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APPROACH CALCULATION OF CERTAIN SPECIFIC INTEGRALS BY INTERPOLATING POLYNOMIALS

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BA'ZI ANIQ INTEGRALLARNI INTERPOLYATSION KO`PHADLAR YORDAMIDA TAQRIBIY HISOBLASH

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Annotatsiya: *Bu maqolada ba'zi aniq integrallarni hisoblashda integral ostidagi funksiyani ratsional funksiya ko`rinishda almashtirib, taqribiy hisoblash sonli usuli keltirib chiqarilgan. Bunday taqribiy hisoblashlar amaliy masalalarni yechishda muhim axamiyatga ega.*

***Kalit so`zlar:*** *singulyar koeffisientli integral, interpolyatsion ko`phad, taqribiy hisoblash, ratsional funksiya.*

ПРИБЛИЖЕННОЕ ВЫЧИСЛЕНИЕ НЕКОТОРЫХ ОПРЕДЕЛЕННЫХ ИНТЕГРАЛОВ С ПОМОЩЬЮ ИНТЕРПОЛЯЦИОННЫХ МНОГОЧЛЕНОВ

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Аннотация: *В этой статье приведён численный метод приближенных вычислений при вычислении некоторых определённых интегралов, заменяя под интегральную функцию в виде рациональной функции. Такие приближенные вычисления имеют важные значения при решения прикладных задач.*

Ключевые слова: *интегралы с сингулярными коеффициентами, интерполяционный полином, приближенный расчет, рациональной функцией.*

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Abstract: *In this study, the method of calculating the integrals was investigated and calculated integrals by inserting the rational functions. We also showed the method for calculating approximate. Such approximate calculations have important meanings when solving an applied problem.*

Keywords: *Integrals with Singular Coefficients, iterpolation Polynomial, Approximate Calculation, rational function.*

Bizga $f(x)\in C^1(a;b)$ va $(a;b)$ oraliqda o`suvchi hamda $f(0)=0$ shartlarni qanoatlantiruvchi funksiya berilgan bo`lsin. Quyidagi aniq integralni taqribiy hisoblashni ko`rib chiqamiz:

$$\int_a^b \frac{f(x)dx}{\left(x^{2m}+c\right)^p}, \qquad (1)$$

bunda $p>0$, $p\neq 1$, $p\neq 2$, $\forall m\in N$, $c>0$ bo`lsin [1].

Dastlab $(a;b)$ oraliqni n ta teng bo'lakka ajratamiz: $(a;b)=\bigcup_{i=0}^{n-1}(a_i;a_{i+1})$, bunda $a_i=a+ih$ – bo`linish nuqtalari, $h=\frac{b-a}{n}$ – oraliqlar uzunligi.

(1) ni quyidagicha yozish mumkin:

$$\int_a^b \frac{f(x)dx}{\left(x^{2m}+c\right)^p}=\sum_{i=0}^{n-1}\int_{a_i}^{a_{i+1}} \frac{f(x)dx}{\left(x^{2m}+c\right)^p}. \qquad (2)$$

(2) tenlikning o`ng tomonidagi $f(x)$ funksiyani ixtiyoriy $(a_i; a_{i+1})$ oraliqda

$$f(x)\approx p_i x^{4m-1}+q_i x^{2m-1} \qquad (*)$$

(*) ko`phad bilan almashtirish bajaraylik [3]. Bu yerda $p_i$ va $q_i$ ixtiyoriy o'zgarmas koeffitsiyentlar. $p_i$ va $q_i$ koeffisientlar musbat bo`lsa, x^{4m-1} va x^{2m-1} funksiyalar o`suvchi ekanligidan yuqoridagi almashtirish o`rinli bo`ladi. Bundan

$$\sum_{i=0}^{n-1}\int_{a_i}^{a_{i+1}} \frac{f(x)dx}{\left(x^{2m}+c\right)^p}\approx\sum_{i=0}^{n-1}\int_{a_i}^{a_{i+1}} \frac{p_i x^{4m-1}+q_i x^{2m-1}dx}{\left(x^{2m}+c\right)^p} \qquad (3)$$

ni olamiz. (3) da integralni ikki qismga ajratamiz:

$$\sum_{i=0}^{n-1}\int_{a_i}^{a_{i+1}} \frac{p_i x^{4m-1}+q_i x^{2m-1}dx}{\left(x^{2m}+c\right)^p}=\sum_{i=0}^{n-1}\int_{a_i}^{a_{i+1}} \frac{p_i x^{4m-1}dx}{\left(x^{2m}+c\right)^p}+\sum_{i=0}^{n-1}\int_{a_i}^{a_{i+1}} \frac{q_i x^{2m-1}dx}{\left(x^{2m}+c\right)^p}. \qquad (4)$$

Quyidagi belgilashlarni kiritamiz:

$$I_1=\int \frac{x^{4m-1}}{\left(x^{2m}+c\right)^p}dx, \qquad I_2=\int \frac{x^{2m-1}}{\left(x^{2m}+c\right)^p}dx$$

I_1 ni bo`laklab integrallash usuli bilan hisoblaymiz:

$$I_1=\int\frac{x^{4m-1}}{\left(x^{2m}+c\right)^p}dx=\left\{\begin{matrix} u=x^{2m}; & du=2mx^{2m-1}dx \\ dv=\dfrac{x^{2m-1}}{\left(x^{2m}+c\right)^p}dx; & v=-\dfrac{1}{(p-1)\left(x^{2m}+c\right)^{p-1}}\end{matrix}\right\}=$$

$$=-\frac{x^{2m}}{2m(p-1)\left(x^{2m}+c\right)^{p-1}}+\frac{1}{p-1}\int\frac{x^{2m-1}}{\left(x^{2m}+c\right)^{p-1}}dx=$$

$$=-\frac{x^{2m}}{2m(p-1)\left(x^{2m}+c\right)^{p-1}}+\frac{1}{2m(p-2)(p-1)\left(x^{2m}+c\right)^{p-2}},$$

ya'ni

$$I_1=\frac{x^{2m}}{2m(p-1)\left(x^{2m}+c\right)^{p-1}}+\frac{1}{2m(p-2)(p-1)\left(x^{2m}+c\right)^{p-2}} \tag{5}$$

tenglik o`rinli [2].

Endi I_2 ni hisoblaymiz:

$$I_2=\int\frac{x^{2m-1}}{\left(x^{2m}+c\right)^p}dx=-\frac{1}{2m(p-1)\left(x^{2m}+c\right)^{p-1}} \tag{6}$$

(5) va (6) tengliklarga ko'ra (4) ni quyidagicha yozish mumkin:

$$\sum_{i=0}^{n-1}\int_{a_i}^{a_{i+1}}\frac{p_ix^{4m-1}+q_ix^{2m-1}dx}{\left(x^{2m}+c\right)^p}=$$

$$=\sum_{i=0}^{n-1}\int_{a_i}^{a_{i+1}}\frac{p_ix^{4m-1}dx}{\left(x^{2m}+c\right)^p}+\sum_{i=0}^{n-1}\int_{a_i}^{a_{i+1}}\frac{q_ix^{2m-1}dx}{\left(x^{2m}+c\right)^p}\approx \tag{4*}$$

$$\approx\sum_{i=0}^{n-1}\left[p_i\left(-\frac{x^{2m}}{2m(p-1)\left(x^{2m}+c\right)^{p-1}}+\frac{1}{2m(p-2)(p-1)\left(x^{2m}+c\right)^{p-2}}\right)-q_i\frac{1}{2m(p-1)\left(x^{2m}+c\right)^{p-1}}\right]\Bigg|_{a_i}^{a_{i+1}}.$$

Endi p_i va q_i noma`lum koeffisiyentlarni topish masalasini qaraylik. (*) almashtirishga ko`ra quyidagi chiziqli tenglamalar sistemasi hosil bo`ladi:

$$\begin{cases} f(a_i)\approx p_ia_i^{4m-1}+q_ia_i^{2m-1} \\ f(a_{i+1})\approx p_ia_{i+1}^{4m-1}+q_ia_i^{2m-1}\end{cases} \tag{7}$$

(7) chiziqli tenglamalar sistemasi yagona yechimga ega, chunki

$$\begin{vmatrix} a_i^{4m-1} & a_i^{2m-1} \\ a_{i+1}^{4m-1} & a_{i+1}^{4m-1}\end{vmatrix}\neq 0 \tag{8}$$

o'rinli ekanligidan, (7) chiziqli tenglamalar sistemasi yechimi

$$\begin{cases} q_i \approx \dfrac{a_i^{4m-1} f\left(a_{i+1}\right) - a_{i+1}^{4m-1} f\left(a_i\right)}{a_i^{2m-1} a_{i+1}^{2m-1}\left(a_i^{2m} - a_{i+1}^{2m-1}\right)} \\ p_i \approx \dfrac{a_i^{2m-1} f\left(a_{i+1}\right) - a_{i+1}^{2m-1} f\left(a_{i+1}\right)}{a_i^{2m-1} a_{i+1}^{2m-1}\left(a_{i+1}^{2m-1} - a_i^{2m}\right)} \end{cases} \tag{9}$$

kelib chiqadi.

(3) taqribiy almashtirish, (4*) va (9) natijalardan (1) integralning taqribiy qiymati kelib chiqadi.

Agar (1) integralda p=1, $\forall m \in N$ bo`lsa,

$$\int_a^b \frac{f(x)dx}{x^{2m}+c} \tag{1*}$$

hosil bo`ladi. U holda I_1 va I_2 integrallar

$$I_1^* = \int \frac{x^{4m-1}}{x^{2m}+c}dx = \frac{x^{2m}+1}{2m}\ln\left(x^{2m}+c\right), \quad I_2^* = \int \frac{x^{2m-1}}{x^{2m}+c}dx = \frac{1}{2m}\ln(x^{2m}+c)$$

ko’rinishga keladi . Bulardan yuqoridagi (*) almashtirishga ko`ra

$$\sum_{i=0}^{n-1}\int_{a_i}^{a_{i+1}} \frac{p_i x^{4m-1} + q_i x^{2m-1}dx}{x^{2m}+c} =$$

$$= \sum_{i=0}^{n-1}\int_{a_i}^{a_{i+1}} \frac{p_i x^{4m-1}dx}{x^{2m}+c} + \sum_{i=0}^{n-1}\int_{a_i}^{a_{i+1}} \frac{q_i x^{2m-1}dx}{x^{2m}+c} \approx$$

$$\approx \sum_{i=0}^{n-1}\left[+p_i\frac{x^{2m}+1}{2m}\ln(x^{2m}+c) + q_i\frac{1}{2m}\ln(x^{2m}+c)\right]\Bigg|_{a_i}^{a_{i+1}}. \tag{4**}$$

Agar (1) integralda p=2, $\forall m \in N$ bo`lsa,

$$\int_a^b \frac{f\left(x\right)dx}{\left(x^{2m}+c\right)^2} \tag{1**}$$

hosil bo`ladi. U holda I_1 va I_2 integrallar

$$I_1^* = \int \frac{x^{4m-1}}{\left(x^{2m}+c\right)^2}dx = -\frac{x^{2m}}{2m\left(x^{2m}+c\right)} + \frac{1}{2m}\ln\left(x^{2m}+c\right),$$

$$I_2^* = \int \frac{x^{2m-1}}{\left(x^{2m}+c\right)^2}dx = -\frac{1}{2m\left(x^{2m}+c\right)}$$

ko’rinishga keladi. Bulardan (3) almashtirishga ko`ra

$$\sum_{i=0}^{n-1}\int_{a_i}^{a_{i+1}}\frac{p_i x^{4m-1}+q_i x^{2m-1}dx}{x^{2m}+c}=$$

$$=\sum_{i=0}^{n-1}\int_{a_i}^{a_{i+1}}\frac{p_i x^{4m-1}dx}{x^{2m}+c}+\sum_{i=0}^{n-1}\int_{a_i}^{a_{i+1}}\frac{q_i x^{2m-1}dx}{x^{2m}+c}\approx$$

$$\approx\sum_{i=0}^{n-1}\left[p_i\left(\frac{1}{2m}\ln\left(x^{2m}+c\right)-\frac{x^{2m}}{2m\left(x^{2m}+c\right)}\right)-q_i\frac{1}{2m\left(x^{2m}+c\right)}\right]\Bigg|_{a_i}^{a_{i+1}}. \quad (4^{***})$$

(1) ko’rinishdagi aniq integralni taqribiy hisoblashda, demak, yuqoridagi belgilashlar yordamida integral ostidagi funksiyani ratsional funksiya bilan almashtirish qilib, aniq integrallarni taqribiy hisoblash mumkin.

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