ADAPTIVE POLE PLACEMENT ALGORITHMS FOR OF NON-MINIMUM-PHASE STOCHASTIC SYSTEMS

J.U. Sevinov
Tashkent State Technical University, Tashkent, Uzbekistan, sevinovjasur@gmail.com

O.H. Boeva
Navoi State Mining Institute, Navoi, Uzbekistan, boyeva-oqila@mail.ru

Follow this and additional works at: https://uzjournals.edu.uz/ijctcm

Part of the Engineering Commons

Recommended Citation
DOI: https://doi.org/10.34920/2020.5-6.38-42
Available at: https://uzjournals.edu.uz/ijctcm/vol2020/iss5/7

This Article is brought to you for free and open access by 2030 Uzbekistan Research Online. It has been accepted for inclusion in Chemical Technology, Control and Management by an authorized editor of 2030 Uzbekistan Research Online. For more information, please contact sh.arkinov@edu.uz.
ADAPTIVE POLE PLACEMENT ALGORITHMS FOR OF NON-MINIMUM-PHASE
STOCHASTIC SYSTEMS

Cover Page Footnote
Tashkent State Technical University, SSC «UZSTROYMATERIALY», SSC «UZKIMYOSANOAT», JV
«SOVPLASTITAL», Agency on Intellectual Property of the Republic of Uzbekistan

This article is available in Chemical Technology, Control and Management: https://uzjournals.edu.uz/ijctcm/vol2020/iss5/7
ADAPTIVE POLE PLACEMENT ALGORITHMS FOR NON-MINIMUM-PHASE STOCHASTIC SYSTEMS

Sevinov J.U.¹[0000-0003-0881-970X], Boeva O.H.²[0000-0001-8176-6806]

¹Tashkent State Technical University, Tashkent, Uzbekistan
jasur.sevinov@tdtu.uz, sevinovjasur@gmail.com.
²Navoi State Mining Institute, Navoi, Uzbekistan
boyeva-oqila@mail.ru.

Abstract. Adaptive pole placement algorithms for non-minimal phase stochastic systems are presented. The characterization of the structure and the set of models of linear dynamic control objects are carried out. As an example, the algorithm of the extended self-tuning controller is considered. To find the estimate of the controller parameters, the recursive least squares method is used. An explicit algorithm is proposed that makes it possible to exclude the operation of matrix inversion, which requires significant computational costs. Pole placement algorithms are given in the presence of white noise at the output. These algorithms belong to the class of stochastic approximation algorithms, the convergence of which is much lower than that of recurrent least squares algorithms. The results of numerical analysis have confirmed their efficiency, which makes it possible to use them in solving applied problems of identification and synthesis of adaptive control systems for technological objects.

Keywords: adaptive controller, non-minimum-phase stochastic systems, pole placement, control object, stable estimation, global stability, system identifiability.

INTRODUCTION

The problem of the synthesis of controllers that ensure the stability of the control object not only with given parameters, but also with parameters containing uncertainty, is very relevant today. This is due to the fact that virtually all control objects operate under conditions of parametric uncertainty. The behavior of a real object operating in conditions of natural noise is characterized by some uncertainty, in addition, people who are characterized by some uncertainty of behavior are usually involved in control systems for complex systems [1,2].

An important property of the control object is the availability of state information. The parameters of objects can change, so the quality of transient processes in the ACS can deteriorate and even become unsatisfactory, and control systems can become unstable. Therefore, it is often required to adjust the parameters of control devices (regulators) in accordance with the changed parameters of the object. To determine the parameters of objects, it is proposed to use identification methods - evaluating the parameters of objects.

The assessment of the fulfillment of the conditions of controllability, observability and non-degeneracy, as well as the quantitative assessment of their degree, are, of course, important stages in the analysis of the properties of objects in the construction of modal control systems with state or input-output regulators. However, it is known [3,4] that even under the condition of a high degree of controllability and observability of some objects, situations may arise when, as a result of solving the problem of synthesizing a modal control system, solutions are obtained that have low parametric roughness.

When managing such objects in conditions of their stability and minimal phase, certain contradictions may arise between the design requirements and the system capabilities, which are resolved...
only by the appearance of non-minimal phase links in its structure or formation of positive feedbacks along
the output coordinate and its derivatives, which significantly degrade the robust properties of the ACS
[2,5,6].

**Problem Definition**

Consider a randomly perturbed linear system with one input and one output, described by the
following difference equation:

\[ A(z^{-1})y_t = B(z^{-1})u_t + C(z^{-1})e_t, \]  \hspace{1cm} (1)

where \( A, B, C \) are polynomials for which \( A(0) \) and \( C(0) \) are equal to one, \( B(0) \) is equal to zero, and \( z^{-1} \)
is the unit backward shift operator.

Signals \( y_t, u_t, e_t \) correspond to output, controlled input, and zero mean white noise type
disturbance.

If we introduce a regulator corresponding to equation [4,7,8]:

\[ Fu(t) + Gy(t) = 0, \]  \hspace{1cm} (2)

for a closed loop, we can obtain equation

\[ T y(t) = F e(t), \]  \hspace{1cm} (3)

that is, the system of poles for a closed loop is determined by the zeros of the selected polynomial \( T \),
provided that \( F \) and \( G \) satisfy the polynomial identity

\[ AF + BG = CT, \]  \hspace{1cm} (4)

at \( F(0) \) and \( T(0) \) equal to one.

Identity (4) corresponds to a unique solution for \( F \) and \( C \), provided that \( A \) and \( B \) are coprime, and \( n_f, n_g \) and \( n_t \) satisfy relations

\[ n_f = n_b - 1, \quad n_g = n_a - 1, \]  \hspace{1cm} (5)

\[ n_t \leq n_u + n_b - n_c - 1. \]  \hspace{1cm} (6)

Under these conditions, the solution to equation (4) is obtained by inverting the matrix of order \( n_u + n_b - 1 \). This is one of the drawbacks of this technique, when the calculations must be repeated at every
discrete moment in time. In order to get away from equation (4), the following selection rule for \( F \) and \( C \)
is accepted:

\[ F = T - PB, \quad G = PA, \]  \hspace{1cm} (7)

which satisfies identity

\[ AF + BG = AT, \]  \hspace{1cm} (8)

for any polynomial \( P \). Then expression (3) is replaced by

\[ ATy(t) = FCe(t), \]  \hspace{1cm} (9)

In the case when the parameters are unknown, the controller (2), (4) should be synthesized based on
the estimates of the parameters for the selected structure of the models [9,10].

As an example, consider the following algorithm for the extended self-tuning controller.

At time \( t \):

1) observe \( y_t \) for system (1);

2) calculate scores \( \hat{y} \) and \( \hat{u} \) using the model

\[ \hat{y}_t = \hat{C} u_t + \hat{e}_t, \]  \hspace{1cm} (10)

3) where \( \hat{C} \) is a selected fixed polynomial;
4) determine \( \hat{F} \) and \( \hat{G} \) from the identity
\[
\hat{F} \hat{F} + \hat{G} \hat{G} = \hat{C} \hat{T}
\] (11)

5) apply control \( u_t, \) satisfying condition
\[
\hat{F}u_t + \hat{G}y_t = 0
\] (12)

6) set \( t \to t+1 \) and return to step (1).

If \( \hat{C} \) is fixed, then recurrent least squares can be used to generate \( \hat{F} \) and \( \hat{G} \). If we take \( \hat{C} \) equal to one, then the algorithm is reduced to the standard adaptive control algorithm, and by appropriately choosing \( \hat{C} \), the convergence rate of the algorithm can be increased.

However, if set to equal in (11), the controller (12) can be selected as
\[
(\hat{F} + \hat{T})u_t = \hat{C} y_t
\] (13)

This corresponds to \( P=-1 \) in (7). Hence, an explicit algorithm can be proposed, which makes it possible to exclude the operation of matrix inversion, which requires significant computational costs.

**Explicit Algorithm.**

At time \( t \):

1. observe \( y_t \) for system (1);
2. calculate scores \( \hat{F} \) and \( \hat{G} \) using the model
\[
\hat{y}_t = \hat{F} u_t + \hat{G} e_t
\] (15)
3. calculate when \( \hat{F} \) and \( \hat{G} \)
\[
\hat{F} = T - P \hat{B}, \quad \hat{G} = P \hat{C}
\] (16)
4. apply control \( u_t \) satisfying the condition
\[
\hat{F}u_t + \hat{G}y_t = 0
\] (17)
5. set \( t \to t+1 \), and return to step (1).

Before performing a full analysis of the stability of the algorithm for, it is useful to investigate possible points of convergence in a more general case and, in particular, to find sufficient conditions for the system to be identifiable, that is, for the validity of (4). These conditions are reduced to the following algorithm [1,2,11]:

If: 1. \( \hat{F} = \hat{A} \) and \( \hat{G} = \hat{B} \);
2. \( \hat{C} \) is resistant;
3. \( T - P \hat{B} \) and \( P \hat{A} \) are mutually simple;
4. \( n_p = 0 \);
5. \( n_t \leq n_b + \min(n_a, n_c) \) then \( \hat{F} \to F \) and \( \hat{G} \to G \), where \( F \) and \( G \) satisfy (4).

It follows from the algorithms that \( \hat{A} \) and \( \hat{B} \) satisfy the polynomial equation
\[
p_0 (A \hat{B} - B \hat{A}) = (C - A)T
\] (17)

If \( C = A \), then the solution (11) will be \( \hat{A} = A \) and \( \hat{B} = B \), open-loop stability is required.

A special case of the pole placement problem is when there is white noise at the output. Consider the option when the algorithm is used in relation to the system
\[
Ay_t = Bu_t + Ae_t
\] (18)
where \( \{ e_i \} \) is a real stochastic process defined on a probability space and satisfying

\[
E[e_t | \mathcal{F}_{t-1}] = 0 ,
\]

\[
E[e_t^2 | \mathcal{F}_{t-1}] = \sigma^2 ,
\]

\[
\lim_{N \to \infty} \sup_{t} \frac{1}{N} \sum_{i=1}^{N} e_i^2 < \infty ,
\]

System (12) can be represented as

\[
y_t = \varphi_{t-1}^{r} \theta_0 + e_t ,
\]

where

\[
\theta_0 = [-a_1 \ldots -a_{n_a} \ b_1 \ldots b_{n_b}]
\]

\[
\varphi_{t-1}^{r} = \left[ y_{t-1} - e_{t-1} \ldots y_{t-n_a} - e_{t-n_a} \ u_{t-1} \ldots u_{t-n_b} \right]
\]

At step (2) of algorithm \( \hat{\theta} \) - the estimate of the vector of model parameters in (14) are formed using the following recursion

\[
\hat{\theta}_t = \hat{\theta}_{t-1} + \frac{\alpha}{r_{t-1}} \hat{\phi}_{t-1} \hat{e}_t \quad (\alpha > 0) ,
\]

\[
\hat{e}_t = y_t - \hat{\varphi}_t^{r} \hat{\theta}_t ,
\]

\[
\hat{\varphi}_t^{r} = \left[ y_{t-1} - \hat{e}_{t-1} \ldots y_{t-n_a} - \hat{e}_{t-n_a} \ u_{t-1} \ldots u_{t-n_b} \right] ,
\]

\[
r_t = r_{t-1} + \varphi_t^{r} \hat{\phi}_t \quad r_0 = 1 .
\]

This class of algorithms belongs to the class of stochastic approximation algorithms. It is known that they converge much slower than recurrent least squares algorithms [5,13-15].

Under the assumptions:

- **A1** the upper limits \( n_a \) and \( n_b \) are known;

- **A2** \( A(z^{-1}) - \frac{1}{2} \alpha \) strictly positive real, the algorithm has the following characteristics with probability equal to one:

  \[
P_1 \| \hat{\theta}_t \| < M_1(\omega) < \infty \quad \text{for all } t ;
\]

  \[
P_2 \sum_{t=1}^{\infty} \| \hat{\theta}_t - \hat{\theta}_{t-1} \|^2 < \infty \quad \text{for any final positive} ;
\]

  \[
P_3 \sum_{t=1}^{\infty} \frac{\xi_t^2}{r_t} < \infty , \quad \text{where } \xi_t = e_t - \hat{e}_t \in \mathcal{F}_{t-1} .
\]

Assumption A2) implies that the open-loop system is stable, i.e.

\[
A2' \quad A(z) = 0 \Rightarrow |z| > 1 .
\]

In addition, P2) assumes

\[
P_2' \lim_{t \to \infty} \| \hat{\theta}_t - \hat{\theta}_{t-1} \| = 0 , \quad \text{almost certainly} .
\]

The characteristics of \( P1-P3 \) do not depend on the way the input signal is generated, except that it must be causal in the sense that \( u_t \in \mathcal{F}_t \).

The controller uses the algorithm (18) and
we will introduce the error signal
\[ \eta_i = e_i - e_{i-1}. \] (30)

**GLOBAL SUSTAINABILITY**

If assumptions A1) and A2) are valid, the algorithm ensures stable behavior of the system in the sense that with a probability equal to one [4,7]:

\[ \lim_{N \to \infty} \sum_{i=1}^{N} y_i^2 < \infty, \] (31)

\[ \lim_{N \to \infty} \sum_{i=1}^{N} \mu_i^2 < \infty, \] (32)

\[ \lim_{N \to \infty} \sum_{i=1}^{N} \varepsilon_i^2 = 0, \] (33)

\[ \lim_{N \to \infty} \sum_{i=1}^{N} \eta_i^2 = 0. \] (34)

**CHARACTERISTICS OF THE CONTROLLER.**

The characteristics of the evaluation process P1) and P2) imply the following characteristics of the controller with a probability equal to one [7]:

C1) \[ |d_t| < M_2(\omega) < \infty \]

for all \( t \);

C2) \[ \sum_{i=r}^{\infty} (d_i - d_{i-r})^2 < \infty \]

for any finite positive integer \( r \), where \( d_t \) denotes any coefficient for any controller polynomial at time \( t \).

**SYSTEM IDENTIFIABILITY.**

The properties of the forecast error (33) assume that the system is identifiable in the sense that the adaptive controller satisfies the condition

\[ \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \left[ \mathcal{I} \left( T_y - \hat{T}_y e_i \right) \right]^2 = 0 \]

If, moreover,

A3) \[ \mathcal{I}(\varepsilon) = 0 \Rightarrow |\varepsilon| > 1 \]

for all \( t > T_i \)

then

\[ \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} (T_y - \hat{T}_y e_i)^2 = 0. \]

The algorithms presented turn out to be effective in solving applied problems of identification and synthesis of adaptive control systems for technological objects under conditions of variability of the object noise covariance matrices and noise of measurement.

**References**


