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RECOVERING OF A STATIONARY EXTERNAL FORCE BY DISTRIBUTIONS

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**Bog`langan protsedura**. Slot qiymati sifatida Lisp II yordamchi dasturida protsedura yoki obyektga yo'naltirilgan dasturlash metodidan foydalanish mumkin. Bog`langan protsedura boshqa freymdan yuborilgan xabar bilan boshlanadi. Ushbu protsedurali bilimlar freym tizimlarida chiqarishni boshqarish vositasidir va ularning yordami bilan har qanday chiqarish mexanizmi amalga oshirilishi mumkin. Bunday bilimlarni tasvirlash va ular bilan intellektual tizimlarni to'ldirish juda qiyin vazifadir, bu Sun`iy intellekt ishlab chiquvchilari uchun qo'shimcha mehnat va vaqtni talab qiladi.

So'nggi yillarda "freym" atamasi ko'pincha "obyektga yo'naltirilgan" atamasi bilan almashinmoqda. Freym shablonini sinf sifatida ko'rish mumkin, freym namunasini esa obyekt sifatida. Obyektga yo'naltirilgan dasturlash tillari (OYD) sinflar va obyektlarni yaratish uchun vositalarni, shuningdek, obyektni qayta ishlash protseduralarini (metodlarini) tavsiflash vositalarini ta'minlaydi. Bog`langan protseduralarni amalga oshirish vositalarini o'z ichiga olmaydigan OYD tillari moslashuvchan mexanizmni tashkil etishga imkon bermaydi, shuning uchun ular ustida ishlab chiqilgan dasturlar obyektga yo'naltirilgan ma'lumotlar bazasini taqdim etadi yoki boshqa ma'lumotlarni qayta ishlash vositalari bilan (masalan, PYTHON tili bilan) integratsiyalashuvni talab qiladi. Shuningdek, freym modeliga asoslangan ixtisoslashtirilgan bilimlarni ifodalash tillari mavjud, ularga misollar: FRL (Frame namoyishi tili), KRL (bilimlarni namoyish etish tili), freym "qobiq" Karra va boshqalar. Shuningdek, freym tipidagi ekspert tizimlari mavjud: AMALIST, TRISTAN, ALTERID, MODIS.

Xulosa. Shunday qilib, freymli modellar quyidagi afzalliklarga ega:

1. Bilimlarni freymlar asosida ifodalash tushunchalarning umumiy iyerarxiyasini aniq shaklda saqlashga imkon beradi.

2. Meros prinsipi xotirani tejashga, tafsilotlar to`liq bo'lmagan taqdirda ham vaziyatni tahlil qilishga imkon beradi.

3. Freym modeli juda universaldir, chunki u haqiqiy dunyo haqida xilma-xil bilimlarni quyidagilar orqali namoyish etishga imkon beradi:

- obyekt va tushunchalarni belgilash uchun foydalaniladigan freym-konstruktsiyalar;
- freym – rollar (mijoz, menejer);
- freym - ssenariylar (bankrotlik, yig‘ilish);
- freym - vaziyatlar (avariya, qurilmaning ishlash tartibi) va boshq.

4. Ilova qilingan protseduralardan foydalanib, freym tizimi har qanday qaror qabul qilish mexanizmini amalga oshirishga imkon beradi.

ADABIYOTLAR

1. Minskiy M. Freymi dlya predstavleniya znaniy / M. Minskiy // Psixologiya mashinnogo zreniya. - M.: Mir, 1978. - S. 72.

2. Fedoruk V.G. Iskusstvenniy intellekt v SAPR: Konspekt lektsiy. URL: http:// rk6.bmstu.ru/ electronic_book/ iisapr/ ai_ lect.html (дата обращения: 02.11.2009).

3. Kryonke D. Teoriya i praktika postroeniya baz dannix. - 8-e izdanie. - SPb.: Piter, 2003.

4. Gavrilova T. A., Xoroshevskiy V.F. Bazi znaniy intellektualnix sistem SPb: Piter, 2001. - 384 s.

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STATSIONAR MUHITDA TASHQI KUCHNI TAQSIMOT ORQALI TIKLASH

ВОССТАНАВЛЕНИЕ ВНЕШНЕЙ СИЛЫ ПО РАСПРЕДЕЛЕНИЮ В СТАЦИОНАРНОЙ СРЕДЕ

RECOVERING OF A STATIONARY EXTERNAL FORCE BY DISTRIBUTIONS

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Tayanch so’zlar: Koshi masalasi, algebraic va transcendent sonlar, o’rama, Rot teoremasi, Fur’e almashtirishi, Dirakning deltasi.

Ключевые слова: задача Коши, алгебраические и трансцендентные числа, свертка, теорема Рота, преобразование Фурье, дельта Дирака.

Key words: Cauchy problem, algebraic and transcendental numbers, convolution, Roth theorem, Fourier transformation, Dirac’s delta.

Ushbu ishda to'lqin tenglamasidagi tashqi kuch statsionar bo'lgan holda uni umumlashgan funksiyalar yordamida tiklash masalasi o'rganilgan. Ushbu muammo transandantal holatlar uchun hal qilinmagan. Buning uchun biroz natija bor va keyingi ishda biz ushbu holatlarni keltiramiz Uning uchun mavjudlik va yagonalik teoremalari isbotlangan.

В статье изучена как построить внешнюю силу на волновом уравнении со стационарным и восстановить его с помощью обобщенных функций. Эта проблема не решена для трансцендентных случаев. У нас есть небольшой результат для этого, и в следующей работе мы принесем эти случаи Кроме того, доказаны теоремы существования и единственности.

In the article there is researched how to construct the external force on the stationary wave equation and restores it with generalized functions. This problem hasn't solved for transcendental cases. We have some a little result for it and the next work we will bring that cases *Thus, the theorems of existence and uniqueness have been proved.*

Set up the main problem. In this work we shall construct the unknown $f(x) \in D'(R^3)$ function by the solution to the Cauchy problem in the domain (R^3, R_+)

$$\begin{cases} u_{tt} - \Delta u = f(x), \\ u(x,0) = \dfrac{\partial u}{\partial t}(x,0) = 0, \end{cases} \qquad (1.1)$$

where u is given distributions from the space $D'(R^3)$ [1].

It is well known, if $f(x) \in C^2(R^3)$, then the solution to the (1.1) Cauchy problem will be given by

$$u(x,t) = \frac{1}{4\pi} \int_{B(x,t)} \frac{f(\xi)}{|x-\xi|} d\xi = \left(\frac{\chi_{B(0,t)}}{|y|} * f \right)(x)$$

where $B(x,t)-$ the ball with radius t and centered at x. Since for any fixed $t > 0$. $\frac{\chi_{B(0,t)}}{|y|}$ is integrable function with compact support, so convolution is well-defined. Moreover for any fixed $t_0 > 0$ the restriction of $u(x,t)$ on the hyperplane $t = t_0$ also well-defined.

The problem of determining $u \in$ from (1.1) with given $f(x)$ is called the direct problem for isotropic media. Actually, in most cases such as water waves or vibrating strings or membranes, the wave equation gives only an approximation to the correct physics that is valid for vibrations of small amplitude. However, it is an easy consequence of Maxwell's equations that the wave equation is satisfied exactly by the components of the classical electromagnetic field in a vacuum.

The following inverse problem can be formulated for (1): to find $f(x) \in D'(R^3)$, if for the solution of the direct problem is known in the $t = t_1$ and $t = t_2$, we know that

$$u(x,t_1) = g_1(x), \; u(x,t_2) = g_2(x), \qquad (1.2)$$

where g_i, $i = 1,2-$ are given functions. The uniqueness of f depends on whether $\frac{t_2}{t_1}$ is rational or not.

Here a definition about algebraic number [3].

Definition. A real number is said to be algebraic number, if it satisfies some algebraic equation with integer coefficients.

It is easy to see that every rational number is an algebraic number. For example, $\frac{5}{7}$ satisfies the equation $5x - 7 = 0$; and this equation is of the prescribed type. More generally, any rational number $\frac{a}{b}$ $(b \neq 0)$, satisfies the equation $bx - a = 0$; and so it is an algebraic number. In the figure, we have listed $\sqrt{2}$ and $\sqrt[3]{7}$ as examples of algebraic numbers. They are algebraic because they satisfy the algebraic equations $x^2-2=0$ and $x^3-7=0$ respectively. The following auxiliary theorem is well-known [3].

Theorem(Roth). If α is non-rational algebraic number, then $\forall\varepsilon>0$ there exists $T(\varepsilon,\alpha)>0,$ such that the inequality

$$\left|\alpha-\frac{p}{q}\right|\geq\frac{T(\varepsilon,\alpha)}{q^{2+\varepsilon}}$$

holds for any $p,q\in Z.$

Besides, there is weak result than Roth.

Lemma. Suppose $\alpha\in C$ is non-rational algebraic number of degree 2 over Q. Then there is a positive number $T,$ depending only on $\alpha,$ such that

$$\left|\alpha-\frac{p}{q}\right|\geq\frac{T(\alpha)}{q^2}$$

for all rational numbers $\frac{p}{q}.$

Now, we bring the main result of this work.

Theorem 1. If $\frac{t_2}{t_1}\in A\setminus Q-$non-rational algebraic number, then f can be uniquely defined.

Lemma. Let $A\setminus Q$ be a set of non-rational algebraic numbers. Then, for $\forall\kappa>0$ there is a positive number $T(k,\alpha)$, such that

$$\frac{T}{|\xi|^{1+\kappa}}\leq\max\{dist(\{|\xi|\},\{\pi k\}),dist(\{\alpha|\xi|\},\{\pi k\})\}\leq\frac{\pi}{2},\quad (1.3)$$

where $\alpha\in A\setminus Q.$

Theorem 2. If $g_1,g_2\in D'(R^3)$ are distributions satisfying the condition $T_1g_2=T_2g_1,$ then there exists an unique distribution $f\in D'(R^3)$ such that $T_1f=g_1$ and $T_2f=g_2,$ where $T_i=\frac{\chi_{B(0,t_i)}}{|y|},i=1,2.$

Proposition. 1) If $\frac{t_2}{t_1}\in Q$ then for

$$f(x)=4\pi r_0\frac{\sin(r_0|x|)}{|x|},\qquad (1.4)$$

with $r_0=\frac{2\pi p}{t_1}=\frac{2\pi q}{t_2},1=(p,q)\in Z^2,$ we have $g_1=g_2=0.$

2) If $g_i\neq0,i=1,2$ and $\frac{t_2}{t_1}\in Q,$ then we can't find the f.

2 Proof of the main results. Proof theorem 1. First of all we assume that $f\in S'(R^3)$ Shwartz class of distribution, then so is $u(x,t_j),j=1,2.$ Now examine the problems solved by using the Fourier transform. Taking the (1.1) system of initial conditions into account, we see that

$$\hat{u}(\xi,t)=\frac{1-\cos(|\xi|t)}{|\xi|^2}\hat{f}(\xi),$$

where $\hat{u}(\cdot,t)$ denotes is the Fourier transform of $u(\cdot,t)$ for each t. Moreover, taking the (1.2) initial conditions into account, we get the following system of equations

$$\begin{cases} \hat{g}_1(\xi) = \dfrac{1-\cos(|\xi| t_1)}{|\xi|^2} \hat{f}(\xi), \\ \hat{g}_2(\xi) = \dfrac{1-\cos(|\xi| t_2)}{|\xi|^2} \hat{f}(\xi). \end{cases} \quad (2.1)$$

Now, if $\hat{g}_1 = \hat{g}_2 = 0,$ then we will show that $\hat{f} = 0.$ In the other words *supp* $\hat{f} = \emptyset.$ Indeed if $\xi^0 \in supp\ \hat{f},$ then from the (2.1) we get

$$1-\cos(|\xi| t_1) = 0,\ 1-\cos(|\xi| t_2) = 0$$

or

$$t_1 = \frac{2\pi}{|\xi|} k_1,\ t_2 = \frac{2\pi}{|\xi|} k_2,\ k_i \in Z,\ i = 1,2.$$

But, this contradicts to $\frac{t_2}{t_1} \in A \setminus Q$ ($A-$all algebraic numbers). So that, from (2.1) we can define a $\hat{f}$ function uniquely by $\hat{g}_i, i = 1,2$ and using calculate one shall get:

$\hat{f}(\xi) = a(\xi)(\hat{g}_1(\xi) + \hat{g}_2(\xi)),$

where $a(\xi) := \dfrac{|\xi|^2}{2\left(\sin^2 \frac{|\xi|}{2} t_1 + \sin^2 \frac{|\xi|}{2} t_2\right)}.$ Now, we will show that the function satisfies the following condition

$$|a(\xi)| \le T(\alpha)(1 + |\xi|^{4+2\kappa}),\ \kappa > 0$$

that is, this function is a distribution. Without loss of generality we take $t_1 = 1,\ t_2 = \alpha \in A \setminus Q$ and show the following inequality

$$\sin^2(|\xi|) + \sin^2(\alpha|\xi|) \ge T|\xi|^{-2-2\kappa}, \quad (2.2)$$

Assume that the $\left||\xi| - \pi k\right| \le \frac{T(k)}{|\xi|^{1+\kappa}}$ inequality holds, here $\kappa > 0$. Now we prove the next auxiliary lemma.

Proof Lemma. We can easily see that, the upper estimate is hold. So that we have to show the lower bound. Actually, for large enough $|\xi|,$ distance between the given sets are not less than $\frac{T}{|\xi|^{1+\kappa}}$ in time, if it holds then

$$\begin{cases} \left||\xi| - \pi k_1\right| < \dfrac{T}{|\xi|^{1+\kappa}} \\ \left|\alpha|\xi| - \pi k_2\right| < \dfrac{T}{|\xi|^{1+\kappa}}. \end{cases} \quad (2.3)$$

But large enough $|\xi|$ and according to the Roth theorem for $0 < \varepsilon < \kappa$

$$\left|\alpha|\xi| - \pi k_2\right| = \left|(|\xi| - \pi k_1)\alpha + \pi\alpha k_1 - \pi k_2\right| > \frac{T_1}{k_1^{1+\varepsilon}} - \frac{T_2}{|\xi|^{1+\kappa}}, \quad (2.4)$$

holds. And to the first inequality of (2.3) we get

$$\frac{1}{\pi}\left(|\xi| - \frac{T}{|\xi|^{1+\kappa}}\right) < k_1 < \left(|\xi| + \frac{T}{|\xi|^{1+\kappa}}\right)$$

and $|\xi| + \frac{T}{|\xi|^{1+\kappa}} \le 2|\xi|$ holds for $|\xi| \succ 1,$ so that for (2.4) we get

$$\left|\alpha|\xi| - \pi k_2\right| \ge \frac{T_\varepsilon}{|\xi|^{1+\varepsilon}}.$$

Now we show the (2.2) estimate. According to the known $\sin x \ge \frac{2x}{\pi},\ x \in [0, \frac{\pi}{2}]$ inequality, we have

$$\sin^2(\alpha|\xi|) = \sin^2(\alpha|\xi| - \pi k) \ge \widetilde{T}\left|\alpha|\xi| - \pi k\right|^2 \ge \frac{T}{|\xi|^{2+2\kappa}}$$

or

$$\sin^2(|\xi|) = \sin^2(|\xi| - \pi k) \geq \frac{T}{|\xi|^{2+2\kappa}}.$$

So that, $\forall \kappa > 0$ the following estimate $|a(\xi)| \leq T(1 + |\xi|^{4+2\kappa})$, holds.

Corollary. If the $\hat{g}_1 + \hat{g}_2$ is given, then $\hat{f}$ is uniquely defined and using from inverse Fourier transform for it, we get

$$f(x) = \breve{a} * (g_1(x) + g_2(x)). \qquad (2.5)$$

Proof theorem 2. Assume that $g_1, g_2 \in D'(R^3)$ are generalized functions satisfying the condition $T_1 g_2 = T_2 g_1$.According to the (2.5) and by commutative of the convolution distribution function one has:

$$T_1(f) = T_1(\breve{a} * (g_1 + g_2)) = T_1(\breve{a} * g_1) + T_1(\breve{a} * g_2) = \breve{a} * T_1 g_1 + \breve{a} * T_1 g_2 =$$
$$= \left(\breve{a} * (T_1 + T_2)\right) * g_1 = g_1,$$

that is $T_1(f) = g_1$. Similarly we get $T_2(f) = g_2$.

So that, the theorem 1 is valid.

Proof Proposition. First part. Let (1.4) holds. Then, according to the system of (2.1), we have

$$\hat{g}_i(\xi) = \frac{1 - \cos(|\xi| t_i)}{|\xi|^2} \hat{f}(\xi), \ i = 1,2$$

or

$$\hat{g}_i(\xi) = \frac{1 - \cos(|\xi| t_i)}{|\xi|^2} \delta(|\xi| - r_0),$$

where, $\delta(\cdot)$ is Dirac's delta on the sphere $\{|\xi| = r_0\}$ and here we get the Fourier transform of $f(\cdot)$. From (2.1) we get

$$\hat{g}_i(\xi) = 0$$

or $g_1(x) = g_2(x) = 0$.

Second part. We can easy conclude using by (2.1). It means the condition $\frac{t_2}{t_1} \in A \setminus Q$ of theorem 1 is essential.

This problem hasn't solved for transcendental cases. We have some a little result for it and the next work we will bring that cases.

REFERENCES

1. Palamodov V.P. Generalized functions and harmonic analysis.[in Russian] Itogi Nauki i Tekhniki. Ser. Sovrem. Probl. Mat. Fund. Napr., 1991, Volume 72. – P. 5-134.

2. Ikromov I.A, Rahmonov A.A. On reconstruction of distributions by spherical means. Uzbek Mathematical Journal, 2019, No 3. – P. 1-10.

3. Ivan Niven. Numbers: Rational and Irrational. (Monograph) University of Oregon, 1961. – P.145.