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PURSUIT PROBLEM IN MOVEMENT WITH ACCELERATION FOR CONTROLS ON CONSTRAINT OF GRANWOLL TYPE

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Cover Page Footnote

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Erratum

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Abstract. *In this work is considered a differtial game of the second order, when control functions of the players satisfies geometric constraints. The proposed method substantiates the parallel approach strategy in this differential game of the second order. The new sufficient solvability conditions are obtained for problem of the pursuit.*

Keywords. *Differential game, geometric constraint, evader, pursuer, strategy of the parallel pursuit, acceleration.*

BOSHQARUVLARI GRONUOLL CHEGARALANISHGA EGA TEZLANISHLI HARAKATDA TUTISH MASALASI

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NamDU Differensial tenglamalar va matematik fizika kafedrasi va FarDU Matematik analiz kafedrasi magistrantlari

Annotatsiya. *Ushbu ma'ruzada boshqaruvlar Gronoull chegaralanishga ega holda ikkinchi tartibli differensial o'yinlar uchun tutish masalasi o'rganiladi. Bunda quvlovchi uchun parallel quvish strategiyasi quriladi va uning yordamida tutish masalasi uchun yetarli shartlar keltiriladi.*

Kalit so`zlar: *Differensial o'yin, geometrik chegaralanish, parallel quvish strategiyasi, quvlovchi, qochuvchi, tezlanish, Granoull chegaralanishli.*

ЗАДАЧА ПЕРЕХВАТА ПРИ ДВИЖЕНИИ СО УСКОРЕНИЕМ И С ОГРАНИЧЕНИЯМИ ГРОНУОЛЛА

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Аннотация. *В работе рассматривается дифференциальная игра второго порядка при ограничениях Гронуолла на управления игроков. При этом предлагается стратегия параллельного преследования для преследователя и при помощи этой стратегии решается задача преследования.*

Ключевые слова: *Дифференциальная игра, ограничение Гронуолла, стратегия параллельного преследования, преследователь, убегающий, ускорения.*

Let **P** and **E** objects with opposite aim be given in the space $\boldsymbol{R^n}$ and their movements are based on the following differential equations and initial conditions

$$\mathbf{P}:\ \ddot{x}=u,\ x_1-kx_0=0,\ |u(t)|^2\le\rho^2+2l\int_0^t|u(s)|^2\,ds,\ (1)$$

$$\mathbf{E}:\ \ddot{y}=v,\ y_1-ky_0=0,\ |v(t)|^2\le\sigma^2+2l\int_0^t|v(s)|^2\,ds,\ (2)$$

where $x,y,u,v\in\boldsymbol{R^n}$; x – a position of **P** object in the space $\boldsymbol{R^n}$, $x_0=x(0),\ x_1=\dot{x}(0)$ – its initial position and velocity respectively at $t=0$; u – a controlled acceleration of the pursuer, mapping $u:[0,\infty)\to\boldsymbol{R^n}$ and it is chosen as a measurable function with respect to t; we denote a set of all measurable functions $u(\cdot)$ such that satisfies the condition $|u(t)|^2\le\rho^2+2l\int_0^t|u(s)|^2\,ds$ by G_P. y – a position of **E** object in $\boldsymbol{R^n}$ space, $y_0=y(0),\ y_1=\dot{y}(0)$ – its initial position and velocity respectively at $t=0$; v – a controlled acceleration of the evader, mapping $v:[0,\infty)\to\boldsymbol{R^n}$ and it is chosen as a measurable function with respect to t; we denote a set of all measurable functions $v(\cdot)$ such that satisfies the condition $|v(t)|^2\le\sigma^2+2l\int_0^t|v(s)|^2\,ds$ by G_E.

Definition 1. For a trio of $(x_0,x_1,u(\cdot)),u(\cdot)\in G_P$, the solution of the equation (1), that is, $x(t)=x_0+x_1t+\int_0^t\int_0^s u(\tau)d\tau ds$ is called a trajectory of the pursuer on interval $t\ge0$.

Definition 2. For a trio of $(y_0,y_1,v(\cdot)),v(\cdot)\in G_E$, the solution of the equation (2), that is, $y(t)=y_0+y_1t+\int_0^t\int_0^s v(\tau)d\tau ds$ is called a trajectory of the evader on interval $t\ge0$.

Definition 3. The pursuit problem for the differential game (1) - (2) is called to be solved if there exists such control function $u^*(\cdot)\in \boldsymbol{G_p}$ of the pursuer for any control function $v(\cdot)\in \boldsymbol{G_E}$ of the evader and the following equality holds at some finite time t^*

$$x(t^*)=y(t^*). \quad (3)$$

Definition 4. For the problem (1)-(2), time T is called a guaranteed pursuit time if it is equal to an upper boundary of all the finite values of pursuit time t^* which satisfies the equality (3).

Definition 5. For the differential game (1) - (2), the following function is called П-strategy of the pursuer ([3]-[4]):

$$u(v)=v-\lambda(v)\xi_0, \quad (4)$$

where $\lambda(v)=(v,\xi_0)+\sqrt{(v,\xi_0)^2+\delta e^{2lt}}$, $\xi_0=\frac{z_0}{|z_0|}$, $\delta=\rho^2-\sigma^2\geq 0$,

(v,ξ_0) is the scalar product of vectors v and ξ_0 in the space $\boldsymbol{R}^n$.

Lemma 1 (Gronwall). Suppose a mapping $\varphi(t):[0,\infty)\to \boldsymbol{R}^n$ is bounded, non-negative and measurable function. Moreover, $l\geq 0$ and $\rho>0$ are constant and for the given if an inequality $|\varphi(t)|^2\leq \rho^2+2l\int_0^t|\varphi(s)|^2\,ds$ holds, then a relation $\varphi(t)\leq \rho e^{lt}$ is always true.

Lemma 2 . If $\rho\geq\sigma$, then the following inequality is true for the function $\lambda(v,\xi_0)$:

$$e^{lt}(\rho-\sigma)\leq\lambda(v,\xi_0)\leq e^{lt}(\rho+\sigma).$$

Theorem. If for the second order differential game (1) – (2) with Gronwall constraint a condition $\rho>\sigma$ is true, then the pursuit problem is solved by П-strategy (4) on interval $(0,t)$ and an approach function between the objects becomes as follows:

$$f(l,t,|z_0|,\rho,\sigma,k)=|z_0|(kt+1)-\frac{\rho-\sigma}{l^2}e^{lt}+\frac{\rho-\sigma}{l^2}+\frac{\rho-\sigma}{l}t$$

Proof. Suppose the pursuer chooses a strategy in the form (4) when the evader chooses any control function $v(\cdot)\in\boldsymbol{G_E}$. Then according to the equations (1) and (2) we define the following Caratheodory's equation

$$\ddot{z}=-\lambda(v(t))\xi_0, \qquad \dot{z}(0)-kz(0)=0,$$

Hence the following solution will be found by the given initial conditions

$$z(t) = z_0(kt+1) - \xi_0 \int_0^t \int_0^s \lambda(v(\tau), \xi_0)\, d\tau ds$$

or

$$|z(t)| = |z_0|(kt+1) - \int_0^t \int_0^s \left((v, \xi_0) + \sqrt{(v, \xi_0)^2 + \delta e^{2lt}} \right) d\tau ds .$$

We form the following inequalities in relation to **Lemma 1**

$$|z(t)| \le |z_0|(kt+1) - \int_0^t \int_0^s e^{l\tau}(\rho - \sigma) d\tau ds \Rightarrow$$

$$|z(t)| \le |z_0|(kt+1) - \frac{\rho-\sigma}{l^2} e^{lt} + \frac{\rho-\sigma}{l^2} + \frac{\rho-\sigma}{l} t$$

We denote

$$f(l, t, |z_0|, \rho, \sigma, k) = |z_0|(kt+1) - \frac{\rho-\sigma}{l^2} e^{lt} + \frac{\rho-\sigma}{l^2} + \frac{\rho-\sigma}{l} t \quad . \ (5)$$

Define a positive solution t^* such that the function (5) equals to zero

$$\frac{\rho-\sigma}{l^2} e^{lt} = |z_0|(kt+1) + \frac{\rho-\sigma}{l^2} + \frac{\rho-\sigma}{l} t .$$

We will form the following equation by simplifying

$$e^{lt} = t\left(\frac{|z_0| k l^2}{\rho - \sigma} + l \right) + \frac{|z_0| l^2}{\rho - \sigma} + 1$$

where $A = \frac{|z_0| k l^2}{\rho - \sigma} + l$, $B = \frac{|z_0| l^2}{\rho - \sigma} + 1$, $B > 1$. Thus, we have the following equation

$$e^{lt} = At + B \quad (6)$$

In order to define a pursuit time we will consider some cases of the equation (5).

1. Let be $A < 0 \Rightarrow k < \frac{\sigma - \rho}{|z_0| l}$. Then the equation (5) has a unique positive solution t^* and this solution is a pursuit time. (Fig-1)

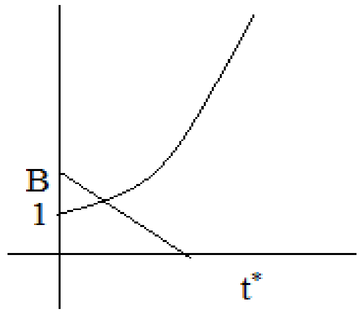

Figure-1

2. Let be $A = 0 \implies k = \dfrac{\sigma - \rho}{|z_0| l}$. Then a solution of the equation (5) is

$$t^* = \frac{\ln\left(\dfrac{|z_0| l^2}{\rho - \sigma} + 1\right)}{l}$$. (Fig-2)

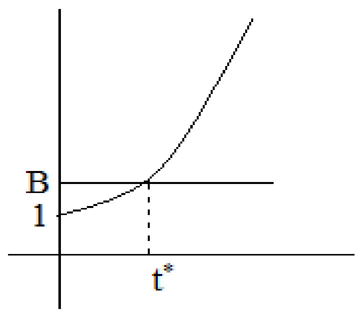

Figure-2

3. Let be $A > 0 \implies k > \dfrac{\sigma - \rho}{|z_0| l}$. Then the equation (5) has a positive solution t^* (Fig-3)

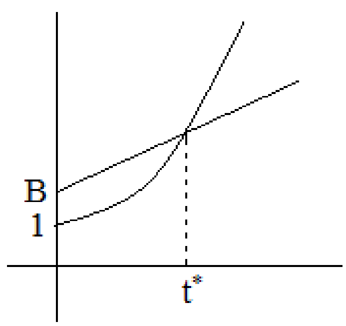

Figure-3

In conclusion, the relation (3) is true at some time t^* according to the inequality $|z(t)| \le f(k, t, \rho, \sigma, l, |z_0|)$ and properties of (5), and it is determined that a relation

$t^* \leq T$ is correct, i.e., the pursuit problem is solved, which completes the proof of the Theorem.

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