EVASION PROBLEM FOR THE SECOND ORDER DIFFERENTIAL GAME

Ulmasjon Boykuzi o'g'li Soyibboev
NamSU masters at the Department of differential equation main teacher at the department of applied mathematics and informatics

Nargiza Tashkinbaevna Umaralieva
NamSU teacher at the department of applied mathematics and informatics

Sabohat Ismoiljon qizi Uralova
NamSU masters at the Department of differential equation and mathematics-physics

Follow this and additional works at: https://uzjournals.edu.uz/namdu

Part of the Physical Sciences and Mathematics Commons

Recommended Citation
Soyibboev, Ulmasjon Boykuzi o'g'li; Umaralieva, Nargiza Tashkinbaevna; and Uralova, Sabohat Ismoiljon qizi (2019) "EVASION PROBLEM FOR THE SECOND ORDER DIFFERENTIAL GAME," Scientific Bulletin of Namangan State University: Vol. 1 : Iss. 4 , Article 2.
Available at: https://uzjournals.edu.uz/namdu/vol1/iss4/2

This Article is brought to you for free and open access by 2030 Uzbekistan Research Online. It has been accepted for inclusion in Scientific Bulletin of Namangan State University by an authorized editor of 2030 Uzbekistan Research Online. For more information, please contact brownman91@mail.ru.
EVASION PROBLEM FOR THE SECOND ORDER DIFFERENTIAL GAME

Cover Page Footnote
???????

Erratum
???????

This article is available in Scientific Bulletin of Namangan State University: https://uzjournals.edu.uz/namdu/vol1/iss4/2
EVASION PROBLEM FOR THE SECOND ORDER DIFFERENTIAL GAME
Soyibboev Ulmasjon Boykuzi o‘g’li
Umaralieva Nargiza Tashkinbaevna
Uralova Sabohat Ismoiljon qizi
NamSU masters at the Department of differential equation and mathematics-physics and main teacher at the department of applied mathematics and informatics

Abstract: In this paper, we study the evasion problem for the second order differential game when the initial positions of moving objects are linearly dependent and controls of the players have geometric constraints. Here the new sufficient solvability conditions for evader will be proposed.

Keywords: Differential game, acceleration, geometric constraint, evader, pursuer, initial positions, strategy.

IKKINCHI TARTIBLI DIFFERENSIAL O‘YIN UCHUN QOCHISH MASALASI
Soyibboyev O‘Imasjon Boyko‘zi o‘g‘li
Umaraliyeva Nargiza Tashkinbayevna
Uralova Saboxat Ismoiljonqizi
NamDU Differensial tenglamalar va matematik fizika kafedrasi magistrantlari va Amaliy matematika va informatika kafedrasi katta o‘qituvchisi


Kalit so‘zlar: Differensial o‘yin, tezlanish, geometrik chegaralanish, qochuvchi, quvlovchi, boshlang‘ich holat, strategiya.

ЗАДАЧА УБЕГАНИЯ ДЛЯ ДИФФЕРЕНЦИАЛЬНЫХ ИГР ВТОРОГО ПОРЯДКА
Сайиббаев Улмасжон Байкузи угли
Умаралиева Наргиза Ташкинбаевна
Уралова Сабохат Исмоилжон кизи,
магистранты NamGU кафедры Дифференциальная уравнения и математической физики учитель кафедры Прикладного математики и информатики

Аннотация: В настоящей работе изучается задача убегания для дифференциальных игр второго порядка, когда начальные состояния и начальные скорости игроков линейно зависимости при геометрических ограничениях науправления. Получены новые достаточные условия разрешимости для задачи убегания.

Ключевые слова: дифференциальная игра, ускорения, геометрическое ограничение, преследователь, убегающий, начальные состояния, стратегия.

Let $P$ and $E$ objects with opposite aim be given in the space $R^n$ and their movements are based on the following differential equations and initial conditions
\[ P: \dot{x} = u, \quad x_1 - kx = 0, |u| \leq \alpha \tag{1} \]
\[ E: \dot{y} = v, \quad y_1 - ky = 0, |v| \leq \beta \tag{2} \]
where \( x, y, u, v \in \mathbb{R}^n \); \( x \) – a position of \( P \) object in the space \( \mathbb{R}^n \), \( x_0 = x(0) \), \( x_1 = \dot{x}(0) \) – its initial position and velocity respectively at \( t = 0 \); \( u \) – a controlled acceleration of the pursuer, mapping \( u: [0, \infty) \rightarrow \mathbb{R}^n \) and it is chosen as a measurable function with respect to time; we denote a set of all measurable functions \( u(\cdot) \) such that satisfies the condition \( |u| \leq \alpha \) by \( U \).
\( y \) – a position of \( E \) object in the space \( \mathbb{R}^n \), \( y_0 = y(0) \), \( y_1 = \dot{y}(0) \) – its initial position and velocity respectively at \( t = 0 \); \( v \) – a controlled acceleration of the evader, mapping \( v: [0, \infty) \rightarrow \mathbb{R}^n \) and it is chosen as a measurable function with respect to time; we denote a set of all measurable functions \( v(\cdot) \) such that satisfies the condition \( |v| \leq \beta \) by \( V \).

**Definition 1.** For a trio of \((x_0, x_1, u(\cdot)), u(\cdot) \in U\), the solution of the equation (1), that is, \( x(t) = x_0 + x_1 t + \int_0^t u(\tau)d\tau ds \) is called a trajectory of the pursuer on interval \( t \geq 0 \).

**Definition 2.** For a trio of \((y_0, y_1, v(\cdot)), v(\cdot) \in V\), the solution of the equation (2), that is, \( y(t) = y_0 + y_1 t + \int_0^t v(\tau)d\tau ds \) is called a trajectory of the evader on interval \( t \geq 0 \).

**Definition 3.** For differential game (1) – (2), an evasion problem is said to be held however, the pursuer chooses any control function \( u(\cdot) \in U \), if there exists \( v^*(\cdot) \in V \) for the evader and the following condition is true for the trajectories \( x(t), y(t) \) that is found according to those control functions [4]:
\[ x(t) \neq y(t), (t \geq 0) \tag{3} \]

To solve the evasion problem we will propose a strategy of the evader as follows:

**Definition 4.** In differential game (1) – (2) we call the strategy of the evader the following function [1]:
\[ v^*(t) = -\frac{z_0}{|z_0|} \beta, t \geq 0 \tag{4} \]
where \( z_0 = x_0 - y_0 \neq 0 \).

**Theorem.** If one of the following conditions holds:
1. \( \alpha = \beta \) and \( k \geq 0 \); or
2. \( \alpha < \beta \) and \( k \in \left( -\frac{2(\beta - \alpha)}{|z_0|}, +\infty \right) \),

then for differential game (1) – (2), the evasion problem is solved by the strategy of the evader (4) and a change function between the objects will be in the following form:
\[
f(t, k, \alpha, \beta, z_0) = \begin{cases} 
|z_0|kt + |z_0|, & \text{if } \alpha = \beta, \\
\frac{\beta - \alpha}{2}t^2 + |z_0|kt + |z_0|, & \text{if } \alpha < \beta.
\end{cases}
\]

**Proof.** Suppose the pursuer chooses any control function \(u(\cdot) \in U\) and the evader chooses the control function (4). Then according to (1) – (2) we have the following solutions:

\[
x(t) = x_0 + tx_1 + \int_0^t u(\tau)d\tau ds,
\]

\[
y(t) = y_0 + ty_1 + \int_0^t y^*(\tau)d\tau ds.
\]

Now write their distinction function:

\[
z(t) = x(t) - y(t) = z_0 + z_1t + \int_0^t u(\tau)d\tau ds + \int_0^t z_0\beta d\tau ds,
\]

where \(z_1 = \dot{x}(0) - \dot{y}(0)\). If we subtract the initial conditions, we form a relation \(z_1 = k z_0\).

From this, we have the following equality:

\[
z(t) = z_0(kt + 1) + \frac{z_0}{|z_0|} \frac{\beta t^2}{2} + \int_0^t u(\tau)d\tau ds
\]

Evaluate the absolute value of this function from low:

\[
|z(t)| \geq \left| z_0(kt + 1) + \frac{z_0}{|z_0|} \frac{\beta t^2}{2} \right| - \int_0^t \left| u(\tau)d\tau ds \right| \geq \left| z_0 \right|(kt + 1) + \left( \beta - \alpha \right) \frac{t^2}{2}.
\]

We will consider as a parametric function the right side of the latest inequality:

\[
f(k, t, \alpha, \beta, z_0) = \left| z_0 \right|(kt + 1) + \frac{\beta - \alpha}{2} t^2. (5)
\]

Now we introduce some simplifications, i.e., \( |z_0| = a, \frac{\beta - \alpha}{2} = \gamma \). Therefore, the function (5) becomes in the following form:

\[
f(k, t, \gamma) = a + akt + \gamma t^2. (6)
\]

Now we will check the function (6) with respect to parameters \( \rho, \sigma \) and \( k \).

1. Let be \( \alpha = \beta \). Then \( f(k, t) = a + akt \) and we will analyze this function in respect of a sign of parameter \( k \):

   1.1. Let be \( k > 0 \). Consequently \( ak > 0 \), and there doesn’t exist a positive solution \( t \) which the function equals to zero. So the evasion holds. (Fig -1)
1.2. Let be $k = 0$. Thus $f(k, t) = a$ and a distance between the pursuer and evader don’t change. So the evasion holds. (Fig-2)

1.3. Let be $k < 0$. Then $ak < 0$. The function $f(t, k)$ has a positive solution $t = -\frac{1}{k}$. So the evasion doesn’t hold. (Fig-3)

2. Let be $\alpha < \beta$. Then $\gamma > 0$ in the function (6).

2.1. Let be $k < 0$. Then $t_0 = -\frac{ak}{2\gamma} > 0$ is a minimum approach point. In order to being the evasion held a discriminant $D = a^2k^2 - 4a\gamma$ must be negative, i.e.,

$$D = a^2k^2 - 4a\gamma < 0 \Rightarrow k^2 < \frac{4\gamma}{a} \Rightarrow k \in \left(-\sqrt{\frac{2(\beta - \alpha)}{|z_0|}}, \sqrt{\frac{2(\beta - \alpha)}{|z_0|}}\right).$$

If we unite the latest interval with the interval $k < 0$, then the evasion holds on interval $k \in \left(-\sqrt{\frac{2(\beta - \alpha)}{|z_0|}}, 0\right).$ (Fig-4)
2.2. Let be $k = 0$. Then $f(t, \gamma) = a + \gamma t^2$ and there doesn’t exist a pursuit time because of $\gamma > 0$. Thus the evasion holds. (Fig-5)

2.3. Let be $k > 0$. Consequently, $t_0 < 0$ in the function (6). Hence there is no a positive solution $t$ which the function equals to zero. So the evasion holds. (Fig-6)

In conclusion, the relation (3) is true in all values of interval $t \geq 0$ according to the inequality $\left|z(t)\right| \geq f(k, t, \alpha, \beta, \left|z_0\right|)$ and properties of (5), that is, the evasion problem is solved, which completes the proof of the Theorem.

References: