RECOVERING OF A STATIONARY EXTERNAL FORCE BY DISTRIBUTIONS

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So`nggi yillarda "freym" atamasi ko`pincha "obyektga yo`naltirilgan" atamasi bilan almashinmoqda. Freym shablonini sifatida ko`rish mumkin, freym namunasini esa obyekt sifatida. Obyektga yo`naltirilgan dasturlash tillari (OYD) sinvlar va obyektlarni yaratish uchun vositalarni, shuningdek, obyektga yo`naltirilgan amalga oshirilgan methodlarini (metodlarini) tavisiflash vositalarni ta`minlaydi. Bog`langan процедураларни amalga oshirish vositalarini o`z ichiga olmaydigan OYD tillari moslashuvchilari mexanizmini tashkil etishga imkon beradi. Shuningdek, freym modeliga asoslangan икситослаштирилган процедураларни ифодалаштириш тилининг таъкидий илмий вақтни таълиқ қилади.

So`nggi yillarda "freym" atamasi ko`pincha "obyektga yo`naltirilgan" atamasi bilan almashinmoqda. Freym shablonini sifatida ko`rish mumkin, freym namunasini esa obyekt sifatida. Obyektga yo`naltirilgan dasturlash tillari (OYD) sinvlar va obyektlarni yaratish uchun vositalarni, shuningdek, obyektga yo`naltirilgan amalga oshirish vositalari mavjud, ularga misollar: FRL (Frame namoyishi tili), KRL (bilimlarni namoyish etish tili), freym "qobiq" Karra va boshqalar. Shuningdek, freym tipidagi ekspert tizimlarni mavjud, ularga misollar: FRL (Frame namoyishi tili), KRL (bilimlarni namoyish etish tili), freym "qobiq" Karra va boshqalar. Shuningdek, freym tipidagi ekspert tizimlarni mavjud: AMALIST, TRISTAN, ALTERID, MODIS.

Xulosa. Shunday qilib, freymlari modellardan quydagi афзалликга эга:
1. Biliimlarni freymlar asosida ifodalash tushunchalarning umumiy iyerarxiyasini aniq shaklda saqlashga imkon beradi.
2. Meros prinipsi xotirani tejashga, tafsilotlar to`liq bo'lmagan taqdirda ham vaziyatni tahlil qilishga imkon beradi.
3. Freym modeli juda universaldir, chunki u haqiqiy dunyo ixtisoslashtirilgan bilimlarni ifodalash uchun qo`shimcha mexanizmni yaratishga imkon beradi:
4. Ilova qilingan процедуралардан foydalanib, freym tizimni har qanday qaror qabul qilish mexanizmini amalga oshirishga imkon beradi.

ADABIYOTLAR

UDC: 517.9

STATSIONAR MUHITDA TASHQI KUCHNI TAQSIMOT ORQALI TIKLASH
ВОССТАНАВЛЕНИЕ ВНЕШНЕЙ СИЛЫ ПО РАСПРЕДЕЛЕНИЮ В СТАЦИОНАРНОЙ СРЕДЕ

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Tayanch so`zlar: Koshi masalasi, algebraic va transcendent sonlar, o`rama, Rot teoremasi, Fur`e almashtirishi, Dirakning deltasi.

Key words: Cauchy problem, algebraic and transcendental numbers, convolution, Roth theorem, Fourier transformation, Dirac's delta.

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Ushbu ishda to'lrin tenglamasidagi tashqi kuch stationar bo'lgan holda uni umumlashgan funksiyalar yordamida tiklash masalasi o'rganilgan. Ushbu muammo transandantal holatlar uchun hal qilinmagan. Buning uchun biroz natija bor va keyingi ishda biz ushbu holatlarni keltiramiz Uning uchun mavjudlik va yagonalik teoremalari isbotlangan.

В статье изучена как построить внешнюю силу на волновом уравнении со стационарным и восстановить его с помощью обобщенных функций. Эта проблема не решена для трансцендентных случаев. У нас есть небольшой результат для этого, и в следующей работе мы принесем эти случаи. Кроме того, доказаны теоремы существования и единственности.

In the article there is researched how to construct the external force on the stationary wave equation and restores it with generalized functions. This problem hasn’t solved for transcendental cases. We have some a little result for it and the next work we will bring that cases. Thus, the theorems of existence and uniqueness have been proved.

Set up the main problem. In this work we shall construct the unknown \( f(x) \in D'(R^3) \) function by the solution to the Cauchy problem in the domain \( (R^3, R) \)

\[
\begin{cases}
    u_t - \Delta u = f(x), \\
    u(x,0) = \frac{\partial u}{\partial t}(x,0) = 0,
\end{cases}
\]

where \( u \) is given distributions from the space \( D'(R^3) \)[1].

It is well known, if \( f(x) \in C^2(R^3) \), then the solution to the (1.1) Cauchy problem will be given by

\[
u(x,t) = \frac{1}{4\pi} \int_{B(x,t)} \frac{f(\xi)}{\|x - \xi\|} d\xi = \left(\frac{\chi_{B(0,t)}}{|y|} * f\right)(x)
\]

where \( B(x,t) \) – the ball with radius \( t \) and centered at \( x \). Since for any fixed \( t > 0 \), \( \chi_{B(0,t)} \) is integrable function with compact support, so convolution is well-defined. Moreover for any fixed \( t_0 > 0 \) the restriction of \( u(x,t) \) on the hyperplane \( t = t_0 \) also well-defined.

The problem of determining \( u \in (1.1) \) with given \( f(x) \) is called the direct problem for isotropic media. Actually, in most cases such as water waves or vibrating strings or membranes, the wave equation gives only an approximation to the correct physics that is valid for vibrations of small amplitude. However, it is an easy consequence of Maxwell’s equations that the wave equation is satisfied exactly by the components of the classical electromagnetic field in a vacuum.

The following inverse problem can be formulated for (1): to find \( f(x) \in D'(R^3) \), if for the solution of the direct problem is known in the \( t = t_1 \) and \( t = t_2 \), we know that

\[
u(x,t_1) = g_1(x), \quad u(x,t_2) = g_2(x),
\]

where \( g_i, i = 1,2 \) – are given functions. The uniqueness of \( f \) depends on whether \( \frac{t_2}{t_1} \) is rational or not.

Here a definition about algebraic number [3].

Definition. A real number is said to be algebraic number, if it satisfies some algebraic equation with integer coefficients.

It is easy to see that every rational number is an algebraic number. For example, \( \frac{5}{7} \) satisfies the equation \( 5x - 7 = 0 \); and this equation is of the prescribed type. More generally, any rational number \( \frac{a}{b} \) (\( b \neq 0 \)), satisfies the equation \( bx - a = 0 \); and so it is an algebraic
number. In the figure, we have listed $\sqrt{2}$ and $\sqrt[7]{3}$ as examples of algebraic numbers. They are algebraic because they satisfy the algebraic equations $x^2 - 2 = 0$ and $x^3 - 7 = 0$ respectively. The following auxiliary theorem is well-known [3].

**Theorem (Roth).** If $\alpha$ is non-rational algebraic number, then $\forall \varepsilon > 0$ there exists $T(\varepsilon, \alpha) > 0$, such that the inequality

$$|\alpha - \frac{p}{q}| \geq \frac{T(\varepsilon, \alpha)}{q^{2+\varepsilon}}$$

holds for any $p, q \in \mathbb{Z}$.

Besides, there is weak result than Roth.

**Lemma.** Suppose $\alpha \in \mathbb{C}$ is non-rational algebraic number of degree 2 over $\mathbb{Q}$. Then there is a positive number $T$, depending only on $\alpha$, such that

$$|\alpha - \frac{p}{q}| \geq \frac{T(\alpha)}{q^2}$$

for all rational numbers $\frac{p}{q}$.

Now, we bring the main result of this work.

**Theorem 1.** If $\frac{t_2}{t_1} \in A \setminus Q$ – non-rational algebraic number, then $f$ can be uniquely defined.

**Lemma.** Let $A \setminus Q$ be a set of non-rational algebraic numbers. Then, for $\forall k > 0$ there is a positive number $T(k, \alpha)$, such that

$$T \leq \max \{\text{dist}([\xi], \{n\}), \text{dist}([\alpha \xi], \{n\})\} \leq \frac{\pi}{2},$$

where $\alpha \in A \setminus Q$.

**Theorem 2.** If $g_1, g_2 \in D'(\mathbb{R}^3)$ are distributions satisfying the condition $T_1 g_2 = T_2 g_1$, then there exists an unique distribution $f \in D'(\mathbb{R}^3)$ such that $T_1 f = g_1$ and $T_2 f = g_2$, where $T_i = \frac{X_{B(0,t_i)}}{|y|}, i = 1, 2$.

**Proposition.** 1) If $\frac{t_2}{t_1} \in Q$ then for

$$f(x) = 4\pi r_0 \frac{\sin(r_0 |x|)}{|x|}.$$  

with $r_0 = \frac{2\pi p}{t_1} = \frac{2\pi q}{t_2}$, $1 = (p, q) \in \mathbb{Z}^2$, we have $g_1 = g_2 = 0$.

2) If $g_i \neq 0, i = 1, 2$ and $\frac{t_2}{t_1} \in Q$, then we can’t find the $f$.

**2 Proof of the main results. Proof theorem 1.** First of all we assume that $f \in S'(\mathbb{R}^3)$ Shwartz class of distribution, then so is $u(x, t), j = 1, 2$. Now examine the problems solved by using the Fourier transform. Taking the (1.1) system of initial conditions into account, we see that

$$\hat{u}(\xi, t) = \frac{1 - \cos(|\xi| t)}{|\xi|^2} \hat{f}(\xi),$$

where $\hat{u}(\cdot, t)$ denotes is the Fourier transform of $u(\cdot, t)$ for each $t$. Moreover, taking the (1.2) initial conditions into account, we get the following system of equations
\[
\begin{align*}
\hat{g}_1(\xi) &= \frac{1 - \cos(\|\xi\| t_1)}{|\xi|^2} \hat{f}(\xi), \\
\hat{g}_2(\xi) &= \frac{1 - \cos(\|\xi\| t_2)}{|\xi|^2} \hat{f}(\xi).
\end{align*}
\] (2.1)

Now, if \( \hat{g}_1 = \hat{g}_2 = 0 \), then we will show that \( \hat{f} = 0 \). In the other words \( \text{supp} \hat{f} = 0 \).

Indeed if \( \xi^0 \in \text{supp} \hat{f} \), then from the (2.1) we get
\[
1 - \cos(\|\xi\| t_1) = 0, \quad 1 - \cos(\|\xi\| t_2) = 0
\]
or
\[
t_1 = \frac{2\pi}{|\xi|} k_1, \quad t_2 = \frac{2\pi}{|\xi|} k_2, \quad k_i \in \mathbb{Z}, \quad i = 1, 2.
\]

But, this contradicts to \( \frac{t_2}{t_1} \in A \setminus Q \) (\( A \) – all algebraic numbers). So that, from (2.1) we can define a \( \hat{f} \) function uniquely by \( \hat{g}_i, \quad i = 1, 2 \) and using calculate one shall get:
\[
\hat{f}(\xi) = \alpha(\xi)(\hat{g}_1(\xi) + \hat{g}_2(\xi)),
\]
where \( \alpha(\xi) := \frac{|\xi|^2}{2(\sin^2 \frac{|\xi|}{2} t_1 + \sin^2 \frac{|\xi|}{2} t_2)} \).

Now, we will show that the function satisfies the following condition
\[
|\alpha(\xi)| \leq T(\alpha)|1 + |\xi||^{2-2\kappa}, \quad \kappa > 0
\]
that is, this function is a distribution. Without loss of generality we take \( t_1 = 1, \quad t_2 = \alpha \in A \setminus Q \) and show the following inequality
\[
\sin^2(|\xi|) + \sin^2(\alpha|\xi|) \geq T|\xi|^{2-2\kappa}, \quad (2.2)
\]
Assume that the \( |\alpha| |\xi| - nk| \leq \frac{T(k)}{|\xi|^{1+\kappa}} \) inequality holds, here \( \kappa > 0 \). Now we prove the next auxiliary lemma.

**Proof Lemma.** We can easily see that, the upper estimate is hold. So that we have to show the lower bound. Actually, for large enough \( |\xi| \), distance between the given sets are not less than \( \frac{T}{|\xi|^{1+\kappa}} \) in time, if it holds then
\[
\begin{align*}
\|\xi|-nk_i|&<\frac{T}{|\xi|^{1+\kappa}} \quad \text{(2.3)} \\
|\alpha| |\xi| - nk_i| &< \frac{T}{|\xi|^{1+\kappa}}.
\end{align*}
\]

But large enough \( |\xi| \) and according to the Roth theorem for \( 0 < \varepsilon < \kappa \)
\[
|\alpha| |\xi| - nk_i| = |(|\xi| - nk_i)\alpha + \pi \varepsilon k_i - nk_i| > \frac{T_1}{k_i^{1+\kappa}} - \frac{T_2}{|\xi|^{1+\kappa}}, \quad (2.4)
\]
holds. And to the first inequality of (2.3) we get
\[
\frac{1}{\pi} \left( |\xi| - \frac{T}{|\xi|^{1+\kappa}} \right) < k_i \leq \left( |\xi| + \frac{T}{|\xi|^{1+\kappa}} \right)
\]
and \( |\xi| + \frac{T}{|\xi|^{1+\kappa}} \leq 2 |\xi| \) holds for \( |\xi| > 1 \), so that for (2.4) we get
\[
|\alpha| |\xi| - nk_i| \geq \frac{T_2}{|\xi|^{1+\kappa}}.
\]

Now we show the (2.2) estimate. According to the known \( \sin x \geq \frac{2x}{\pi}, \quad x \in [0, \frac{\pi}{2}] \) inequality, we have
\[
\sin^2(\alpha |\xi|) = \sin^2(\alpha |\xi| - nk) \geq \bar{T}|\alpha| |\xi| - nk|^2 \geq \frac{T}{|\xi|^{2+2\kappa}}
\]
or
\[
\sin^2(|\xi|) = \sin^2(|\xi| - \pi \kappa) \geq \frac{T}{|\xi|^{2+2\kappa}}.
\]
So that, \( \forall \kappa > 0 \) the following estimate \(|a(\xi)| \leq T(1+|\xi|^{4+2\kappa})\), holds.

**Corollary.** If the \( \hat{g}_1 + \hat{g}_2 \) is given, then \( \hat{f} \) is uniquely defined and using from inverse Fourier transform for it, we get

\[
\hat{f}(x) = \tilde{\alpha} \ast (g_1(x) + g_2(x)). \quad (2.5)
\]

**Proof theorem 2.** Assume that \( g_1, g_2 \in D'(R^3) \) are generalized functions satisfying the condition \( T_1 g_2 = T_2 g_1 \). According to the (2.5) and by commutative of the convolution distribution function one has:

\[
\begin{align*}
T_1(f) &= T_1(\tilde{\alpha} \ast (g_1 + g_2)) = T_1(\tilde{\alpha} \ast g_1) + T_1(\tilde{\alpha} \ast g_2)
= \tilde{\alpha} \ast T_1 g_1 + \tilde{\alpha} \ast T_1 g_2
= (\tilde{\alpha} \ast (T_1 + T_2)) \ast g_1 = g_1,
\end{align*}
\]

that is \( T_1(f) = g_1 \). Similarly we get \( T_1(f) = g_2 \).

So that, the theorem 1 is valid.

**Proof Proposition. First part.** Let (1.4) holds. Then, according to the system of (2.1), we have

\[
\hat{g}_i(\xi) = \frac{1 - \cos(|\xi| t_i)}{|\xi|^2} \hat{f}(\xi), \quad i = 1, 2
\]

or

\[
\hat{g}_i(\xi) = \frac{1 - \cos(|\xi| t_i)}{|\xi|^2} \delta(|\xi| - r_0),
\]

where, \( \delta(\cdot) \) is Dirac’s delta on the sphere \( \{ |\xi| = r_0 \} \) and here we get the Fourier transform of \( f(\cdot) \). From (2.1) we get

\[
\hat{g}_i(\xi) = 0
\]

or \( g_1(x) = g_2(x) = 0 \).

**Second part.** We can easy conclude using by (2.1). It means the condition \( t_2 \neq t_1 \in A \setminus Q \) of theorem 1 is essential.

This problem hasn’t solved for transcendental cases. We have some a little result for it and the next work we will bring that cases.

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