Scientific Bulletin of Namangan State University

Volume 1 | Issue 3 Article 1

7-10-2019

EVASION PROBLEM FOR MOVEMENTS WITH ACCELERATION ON CONSTRAINT OF GRANWOLL TYPE

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Erratum

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01.00.00 ФИЗИКА-МАТЕМАТИКА ФАНЛАРИ
ФИЗИКО-МАТЕМАТИЧЕСКИЕ НАУКИ
PHYSICAL AND MATHEMATICAL SCIENCES

EVASION PROBLEM FOR MOVEMENTS WITH ACCELERATION ON CONSTRAINT OF GRANWOLL TYPE

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Abstract. *In theory of differential games, the problems put geometric, integral and their being together constraints to controls were studied sufficiently. In this paper, the evasion problem of the second order differential game will be stuided in which case new control classes with the name of constraint of Gronwall type have been introduced to control functions.*

Key words: *Differential game, acceleration, Gronwall’s lemma, Gronwall constraint, evader, pursuer, initial position.*

GRONUOLL TIPIDAGI CHEGARALANISHGA EGA TEZLANISHLI HARAKATLAR UCHUN QOCHISH MASALASI

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Annotatsiya. *Differensial o‘yinlar nazariyasida boshqaruvlarga geometrik, integral va ularning birgalikdagi chegaralanishlari qo‘yilgan masalalar yetarlicha o‘rganilgan. Ushbu maqolada boshqaruv funksiyalariga Gronuoll tipidagi chegaralanish nomi bilan yangi boshqaruv sinflari kiritilgan holda ikkinchi tartibli differensial o‘yinning qochish masalasi o‘rganilgan.*

Kalit so‘zlar: *Differensial o‘yin, tezlanish, Gronuoll lemmasi, Gronuoll chegaralanish, qochuvchi, quvlovchi, boshlang‘ich holat.*

ЗАДАЧА УБИГАНИЯ ПРИ ДВИЖЕНИИ СО УСКОРЕНИЕМ И С ОГРАНИЧЕНИЯМИ ГРОНУОЛЛА

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Аннотация. *В теории дифференциальных играх достаточно изучены задачи задающего при управления геометрического, интегрального и их совместных ограничениях. В работе изучается задача убегания для дифференциальных игр второго порядка, когда начальные состояния и начальные скорости игроков линейно зависимы при ограничениях Гронуолла на управления.*

Ключевые слова: *Дифференциальная игра, ограничение Гронуолла, лемма Гронуолла, преследователь, убегающий, ускорения, начальные состояния.*

Let **P** and **E** objects with opposite aim be given in the space $\boldsymbol{R^n}$ and their movements are based on the following differential equations and initial conditions

$$\mathbf{P}: \ \ddot{x} = u, \ x_1 - kx_0 = 0, \ |u(t)|^2 \le \rho^2 + 2l\int_0^t |u(s)|^2\, ds, \quad (1)$$

$$\mathbf{E}: \ \ddot{y} = v, \ y_1 - ky_0 = 0, \ |v(t)|^2 \le \sigma^2 + 2l\int_0^t |v(s)|^2\, ds, \quad (2)$$

where $x, y, u, v \in \boldsymbol{R^n}$; x – a position of **P** object in the space $\boldsymbol{R^n}$, $x_0 = x(0),\ x_1 = \dot{x}(0)$ – its initial position and velocity respectively at $t = 0$; u – a controlled acceleration of the pursuer, mapping $u:[0,\infty) \to \boldsymbol{R^n}$ and it is chosen as a measurable function with respect to t; we denote a set of all measurable functions $u(\cdot)$ such that satisfies the condition $|u(t)|^2 \le \rho^2 + 2l\int_0^t |u(s)|^2\, ds$ by G_P. y – a position of **E** object in the space $\boldsymbol{R^n}$, $y_0 = y(0),\ y_1 = \dot{y}(0)$ – its initial position and velocity respectively at $t = 0$; v – a controlled acceleration of the evader, mapping $v:[0,\infty) \to \boldsymbol{R^n}$ and it is chosen as a measurable function with respect to t; we denote a set of all measurable functions $v(\cdot)$ such that satisfies the condition $|v(t)|^2 \le \sigma^2 + 2l\int_0^t |v(s)|^2\, ds$ by G_E.

Definition 1. For a trio of $\left(x_0, x_1, u(\cdot)\right), u(\cdot) \in G_P$, the solution of the equation (1), that is, $x(t) = x_0 + x_1 t + \int_0^t \int_0^s u(\tau) d\tau ds$ is called a trajectory of the pursuer on interval $t \geq 0$.

Definition 2. For a trio of $\left(y_0, y_1, v(\cdot)\right), v(\cdot) \in G_E$, the solution of the equation (2), that is, $y(t) = y_0 + y_1 t + \int_0^t \int_0^s v(\tau) d\tau ds$ is called a trajectory of the evader on interval $t \geq 0$.

Definition 3. For differential game (1) – (2), the evasion problem is said to be held however, the pursuer chooses any control function $\forall u(\cdot) \in G_P$, if there exists $\exists v^*(\cdot) \in G_E$ for the evader and the following condition is true for the tracjectories $x(t)$, $y(t)$ that is found according to those control functions:

$$x(t) \neq y(t), (t \geq 0) \quad (3)$$

To solve the evasion problem we will propose a strategy of the evader as follows:

Definition 4. In differential game (1) – (2) we call the strategy of the evader the following function [2]:

$$v^*(t) = -\sigma e^{lt} \xi_0, (t \geq 0) \ , (4)$$

where $\xi_0 = \dfrac{x_0 - y_0}{\left|x_0 - y_0\right|}$.

Lemma (Gronwall). Suppose a mapping $\varphi(t) : [0, \infty) \to \boldsymbol{R^n}$ is bounded, non-negative and measurable function. Moreover, $l \geq 0$ and $\rho > 0$ are constant and for the given if an inequality $\left|\varphi(t)\right|^2 \leq \rho^2 + 2l \int_0^t \left|\varphi(s)\right|^2 ds$ holds, then a relation $\varphi(t) \leq \rho e^{lt}$ is always true [1] – [4].

Theorem. If one of the following conditions holds:

1. $\rho = \sigma$ and $k > 0$; or 2. $\rho < \sigma$ and $k \geq \dfrac{(\sigma - \rho)l}{|z_0|}$; or 3. $\rho < \sigma$ and $k < 0$,

$$1 + \frac{|z_0| l^2}{\rho - \sigma} < \left(\frac{|z_0| kl}{\rho - \sigma} + 1\right) \cdot \left(1 - \ln\left(\frac{|z_0| kl}{\rho - \sigma} + 1\right)\right)$$

then for the differential game (1) – (2), the evasion problem will be solved by the strategy of the evader (4) and a change function between objects will be in the following form:

$$f(t,k,\rho,\sigma,|z_0|)=\begin{cases}|z_0|kt+|z_0|, & if \ \ \rho=\sigma,\\ |z_0|(kt+1)+\dfrac{(\sigma-\rho)}{l^2}e^{lt}+\dfrac{(\rho-\sigma)}{l}t+\dfrac{\rho-\sigma}{l^2}, & if \ \ \rho<\sigma.\end{cases}$$

Proof. Assume that the pursuer chooses any control function $u(\cdot)\in G_P$ and the evader choose the control function (4). Then according to the equations (1) – (2) we form the following solutions:

$$x(t)=x_0+x_1t+\int_0^t\int_0^s u(\tau)d\tau ds \quad (5)$$

$$y(t)=y_0+y_1t+\int_0^t\int_0^s v^*(\tau)d\tau ds. \quad (6)$$

We denote their distinction function as follows

$$z(t)=x(t)-y(t).$$

According to (5) and (6) we have the function

$$z(t)=z_0+z_1t+\int_0^t\int_0^s u(\tau)d\tau ds+\int_0^t\int_0^s \sigma e^{l\tau}d\tau ds\xi_0$$

where $z_0=x_0-y_0, z_1=x_1-y_1, \xi_0=\dfrac{z_0}{|z_0|}$.

subtracting the initial conditions, we have an equality $z_1=kz_0$. Hence we form a function

$$z(t)=z_0(kt+1)+\left[\frac{\sigma}{l^2}\left(e^{lt}-1\right)-\frac{\sigma}{l}t\right]\xi_0+\int_0^t\int_0^s u(\tau)d\tau ds.$$

Evaluate the absolute value of this function from low:

$$|z(t)|=\left|z_0(kt+1)+\left[\frac{\sigma}{l^2}\left(e^{lt}-1\right)-\frac{\sigma}{l}t\right]\xi_0+\int_0^t\int_0^s u(\tau)d\tau ds\right|\geq$$

$$\left|z_0(kt+1)+\left[\frac{\sigma}{l^2}\left(e^{lt}-1\right)-\frac{\sigma}{l}t\right]\xi_0\right|-\left|\int_0^t\int_0^s u(\tau)d\tau ds\right|\geq|z_0|(kt+1)+\frac{\sigma}{l^2}(e^{lt}-1)-\frac{\sigma}{l}t-$$

$$-\int_0^t\int_0^s |u(\tau)|d\tau ds.$$

we apply the above Gronwall's lemma to the latest function inside the integral:

$$|z(t)|\geq|z_0|(kt+1)+\frac{\sigma}{l^2}(e^{lt}-1)-\frac{\sigma}{l}t-\int_0^t\int_0^s\rho e^{l\tau}d\tau ds=|z_0|(kt+1)+\frac{\sigma}{l^2}(e^{lt}-1)-\frac{\sigma}{l}t-$$

$$-\frac{\rho}{l^2}(e^{lt}-1)+\frac{\rho}{l}t=|z_0|(kt+1)+\frac{(\sigma-\rho)}{l^2}e^{lt}+\frac{(\rho-\sigma)}{l}t+\frac{\rho-\sigma}{l^2}$$

We will consider as a parametric function the right side of the latest inequality:

$$f(k,t,\rho,\sigma,l,|z_0|)=|z_0|(kt+1)+\frac{(\sigma-\rho)}{l^2}e^{lt}+\frac{(\rho-\sigma)}{l}t+\frac{\rho-\sigma}{l^2}. \quad (5)$$

We introduce some simplifications, i.e., $|z_0|=a$, $\frac{\rho-\sigma}{l^2}=\gamma$. Therefore, the function (5) becomes in the following form:

$$f(k,t,\gamma,l,a)=a(kt+1)+\gamma e^{lt}+\frac{\gamma}{l}t+\gamma \quad (6)$$

1. Let be $\rho=\sigma$. Then $f(k,t,a)=a(kt+1)$ and find a value t^* such that this function turns zero:

$$t^*=-\frac{1}{k}.$$

Here there doesn't exist the positive solution $t^*>0$ when $k>0$.

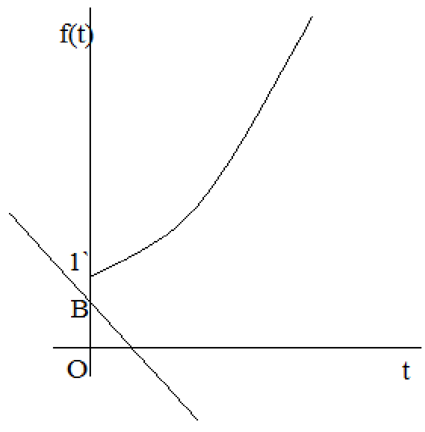
2. Let be $\rho<\sigma$. Equalizing the right side of the function (6) to zero, and if both sides of the function is divided to parameter γ, the following equation forms:

$$\left(\frac{|z_0|k}{\gamma}+l\right)t+1+\frac{|z_0|}{\gamma}=e^{lt} \quad (7)$$

from $\gamma<0$, $B=1+\frac{|z_0|}{\gamma}<1$ and if we denote $A=\frac{|z_0|k}{\gamma}+l\leq 0$

$\Rightarrow\ k\geq\frac{(\sigma-\rho)l}{|z_0|}$, then the equation (7) doesn't have a solution on the interval

$k\geq\frac{(\sigma-\rho)l}{|z_0|}$. (Fig-1)



Figure–1

Now let be $A = \dfrac{|z_0|k}{\gamma} + l > 0 \;\Rightarrow\; k < \dfrac{(\sigma-\rho)l}{|z_0|}$.

Define the equation of the tangent for function $f(l,t) = e^{lt}$ $f(l,t,t_0) = e^{lt_0} + le^{lt_0}(t-t_0) = e^{lt_0}(1-lt_0) + le^{lt_0}t = A_1 t + B_1$.

Now we find conditions such that the function on the left side of (7) is parallel to and lies down the tangent:

$$A = A_1 \;\Rightarrow\; \frac{|z_0|k}{\gamma} + l = le^{lt_0} \;\Rightarrow\; t_0 = \frac{1}{l}\ln\left(\frac{|z_0|k}{\gamma l} + 1\right),$$

where k is a parameter and it must be on the interval $k < 0$. Because

$$\frac{|z_0|k}{\gamma l} + 1 > 1 \;\Rightarrow\; \frac{|z_0|k}{\gamma l} > 0 \;\Rightarrow\; k < 0.$$

In addition,

$$B < B_1 \;\Rightarrow\; 1 + \frac{|z_0|}{\gamma} < e^{lt_0}(1-lt_0) = \left(\frac{|z_0|k}{\gamma l} + 1\right)\left(1 - \ln\left(\frac{|z_0|k}{\gamma l} + 1\right)\right) \;\Rightarrow$$

$$1 + \frac{|z_0|l^2}{\rho - \sigma} < \left(\frac{|z_0|kl}{\rho - \sigma} + 1\right) \cdot \left(1 - \ln\left(\frac{|z_0|kl}{\rho - \sigma} + 1\right)\right). \text{ (Fig} - 2)$$

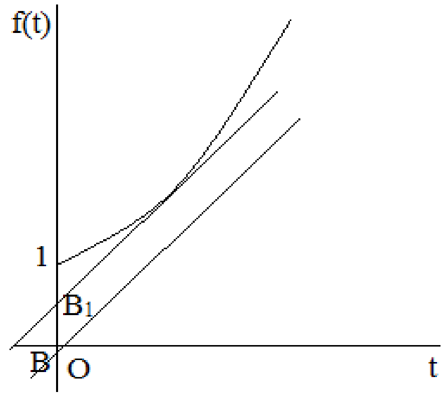


Figure – 2

In conclusion, the relation (3) is true in all values of interval $t \geq 0$ according to the inequality $|z(t)| \geq f(k,t,\alpha,\beta,|z_0|)$ and properties of (5), i.e., the evasion problem is solved, which completes the proof of the Theorem.

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