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DECOMPOSITION METHOD FOR MODELING AND RESEARCH OF PULSE-FREQUENCY CONTROL SYSTEMS

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DECOMPOSITION METHOD FOR MODELING AND RESEARCH OF PULSE-FREQUENCY CONTROL SYSTEMS

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Abstract. *The pulse-frequency control systems are widely used in radar, space industry, for control of technological processes and robots and many other areas. Today, there are various approximate and accurate methods for the analysis and synthesis of pulse-frequency systems (PFS). However, the area of the practical application of the existed methods is mainly limited to single-variable systems. The classical methods provide the consideration of the initial structures of pulse-frequency systems as a whole. This article proposes the decomposition method for modeling and research pulse-frequency automatic control systems. The method is based on the mathematical apparatus of signal-flow graphs. We can use the method for analysis and synthesis of both single-variable and multivariable automatic control systems with pulse-frequency modulation.*

Key words: *pulse-frequency modulation, pulse-frequency system, multivariable system, nonlinearity, decomposition method, analysis, signal-flow graph.*

The features of the decomposition approach

In this part of the article, we will reveal the features of the decomposition approach on the example of a linear pulse system.

As an illustration, Figure 1a shows a single-variable pulse system. The matrices $\overrightarrow{A}$, $\vec{B}$ of the weight coefficients a_{ij}, b_{ij} of the transition functions define the continuous parts of the system. Figure 1b shows the signal-flow graph of this system.

In the case of the pulse transient function *A(t,τ)б*, coefficient a_{ij} means the weight coefficient determined at the *i*-th instant by the pulse transient function, caused by the *j*-th output pulse. In the graph model, transfer a_{ij} means all edges directed to vertex *i* from vertex *j*. Double indexing of the weight coefficients (transfers of graph edges) is formed from one pulse transient function.

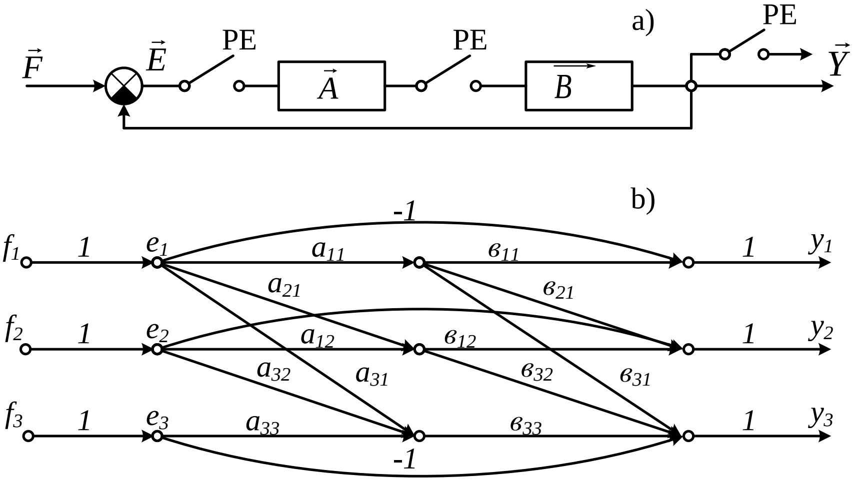


Figure 1. Closed-loop discrete system (a), signal-flow graph (b)

It is important to emphasize that dynamic graphs play a dual role. Firstly, directly on their basis, it is possible to solve the problems of the calculating transients, do analysis and synthesis of the discrete systems. Second, they are the basis for deriving recurrent equations, where the parametric and structural complexities of the original systems are largely excluded.

So, directly from the system graph (Figure 1b) we get:

$$y_1 = \frac{a_{11}b_{11}}{1+a_{11}b_{11}} \cdot f_1;\ e_1 = f_1 - y_1;\ y_2 = \frac{a_{11}b_{21}e_1 + a_{21}b_{22}e_1 + a_{22}b_{22}f_2}{1+a_{22}b_{22}};$$
$$e_2 = f_2 - y_2;$$
$$y_3 = \frac{a_{11}b_{31}e_1 + a_{21}b_{32}e_1 + a_{31}b_{33}e_1 + a_{22}b_{32}e_2 + a_{32}b_{33}e_2 + a_{33}b_{33}f_3}{1+a_{33}b_{33}}; \quad (1)$$
$$e_3 = f_3 - y_3.$$

. . . .

Analyzing the expressions for y_1, y_2, y_3, one can easily derive a recurrence relation for y_n, which allows to calculate the values of the output discrete for the instants without further referring to the graph.

$$y_n = \sum_{i=1}^{n-1} \frac{\langle \vec{a}_{ii,ni} \times \vec{a}_{ii,nn} \rangle \cdot e_i}{1+a_{nn}b_{nn}} + \frac{f_n \cdot a_{nn}b_{nn}}{1+a_{nn}b_{nn}};\ e_i = f_i - y_i. \quad (2)$$

Using the signal-flow graphs, we decompose the system at the level of dynamic processes. From equation (1), we can see that only one feedback loop is involved at the each step of the calculation. The rest of the feedback loops reflect either the history of the processes or the future development of the processes.

Decomposition processes based on signal-flow graphs simaltenuously allow achieving maximum simplification of the computation scheme due to the completeness taking into account the real physical features of plants characteristics. So, for example, the initial values of impulse transients functions (weight functions) of the overwhelming majority of plants at the instants of the application of the actions are equal to zero. Due to this real property of the plants, all feedback loops signal-flow graphs are broken. That is, there is a secondary decomposition of graph models of dynamic processes. Therefore, taking into account a_{ii}=0, b_{ii}=0, we can transform the signal-flow graph (Figure 1b) into the graph shown in Figure 2.

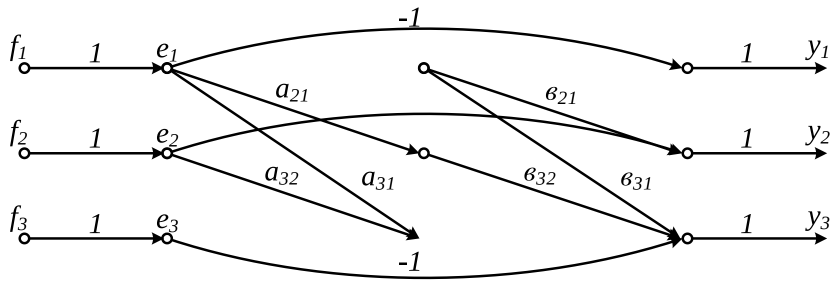


Figure 2. Signal-flow graph with of the edge transfers a_{ii}=0, b_{ii}=0

In this case, we simplify the calculation of the output coordinate largely:

$y_1 = 0; \qquad e_1 = f_1 - y_1;$

$y_2 = 0; \qquad e_2 = f_2 - y_2;$

$y_3 = a_{12} \cdot b_{32} \cdot e_1; \quad e_3 = f_3 - y_3;$

$y_4 = a_{21} \cdot b_{42} \cdot e_1 + a_{31} \cdot b_{43} \cdot e_1 + a_{32} \cdot b_{43} \cdot e_2; \; e_4 = f_4 - y_4;$

. . . .

In the conclusion of the article introductory section, we note, the classical methods of studying linear discrete systems (matrix methods, the *Z*-transform method or the equivalent *D*-transform method [1-3, 5]) do not provide opportunities for the natural decomposition of processes of pulse systems.

For example, when using matrix methods, the vector of the system output discrete (Figure 1a) is determined "in general" from the ratio

$$\vec{Y} = [\vec{J} + \vec{B}\vec{A}]^{-1} \vec{B}\vec{A}\vec{F} \,, \tag{3}$$

and when we use *Z*-transformation, the output value is also determined from the expression "in general"

$$y[n] = Z^{-1} \left\{ \frac{A_1(z) \cdot A_2(z)}{1 + A_1(z) \cdot A_2(z)} \cdot F(z) \right\}. \tag{4}$$

This circumstance is a source of fundamental difficulties in the study of even linear single-variable multi-rate systems and linear multivariable systems. This note is especially true for nonlinear pulse systems [1-17].

Nonlinear systems with pulse frequency modulation

A single-variable nonlinear automatic control system with pulse-frequency modulation represents as a series connection of a pulse-frequency modulator (PFM) and a linear continuous part with a transfer function *W*(*p*) (Figure 3).

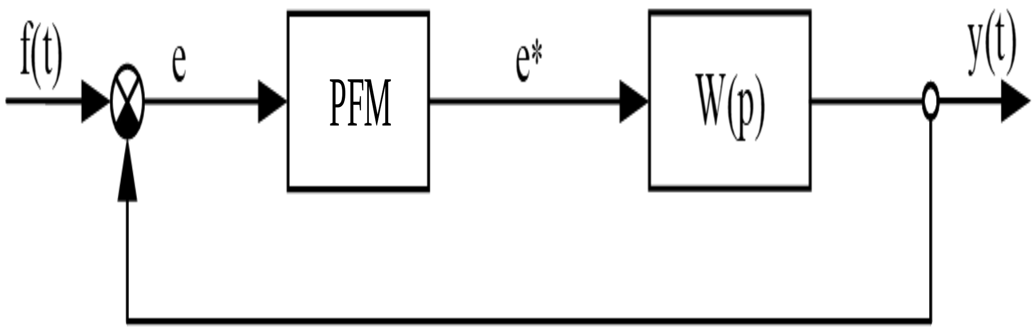
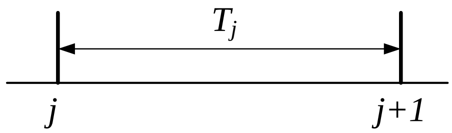


Figure 3. A single-variable nonlinear automatic control system with pulse-frequency modulation

The system dynamic processes calculation falls into two stages. At the first stage, assuming that there is an ordinary linear system, we form the graph model. At the second stage, according to the following algorithm, we carry out the calculation of dynamic processes. Let us introduce the notation.

T_j is the duration of the j-th time interval or otherwise the shift between the j-th and $(j+1)$-th output pulses of the modulator



$$T_j = \psi\left[e_j\left(t_j\right)\right], \tag{5}$$

t_j is the moment when the j-th pulse appears.

$$t_n = \sum_{j=0}^{n-1} T_j, \tag{6}$$

$$e_j = f_j - y_j \text{ is the error signal,} \tag{7}$$

$$e_j^* = b\, Sign\left(e_j\right), \tag{8}$$

b is a constant, in particular, it can be equal to 1.

Algorithm

18. If the pulse transient function of the reduced continuous part of the system $W_{\text{ПНЧ}}(t)$ is determined, then we take into account the real pulse duration τ:

$$W_{\text{ПНЧ}}(t) = L^{-1}\left\{\frac{1-e^{-p\tau}}{p} \cdot W(p)\right\}. \tag{9}$$

If there is an ideal pulse modulator with output unit pulses of type $\delta(t)$, then we determine a pulse transient function $W(t)$:

$$W(t) = L^{-1}\{W(p)\}. \tag{10}$$

19. We build the signal-flow graph of the system.
20. Taking into account the pulse repetition periods at the output of the pulse-frequency modulator, step-by-step, we determine the graph edges transmissions.
21. Step by step we determine the values:

$$y_j;\ \ e_j = f_j - y_j; \qquad e_j^* = Sign\, e_j;\ T_j; \qquad t_j.$$

Example

We will show the application of the algorithm for the system (Figure 3) with the following parameter values:

5. PFM generates pulses with duration τ = 0.1;
6. $W(p) = \frac{5}{p(p+0,5)}$;
7. PFM is replaced by the holder $W_{\text{Ф}}(p) = \frac{1-e^{-p\tau}}{p}$;
8. The transfer function of the reduced continuous part is

   $W_{\text{пнч}}(p) = \frac{1-e^{-p\tau}}{p} \cdot \frac{5}{p(p+0,5)} = \frac{5}{p^2(p+0,5)} - \frac{5}{p^2(p+0,5)} e^{-p\tau}$;

1 $T = \frac{2}{|e(t)|+0,1}$;

2 $f(t) = 4$;

3 $W_{\text{ПНЧ}}(t) = L^{-1}\{W_{\text{ПНЧ}}(p)\} = L^{-1}\left\{\frac{5}{P^2(p+0,5)}\right\} - L^{-1}\left\{\frac{5}{p^2(p+0,5)} \cdot e^{-p\tau}\right\}$;

$$W_{\text{ПНЧ}} = 10\left(t - 2 + 2e^{-\frac{t}{2}}\right) - 10\left[(t-\tau)\ - 2 + 2e^{-\frac{t-\tau}{2}}\right] =$$

$$= 10t - 20 + 20e^{-\frac{t}{2}} - 10t + 10\tau + 20 + 20\, e^{-\frac{t-\tau}{2}} = 20e^{-\frac{t}{2}}\left(1 - e^{\frac{\tau}{2}}\right) + 10\tau =$$

$$= 20e^{-\frac{t}{2}}\left(1 - e^{\frac{0,1}{2}}\right) + 10\tau = 20e^{-\frac{t}{2}}(1 - 1,0513) + 10 \cdot 0,1;$$

$$W_{\text{ПНЧ}} = \left(1 - 1,026e^{-\frac{t}{2}}\right).$$

In what follows, we will omit the subscript "PFM". It is necessary to calculate the transients for the PFS with the above parameters.

Solution

We build the signal-flow graph of the system for several instants (Figure 4). Let us calculate the discrete values of the transient y (t).

First step:

$$y_1 = 0;\ \ e_1 = f_1 - y_1 = 4;\ e_1^* = Sign\, e_1 = 1;\ \ t_1 = 0;$$

$$T_1 = \frac{2}{|e_1| + 0{,}1} = \frac{2}{4{,}1} = 0{,}488; \;\; t_2 = T_1 = 0{,}488.$$

Second step:

$$y_2 = a_{21} \cdot e_1^*;$$

$$a_{21} = W(t_2) = W(T_1) = 1 - 1{,}026 \cdot e^{-\frac{T_1}{2}} = 1 - 1{,}026 \cdot e^{-0{,}244} = 0{,}194;$$

$$y_2 = 0{,}194 \cdot 1 = 0{,}194;$$

$$e_2 = f_2 - y_2 = 4 - 0{,}194 = 3{,}806; \;\; e_2^* = 1;$$

$$T_2 = \frac{2}{|e_2| + 0{,}1} = \frac{2}{3{,}806} = 0{,}512;$$

$$t_3 = T_1 + T_2 = 0{,}488 + 0{,}512 = 1.$$

Third step:

$$y_3 = a_{31} \cdot e_1^* + a_{32} \cdot e_2^*;$$

$$a_{31} = W(t_3) = W(T_1 + T_2) = 1 - 1{,}026 e^{-\frac{1}{2}} = 1 - 1{,}026 \cdot 0{,}606 = 0{,}37;$$

$$a_{32} = W(T_2) = 1 - 1{,}026 e^{-\frac{T_2}{2}} = 1 - 1{,}026 e^{-\frac{0{,}512}{2}} =$$

$$= 1 - 1{,}026 \cdot 0{,}777 = 0{,}202;$$

$$y_3 = 0{,}37 \cdot 1 + 0{,}202 \cdot 1 = 0{,}572;$$

$$e_3 = f_3 - y_3 = 4 - 0{,}572 = 3{,}428;$$

$$e_3^* = 1 \cdot Sign\, e_3 = 1;$$

$$T_3 = \frac{2}{|e_3| + 0{,}1} = \frac{2}{3{,}528} = 0{,}566;$$

$$t_4 = T_1 + T_2 + T_3 = 0{,}488 + 0{,}512 + 0{,}566 = 1{,}566.$$

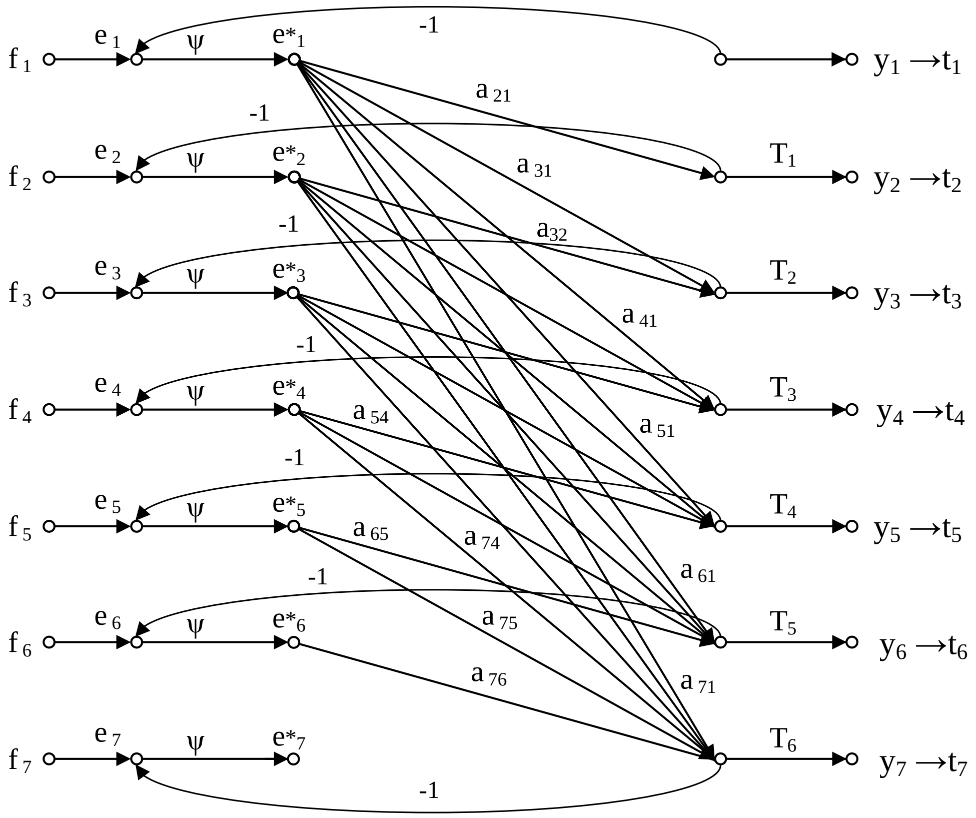


Figure 4. Signal-flow graph of a second-order pulse-frequency system

Fourth step:

$$y_4 = a_{41} \cdot e_1^* + a_{42} \cdot e_2^* + a_{43} \cdot e_3^*;$$

$$a_{41} = W(t_4) = 1 - 1{,}026e^{-\frac{t_4}{2}} = 1 - 1{,}026\, e^{-\frac{1{,}566}{2}} =$$

$$= 1 - 1{,}026 \cdot 0{,}458 = 0{,}53;$$

$$a_{42} = W(T_2 + T_3) = 1 - 1{,}026e^{-\frac{T_2+T_3}{2}} = 1 - 1{,}026e^{-\frac{-0{,}539}{2}} =$$

$$= 1 - 1{,}026 \cdot 0{,}582 = 0{,}402;$$

$$a_{43} = W(T_3) = 1 - 1{,}026e^{-\frac{T_3}{2}} = 1 - 1{,}026\, e^{-\frac{0{,}566}{2}} = 0{,}225;$$

$$y_4 = 0{,}53 \cdot 1 + 0{,}402 \cdot 1 + 0{,}225 \cdot 1 = 1{,}157;$$

$$e_4 = f_4 - y_4 = 4 - 1{,}157 = 2{,}843;$$

$$e_4^* = 1;$$

$$T_4 = \frac{2}{|e_4| + 0{,}1} = \frac{2}{2{,}943} = 0{,}68;$$

$$t_5 = T_1 + T_2 + T_3 + T_4 = 0{,}488 + 0{,}512 + 1{,}172 + 0{,}566 + 0{,}68 = 2{,}247.$$

From the analysis of these calculation stages, we can easily obtain the recurrence relation for calculating the discrete values of the output coordinate

$$y_n = \sum_{i=1}^{n-1} a_{n,i} \cdot e_i, \tag{11}$$

where $a_{n,i}$ is the discrete values of the pulse transition function $W(t)$ at the instant n; i is the time of application of a pulse $\delta(t)$ to the input of the continuous part; e_i is the error signal at the i-th moment of time.

$$a_{n,i} = \exp\left(-\sum_{i=1}^{n-1} T_i\right). \tag{12}$$

According to (11), (12) in Table 1 are summarized the calculation results, and in Figure 5 is shown the process curve.

Table 1

	f	*y*	*e*	*e**	*T*	*t*
1	4	0	4	1	0,488	0
2	4	0,194	3,806	1	0,512	0,488
3	4	0,572	3,428	1	0,566	1
4	4	1,157	2,843	1	0,68	1,567
5	4	1,96	2,04	1	0,93	2,247
6	4	3,085	0,915	1	1,97	3,176
7	4	4,899	-0,899	-1	2	5,146
8	4	4,968	-0,968	-1	1,87	7,146
9	4	4,388	-0,388	-1	4,09	9,016
10	4	3	1	1	1,81	13,1
11	4	4,44	-0,44	-1	3,703	14,91

12	4	3,1	0,9	1	2	18,61
13	4	3,655	0,345	1	4,49	20,61
14	4	4,85	-0,85	-1	2,1	25,1
15	4	4,31	0,31	-1	4,87	27,2
16	4	3,121	0,879	1	2,04	32
17	4	3,079	0,321	1	4,75	34,4
18	4	4,866	-0,866	-1	2,07	39,15

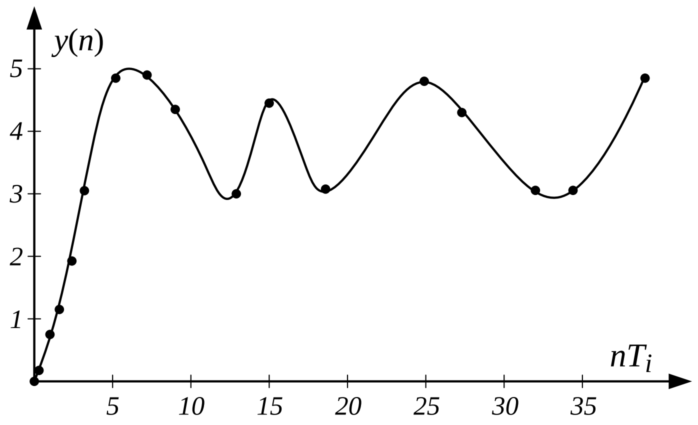


Figure 5. The output process curve

Conclusion

The article offers an accurate algorithm for calculating transients in nonlinear systems with pulse-frequency modulation based on the decomposition approach and the use of signal-flow graphs. The calculation algorithm is equally applicable to the stationary and non-stationary pulse-frequency systems.

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RESEARCH OF THE OPTICAL PROPERTIES OF THE RARE-EARTH METAL COATINGS

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Abstract. *The article discusses the results of a study of the optical properties of coatings with rare-earth elements of the optical materials. The process of photoluminescence is shown using lithium niobate as an example. Photoluminescence spectra for silicon samples are presented, as well as a photoluminescence intensity spectrum depending on wavelength for silicate soda glass samples implanted with erbium ions. The dependence of photoluminescence intensity and lifetime is shown for samples of polycrystalline films used in optical waveguides. The studies of the intensity and lifetime of photoluminescence doped with rare-earth elements of optical materials are carried out.*