order, when control functions of the players satisfies geometric constraints. The proposed method substantiates the parallel approach strategy in this differential game of the second order. The new sufficient solvability conditions are obtained for problem of the pursuit.*

Keywords. *Differential game, geometric constraint, evader, pursuer, strategy of the parallel pursuit, acceleration.*

IKKINCHI TARTIBLI DIFFERENSIAL O'YIN UCHUN PARALLEL QUVISH STRATEGIYASI

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Annotatsiya. *Ushbu maqolada boshqaruvlar geometrik chegaralanishga ega hol uchun ikkinchi tartibli differensial o'yinlar o'rganiladi. Bunda quvlovchi uchun parallel quvish strategiyasi quriladi va uning yordamida tutish masalasi yechiladi.*

Kalit so`zlar: *differensial o'yin, geometrik chegaralanish, parallel quvish strategiyasi, quvlovchi, qochuvchi, tezlanish.*

СТРАТЕГИЯ ПАРАЛЛЕЛЬНОГО ПРЕСЛЕДОВАНИЯ ДЛЯ ДИФФЕРЕНЦИАЛЬНОЙ ИГРЫ ВТОРОГО ПОРЯДКА

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Аннотация. *В работе рассматривается дифференциальная игра второго порядка при геометрических ограничениях на управления игроков. При этом предлагается стратегия параллельного преследования для преследователя и при помощи этой стратегии решается задача преследования.*

Ключевые слова: *Дифференциальная игра, геометрическое ограничение, стратегия параллельного преследования, преследователь, убегающий, ускорения.*

Let $\mathbf{P}$ and $\mathbf{E}$ objects with opposite aim be given in the space R^n and their movements based on the following differential equations and initial conditions

$$\mathbf{P}:\quad \ddot{x}=u,\quad x_1-kx_0=0,\quad |u|\le\alpha, \tag{1}$$

$$\mathbf{E}:\quad \ddot{y}=v,\quad y_1-ky_0=0,\quad |v|\le\beta, \tag{2}$$

where $x,y,u,v\in \boldsymbol{R}^n$; x – a position of **P** object in the space $\boldsymbol{R}^n$, $x_0=x(0),\ x_1=\dot{x}(0)$ – its initial position and velocity respectively at $t=0;$ u – being a controlled acceleration of the pursuer, mapping $u:[0,\infty)\to \boldsymbol{R}^n$ and it is chosen as a measurable function with respect to t. We denote a set of all measurable functions $u(\cdot)$ such that satisfies the condition $|u|\le\alpha$ by $\boldsymbol{U}$. y – a position of **E** object in the space $\boldsymbol{R}^n$, $y_0=y(0),\ y_1=\dot{y}(0)$ – its initial position and velocity respectively at $t=0;$ v – being a controlled acceleration of the evader, mapping $v:[0,\infty)\to \boldsymbol{R}^n$ and it is chosen as a measurable function with respect to t. We denote a set of all measurable functions $v(\cdot)$ such that satisfies the condition $|v|\le\beta$ by $\boldsymbol{V}$.

Definition 1. For a trio of $(x_0,x_1,u(\cdot)), u(\cdot)\in\boldsymbol{U}$, the solution of the equation (1), that is, $x(t)=x_0+x_1t+\int\limits_0^t\int\limits_0^s u(\tau)d\tau ds$ is called a trajectory of the pursuer on interval $t\ge 0$.

Definition 2. For a trio of $(y_0,y_1,v(\cdot)),\ v(\cdot)\in\boldsymbol{V}$, the solution of the equation (2), that is, $y(t)=y_0+y_1t+\int\limits_0^t\int\limits_0^s v(\tau)d\tau ds$ is called a trajectory of the evader on interval $t\ge 0$.

Definition 3. The pursuit problem for the differential game (1) - (2) is called to be solved if there exists such control function $u^*(\cdot)\in\boldsymbol{U}$ of the pursuer for any control function $v(\cdot)\in V$ of the evader and the following equality is carried out at some finite time t^*

$$x(t^*)=y(t^*). \tag{3}$$

Definition 4. For the problem (1)-(2), time T is called a guaranteed pursuit time if it is equal to an upper boundary of all the finite values of pursuit time t^* satisfying the equality (3).

Definition 5. For the differential game (1) - (2), the following function is called П-strategy of the pursuer ([1]-[2]):

$$u(v)=v-\lambda(v)\xi_0, \tag{4}$$

where $\lambda(v,\xi_0)=(v,\xi_0)+\sqrt{(v,\xi_0)^2+\alpha^2-|v|^2}$, $\xi_0=z_0/|z_0|$,

(v,ξ_0) is the scalar product of the vectors v and ξ_0 in the space $\boldsymbol{R}^n$.

Property 1. If $\alpha\ge\beta$, then a function $\lambda(v,\xi_0)$ is continuous, nonnegative and defined for all v such that satisfies the inequality $|v|\le\beta$.

Property 2. If $\alpha \geq \beta$, then the following inequality is true for the function $\lambda(v, \xi_0)$:

$$\alpha - |v| \leq \lambda(v, \xi_0) \leq \alpha + |v|.$$

Theorem. If one of the following conditions holds for the second order differential game (1) – (2), that is, 1. $\alpha = \beta$ and $k < 0$; or 2. $\alpha > \beta$ and $k \in R$, then by virtue of strategy (4) a guaranteed pursuit time becomes as follows

$$T = \begin{cases} \left(|z_0|k + \sqrt{|z_0|^2 k^2 + 2|z_0|(\alpha - \beta)}\right)/(\alpha - \beta), & \text{if } k \neq 0 \text{ and } \alpha > \beta, \\ -1/k, & \text{if } k < 0 \text{ and } \alpha = \beta, \\ \sqrt{2|z_0|/(\alpha - \beta)}, & \text{if } k = 0 \text{ and } \alpha > \beta. \end{cases}$$

Proof. Suppose the pursuer chooses the strategy in the form (4) when the evader chooses any control function $v(\cdot) \in V$. Then, according to the equations (1) – (2), we have the following Caratheodory's equation:

$$\ddot{z} = -\lambda(v(t))\xi_0, \quad \dot{z}(0) - kz(0) = 0$$

Thus the following solution will be found by the given initial conditions

$$z(t) = z_0(kt+1) - \xi_0 \int_0^t \int_0^s \lambda(v(\tau), \xi_0)\, d\tau ds$$

or

$$|z(t)| = |z_0|(kt+1) - \int_0^t \int_0^s \left((v(\tau), \xi_0) + \sqrt{(v(\tau), \xi_0)^2 + \alpha^2 - |v(\tau)|^2}\right) d\tau ds.$$

According to the properties 1- 2, we will form the following inequalities

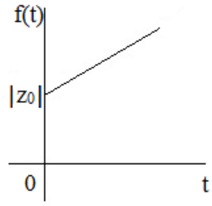
$$|z(t)| \leq |z_0|(kt+1) - \int_0^t \int_0^s (\alpha - |v(\tau)|)\, d\tau ds \quad \Rightarrow$$

$$|z(t)| \leq |z_0|(kt+1) + t^2(\beta - \alpha)/2.$$

We denote

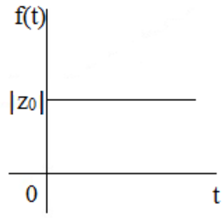
$$f(t, a, k, \alpha, \beta) = a(kt+1) - \frac{t^2}{2}(\alpha - \beta), \; a = |z_0|. \tag{5}$$

1. Let be $\alpha = \beta$.
   1.1. If $k > 0$, then $f(t, a, k, \alpha, \beta) = a(kt+1)$ and it is increasing function (Fig-1)



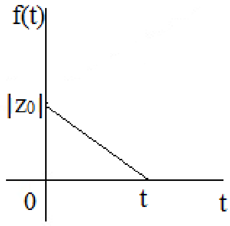
(Figure-1)

1.2. If $k=0$, then $f(t,a,k,\alpha,\beta)=|z_0|$ is constant function (Fig-2)



(Figure-2)

1.3. If $k<0$, then the function (5) is decreasing and it equals to zero at the time $t^*=-\dfrac{1}{k}$ (Fig-3)

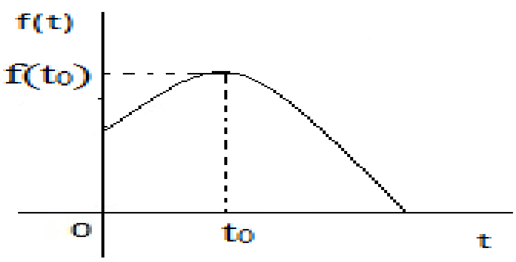


(Figure-3)

2. Let be $\alpha>\beta$.

2.1. If $k>0$, then the function (5) is equal to zero at the time

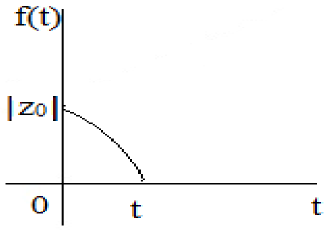
$$T_> = \left(|z_0|k+\sqrt{|z_0|^2 k^2+2|z_0|(\alpha-\beta)}\right)/(\alpha-\beta) \quad \text{(Fig-4)}$$



(Figure-4)

Maximal value of the function (5) is $f(t_0)=\left(2|z_0|(\alpha-\beta)+|z_0|^2 k^2\right)/2(\alpha-\beta)$ at moment $t_0=|z_0|k/(\alpha-\beta)$.

2.2. If $k<0$, then the function (5) decreases monotonically and this function turns to zero at time $T_<$ as in the case 2.1 (Fig-5)



(Figure-5)

2.3. If $k=0$, then $f(t,a,k,\alpha,\beta)=a-\frac{t^2}{2}(\alpha-\beta)$ and the pursuit time equals to $T_0=\sqrt{\frac{2|z_0|}{\alpha-\beta}}$.

In conclusion, the relation (3) is true at some time t^* according to the inequality $|z(t)|\le|z_0|(kt+1)+t^2(\beta-\alpha)/2$ and properties of (5), and it is determined that a relation $t^*\le T$ is correct, i.e., the pursuit problem is solved, which completes the proof of the Theorem.

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