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THE PROBLEM OF OBSERVING THE DIFFUSION PROCESS.

Mahammadi Jabborovich Rustamov
Professor of Jizzakh branch of the Uzbek State National University named after Mirzo Ulugbek, rustamov@mail.ru

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Rustamov Mahammadi Jabborovich.

Professor of Jizzakh branch of the Uzbek State National University named after Mirzo Ulugbek

E-mail address: rustamov@mail.ru

Abstract: The article discusses the determination of concentration in the diffusion process by observing a change in concentration at a particular point in the body. Using the principle of dualism, the problems of control and observation are reduced to the solution of question conditional extremum problems.

Keywords: recovery, diffusion, concentration dualism, control, observation, extremum, rate of change, measurement, equation, point, inverse (compound) operator.

INTRODUCTION

In recent decades, the theory of control and observation in dynamic systems has received great development [2,4,7,9,17,44,52,21,24-27,35,38,40,43] which, in essence, is also the theory of specific inverse problems (in relation to classical initial-boundary problems). Well-developed qualitative and numerical methods for this theory, especially in linear systems [4,6]. The combination of these directions gives a powerful arsenal of methods and studies of inverse problems. However, the issue of constructing efficient computational procedures, as a rule, can be solved only in a narrow class of models, taking into account all the features of finite connections, according to which the specified characteristics are studied (restored, estimated). In this sense, the problem of developing efficient methods for solving inverse problems remains relevant.

1. **Determine the concentration in the diffusion process by observing the change in concentration at a point.**

Let the diffusion process take place in the direction of thickness between plates of infinite length in space (if the distance between them is S = 1). (S). In this case, the process can be viewed along a rod located orthogonal to the plates. Let the concentration at S be expressed by the function T (x, t) over time t. In this case (0≤x≤1,0≤t≤t $^{\rightarrow}$), t $^{\rightarrow}$-fixed point. In this case, the function T (x, t) at t> 0 and [0,1]

$\frac{\vartheta^2 T(x,t)}{\vartheta T} = a\frac{\partial^2 T(x,t)}{\partial x^2}$; (x, t)∈Π (1)

Subject to the equation. Where P = ((0,1) x (0, t $^{\rightarrow}$)) a is the diffusion coefficient. The following diffusion condition is considered at the end of the rod.

$$\mu\frac{\partial T(1,t)}{\partial x} = \alpha[U(t) - T(1,t)],\ t \in [0,\vec{t}],$$

$$\mu\frac{\partial T(0,t)}{\partial x} = 0, \qquad t \in [0,\vec{t}]$$

Here μ is the coefficient of humidity, α is the coefficient of proportionality between humidity and the external environment. The concentration of the external environment is called the regulatory effect or control. For equations (1) and (2) to have a solution, the initial T (x, 0) or final T (x, t $^{\rightarrow}$) diffusion state must also be known (single-valued). However, these quantities cannot always be determined using measuring instruments.

Suppose that in the process of diffusion it is possible to determine the state of diffusion at some points of the rod x = x $^{\rightarrow}$∈ (0,1], T (x $^{\rightarrow}$, t) according to the change in the point x = (x) $^{\rightarrow}$ and using the diffusion law (1) - (2) Determining the state of diffusion at time t $^{\rightarrow}$ is the main problem (problem to be determined). y (t) diffusion

$T(\vec{x},t) = y(t);\ \ t \in [0,\vec{t}]$ (3)

diffusion is called the measured magnitude.

Problem - 1. To determine the function T (x, t $^{\rightarrow}$), x∈ [0,1] through the constants α, γ, μ - y, function y (t) and (1) - (3).

Let q (x) be a function (x) ∈S ^ 1 (0,1).

Issue - 2. In terms of the first issue

$Z_q = \int_0^1 q(x) \cdot T(x,\vec{t})dx$ (4)

find the size. The type of problem solution $q(x) = q_i(x), i = (1,2,\dots,h,..)$ allows you to find T (x, t $^{\rightarrow}$) using the base L_2 (0,1) functions ((4) using projection). For this reason, we will only consider issue 2 in the queue. Consider the case x $^{\rightarrow}$ = 1.

2. **Projection simulation (identification).**

Assuming 0 <x $^{\rightarrow}$ <1, we choose (4) satisfying the expressions (1) - (3) in the following form:

$$Z_q = \int_0^{\vec{t}} [k(t)y(t) + \varphi(t)u(t)]dt, \quad (5)$$

where k (t) and φ (t) are hitherto unknown functions derived from L_2 (0, t $^{\rightarrow}$). According to the technique of the theory of observation in linear (equations) [2,3], we equate the functional (K, φ) so that the following equation is satisfied in the expressions (1) - (3).

$$\int_0^1 q(x) \cdot T(x,\vec{t})dx = \int_0^{\vec{t}} [k(t)y(t) + \varphi(t)u(t)]dt \quad (6)$$

In the solutions of (1) we obtain the following identity.

$$\int_0^1 \int_0^{\vec{t}} \psi(x,t) \cdot \left[\frac{\partial T(x,t)}{\partial x} - a\frac{\partial^2 T(x,t)}{\partial \alpha^2}\right] dxdt \equiv 0 \quad (7)$$

Where ψ (x, t) is an arbitrary function. $\psi \in C_{t,x}^{1,2}(\Pi)$

$$\Pi = \{([0,x] \cdot [0,\vec{t}]) \cup ([\vec{x},1] \cdot [0,\vec{t}])\}.$$

(6) - we add the expression (7) and integrate (2), (3) into a fraction. The resulting expression looks like this.

$$Z_q = \int_0^1 q(x) \cdot T(x,\vec{t})dx = \int_0^F K(t) \cdot T(\vec{x},t)dx + \int_0^F \varphi(t)u(t)\,dt -$$
$$\int_0^F \frac{a\alpha}{\lambda}\psi(1,t)\alpha u(t)dt - \int_0^1 \varphi(x,0) \cdot T(x,0)dx - a\int_0^F \frac{\partial\psi(0,t)}{\partial x} T(0,t)dt -$$
$$a\int_0^F \left(\frac{\partial\psi(1,t)}{\partial x} + \frac{\alpha}{\lambda}\varphi(1,t)\right) \cdot T(1,t)dt + \int_0^1 \psi(x,t) \cdot T(x,\vec{t})dx - \int_0^1 \int_0^F \left[\frac{\partial\psi}{\partial t} + a\frac{\partial^2\psi}{\partial x}\right] \cdot$$
$$T(x,t)dxdt$$

(8)

(8) We equate the coefficients before T (x, t). The result is the following system for φ (x, t).

$$\frac{\partial\psi(x,t)}{\partial t} + a\frac{\partial^2\psi(x,t)}{\partial x^2} = 0 \quad (x,t) \in \Pi(9)$$

$\psi(x,0) = 0, \quad (10); \ \psi(x,\vec{t}) = 0, x \in [0,1] \ (11)$

$\frac{\partial\psi(0,t)}{\partial\alpha} = 0, \ t \in [0,\vec{t}], \ (12)$

$\frac{a\alpha}{x}\psi[1,t] + \frac{\partial\psi(1,t)}{\partial\alpha} = K(t), \ t \in [0,\vec{t}], \ (13)$

Thus a boundary value problem (9) - (13) is formed for φ (x, t). Let y be the solution of y to some K (t). In this case, expression (8) takes the following form:

$$0 = \int_0^F u(t) \cdot \left[\varphi(t) - \frac{a\alpha}{\mu}\psi(1,t)\right] dx$$

From this we conclude that the expression (6) in the functions satisfying (1) - (3) is arbitrary for u (t).

$\varphi(t) = \frac{at}{\mu}\psi(1,t)$ (14)

the fulfillment of the equation is sufficient.

Theorem: (14) is derived from the boundary solution (9) - (8) so that the satisfying expression of equation (1) - (3) is reasonable.

3. **Computational aspect.**

(9) - (15) Solve the problem. Let it be known that the solution of the system (1) - (2) depends on T (x, t) M∈L. L- Linear set in Y_2 (P). u (t) - let the control function be known. Let us take some known functions φ (t) and K (t). Let them approximate the boundary condition (9) - (11), (14), (15). In that case, let there be the following differences.

$$\frac{\partial\tilde{\psi}}{\partial t} + a\frac{\partial^2\tilde{\psi}}{\partial x^2} = \tau(x,t), \quad (x,t) \in \Pi$$

$$\tilde{\psi}(x,0) = \tau_0(x), \quad x \in [0,1]$$

$$\psi(x,\vec{t}) = \tau(x), \qquad x \in [0,t],$$

$$\frac{\partial\psi(0,t)}{\partial x} = \tau^{(0)}, \qquad t \in [0,\vec{t}],$$

$$\frac{a\alpha}{\lambda}\psi(1,t) + \frac{\partial\psi(1,t)}{\partial x} - K(t) = \tau^{(1)}(t), \qquad t \in [0,t]$$

In this view (x, t), K̃ (t) in this view, (16) has the following error according to formula (8).

$$R(\tilde{\varphi},K,T)=\int\int\tau(x,t)dxdt+\int_0^{\vec{t}}\tau_0(x)dt+\int_0^{\vec{t}}\tau_1(t)dt+\int_0^{\vec{t}}\tau^{(1)}(t)+\int_0^{\vec{t}}\tau^0(t)dt$$

(18)

Thus, to increase the accuracy of formula (6), it is necessary to minimize the magnitudes R $(\tilde{\varphi},\tilde{K})$ by choosing $\tilde{\varphi},\tilde{K}$.

The practical way to minimize this value is to have a set of continuous functions at L = Y_2 (P) and M – P. They are continuous.

$$\frac{\partial T(\tilde{x},t)}{\partial x},t\in[0,t]$$

have a product and satisfy (19). In this case q_i> 0 is the coefficient of weight. Then the error (18) is equal to the following

$$R(\tilde{\psi},\tilde{K})=\bar{\bar{C}}\sqrt{J} \quad (20)$$

minimizing these functions by $\tilde{\varphi}$, K̃ (20)

$$\tilde{K}=\sum_{i=1}^{n}K_i(t)\alpha_i, \qquad \tilde{\psi}=\sum_{j=1}^{n}\psi_i(x)\alpha_j$$

we minimize that.

The choice of the method of minimizing this assessment in practice depends on the M and L sets. M, - let in the following view.

$$M=\left\{T(x,t);T(x,t)\in L_2(\Pi),\ \ \|\Gamma\|_\rho^2\le\bar{T}^2\right\} \qquad (19)$$

Here $\|T\|_\rho^2=\rho_0\iint_\Pi T^2(x,t)d\times dt\,\rho_1\int_0^1T^2(x,0)dx+\rho_2\int_0^1T^2(x,\bar{t})dx+$

$$+\rho_3\int_0^{\bar{t}}T^2(0,t)dt+\rho_4\int_0^{\bar{t}}T^2(1,t)dt+\rho_5\int_0^{\bar{t}}T^2(\bar{x},t)dt+\rho_6\int_0^{\bar{t}}\left[\frac{\partial T(\bar{x},t)}{\partial x}\right]^2dt+$$

$$+\rho_7\int_0^{\bar{t}}\left[\frac{\partial T(\bar{\bar{x}},t)}{\partial x}\right]^2dt+\rho_8\int_0^{\bar{t}}T^2(\bar{\bar{x}},t)dt.$$

In this case, there are integrals in the specified limits for the function T (x, t). In this case (16) the value of the expression according to the Koshi-Bunyakovsky inequality (17) is as follows. We express the functions $\tilde{K}(\cdot)$ and $\tilde{\psi}$ as follows.

$$\tilde{K}(t) = \sum_{i=1}^{n} \tilde{K}_i(t), \quad \tilde{\psi}(x,t) = \sum_{i=1}^{n} \widetilde{\psi_1}(x,t), \qquad g(t) = \sum_{i=1}^{n} \alpha_1 g_i(t), \quad (21)$$

$$R(\tilde{\psi},\tilde{K}) = \bar{T}^2 \Big(\frac{1}{\rho_0} \iint_{\Pi} r^2(x,t)\,dxdt + \frac{1}{\rho_1}\int_0^1 r_0^2(x)dx + \frac{1}{\rho_2}\int_0^1 r_1(x)dx +$$

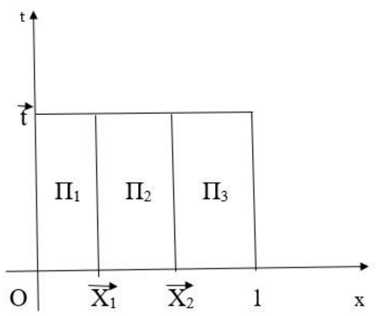
$$+\frac{1}{\rho_3}\int_0^{\bar{t}} r^{(0)^2}(t)dt + \frac{1}{\rho_4}\int_0^{\bar{t}} r^{(1)^2}(t)dt + \frac{1}{\rho_5}\int_0^{\bar{t}} r_2^2(t)dt + \frac{1}{\rho_6}\int_0^{\bar{t}} r^{(2)^2}(t)dt +$$

$$+\frac{1}{\rho_7}\int_0^{\bar{t}} r^{(3)^2}(t)dt + \frac{1}{\rho_8}\int_0^{\bar{t}} r^{(4)^2}(t)dt)^{1/2} \qquad (20)$$

In this case $\{\tilde{K}_i(t)\}$, $\{\tilde{\psi}_i(x,t)\}$, $\{g_i(t)\}$ is a system of basic functions.

To solve the functions $\tilde{\psi}_i(x,t)$, $\tilde{K}_i(t)$, $g_i(t)$ we solve the system (8), (10) - (11), (14). To do this, use (1) $\Pi_1 = (0,\bar{x}) \times (0,\bar{t})$ and $\Pi_2 = (\bar{x},\bar{\bar{x}}) \times (0,\bar{t})$ $\Pi_3 = (\bar{\bar{x}},1) \times (0,\bar{t})$ we solve in right rectangles.

$$\tilde{\psi}(x,t) = \begin{cases} (D\cos\omega x + \bar{D}\sin\omega x)e^{a\omega_1^2 t}, & \Pi_1 \quad \text{да} \\ (D_1\cos\omega x + D_2\sin\omega x)e^{a\omega_2^2 t}, & \Pi_2 \quad \text{да} \\ (D_1^1\cos\omega x + D_2^1\sin\omega x)e^{a\omega_3^2 t}, & \Pi_3 \quad \text{да} \end{cases}$$



We apply the condition (10) on the line x = 0. In this case (22) we apply the condition (11) on the line D¯= 0 x = 1. In this case, the following condition is formed in solutions (22).

$$\frac{a\alpha}{\mu}(D_1^1 \cos\omega + D_2 \sin\omega) = \omega(D_1^1 \sin\omega - D_2^1 \cos\omega) \qquad (23)$$

ψ (x, t) solution Π_1 and Π_2 on the line x = x¯in right-angled rectangles, (14) under the condition of continuity

$$D\cos\omega\bar{x} = D_1\cos\omega\bar{x} + D_2\sin\omega\bar{x} \qquad (24)$$

In turn, we coordinate the solutions (23) and (24) on the line x = x¯at Π_2 and Π_3.

$$D_1\cos\omega\bar{\bar{x}} + D_2\sin\omega\bar{\bar{x}} = D_1^1\cos\omega\bar{\bar{x}} + D_2^1\sin\omega\bar{\bar{x}} \qquad (25)$$

From here (1) the following solution is obtained.

$$\psi(x,t) = \begin{cases} D\cos\omega x \cdot e^{a\omega^2 t}, & (x,t) \in \Pi_1, \\ (D_1\cos\omega x + D_2\sin\omega x)e^{a\omega^2 t}, & (x,t) \in \Pi_2, \\ (D_1^1\cos\omega x + D_2^1\sin\omega x)e^{a\omega^2 t}, & (x,t) \in \Pi_3 \end{cases} \qquad (26)$$

Here D, D_1, D_2, D_1 ^ 1, D_2 ^ 1, ω must satisfy the following equations.

$$\psi(x,t) \begin{cases} D\cos\omega\bar{x} = D_1\cos\omega\bar{x} + D_2\sin\omega\bar{x}, & \Leftarrow (14, 26_{1,2}) \\ D_1\cos\omega\bar{\bar{x}} + D_2\sin\omega\bar{\bar{x}} = D_1^1\cos\omega\bar{\bar{x}} + D_2^1\sin\omega\bar{\bar{x}}, & \Leftarrow (14, 26_{1,3}) \\ \frac{a\alpha}{\mu}(D_1^1\cos\omega + D_2^1\sin\omega) = \omega(D_1^1\sin\omega - D_2^1\cos\omega) & \Leftarrow (11, 26_2) \end{cases} \qquad (27)$$

From these solutions of (1) we construct the functions K (t) and g (t) according to the conditions (12) and (13).

$$\begin{cases} K(t) = a\omega \cdot [(D_1 - D)\sin\omega\bar{x} - D_2\cos\omega\bar{x}], \\ g(t) = a\omega \cdot [(D_1^1 - D)\sin\omega\bar{\bar{x}} + (D_2 - D_2^1)\cos\omega\bar{\bar{x}}]. \end{cases} \qquad (28)$$

(28) system solutions and

$$g(D,\omega) = (D_1^1 - D)\sin\omega\bar{\bar{x}} + (D_2 - \bar{D}_2)\cos\omega\bar{\bar{x}} \neq 0 \qquad (29)$$

A set of conditional variables

(D ^ ((2)), D_1 ^ ((2)), D_2 ^ ((2)), D_1 ^ 1 (1), D_2 ^ 1 (2), ω_i).

We define the corresponding functions (26), (28) as follows.

$$\widetilde{\psi_1}(x,t) = e^{a\omega^2 it} \cdot \eta_i(x), \quad K_i(t) = e^{a\omega^2 i} \cdot \eta(\bar{x}),$$

$$g_i(t) = e^{a\omega^2 i} \cdot \eta_i(\bar{\bar{x}}) \qquad (30)$$

We put these functions in the sum of (21). Thus, by choosing ψ (x, t) in the form (26), we satisfied the equations (8), (10) (14). In that case, if we put (21) and (30) in (20), the differences other than $\eta_0(x)$ and η_1 become zero. With these differences we write the equation error (6).

$$R^2 = \overline{T}^2 \left\{ \frac{1}{\rho_1} \left(\int_0^{\bar{x}} \left[\sum_{i=1}^{n} \alpha_i D^{(i)} \cdot \cos \omega i x \right]^2 dx + \int_{\bar{x}}^{\bar{\bar{x}}} \left[\sum_{i=1}^{n} \alpha_i \left(D_1^{(i)} \cdot \cos \omega_i x + \right. \right. \right. \right.$$

$$\left. +D_2^{(i)}, \sin \omega_i x) \right]^2 dx + \int_{\bar{\bar{x}}}^{1} \left[\sum_{i=1}^{n} \alpha_i \left(D_1'^{(i)} \cdot \cos \omega_i x + \right. \right.$$

$$\left. +D_2'^{(i)} \cdot \sin \omega_i x) \right]^2 dx) + \frac{1}{\rho_i} \left(\int_0^{\bar{x}} \left[\sum_{i=1}^{n} \alpha_i D^{(i)} \cdot \gamma_i \cos \omega_i x \right]^2 dx + \int_{\bar{x}}^{\bar{\bar{x}}} \left[\sum_{i=1}^{n} a_i \gamma_i \right. \right.$$

$$\left(D_1^{(2)} \cdot \cos \omega_i x + D_2^{(i)} \cdot \sin \omega_i x) \right]^2 dx +$$

$$\int_{\bar{\bar{x}}}^{1} \left[\sum_{i=1}^{n} \alpha_i \gamma_i \left(D_1'^{(i)} \cdot \cos \omega_i x + D_2^{(i)} \cdot \sin \omega_i x) \right]^2 dx) \right\}$$

Here $\gamma_i = \exp\left(a \omega_i^2 \bar{t}\right)$ We minimize this error.

Its sufficient condition

$$\frac{\partial R^2 (\tilde{\psi}^{(n)}), \tilde{K}^{(n)}}{\partial \alpha_j} = 0$$

$$\frac{\partial R^2 (\tilde{\psi}^{(n)}, \tilde{K}^{(n)})}{\partial \alpha_j} = 2\bar{T}^2 \sum_{i=1}^{n} \alpha_i \left(\frac{1}{\rho_1} + \frac{\gamma_i \cdot \gamma_j}{\rho_2} \right) \cdot \left[D^{(i)} D^{(j)} \int_0^{\bar{x}} \cos \omega_i x \cdot \cos \omega_j x dx + \right.$$

$$+ \int_{\bar{x}}^{\bar{x}} \left(D_{,}^{(i)} \cos \omega_i x + D_2^{(i)} \sin \omega_i x \right) \cdot \left(D_{,}'^{(j)} \cdot \cos \omega_j x + D_2^{(j)} \cdot \sin \omega_j x \right) dx +$$

$$\left. + \int_{\bar{x}}^{1} \left(D_{,}'^{(i)} \cos \omega_i x + D_2'^{(j)} \cdot \sin \omega_i x \right) \cdot \left(D_{,}'^{(i)} \cos \omega_j x + D_2'^{(j)} \cdot \sin \omega_j x \right) dx \right] \equiv 0$$

This leads to a linear equation that must be solved.

Here T (x, t) - (1) in the solution of the heat dissipation equation $C \cdot \cos \omega_j \bar{\bar{x}}$ is not observed. It is zero. $\cos \omega_j \bar{x} = 0$

In this case, K_2 (t) and ph_2 (x, t) are basic functions. That is, $K_2(t)$ and $\varphi_2(x,t)$are generalized polynomials. We replace the problem of minimization (18) with the extremum of the real variable α_i, β_iни $m+n$ function.

$\min J\ (\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n, \beta_1, \beta_2, \beta_3, \dots, \beta_m,)$

We solve this problem for $m = n, \alpha_i\ = \beta_i$. By separating the variables in the solution of (9), we see the following functions.

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