Application of power transformations to algebraic equations

Zafar Xaydarov  
*Samarkand State University, Uzbekistan, zafarxx@gmail.com*

Follow this and additional works at: [https://uzjournals.edu.uz/samdu](https://uzjournals.edu.uz/samdu)

Part of the Life Sciences Commons, and the Physical Sciences and Mathematics Commons

**Recommended Citation**


Available at: [https://uzjournals.edu.uz/samdu/vol2021/iss1/2](https://uzjournals.edu.uz/samdu/vol2021/iss1/2)

This Article is brought to you for free and open access by 2030 Uzbekistan Research Online. It has been accepted for inclusion in Scientific Journal of Samarkand University by an authorized editor of 2030 Uzbekistan Research Online. For more information, please contact sh.erkinov@edu.uz.
UDC: 517.9

*Application of power transformations to algebraic equations*

Z.Kh. Khaydarov

*Samarkand State University, zafar-xaydarov@mail.ru*

**Annotation.** In this paper, we consider the method of power transformations in algebraic equations, for their simplification (shortening) corresponding to some face of the n-th order. An example of its use for some algebraic equation is also given in the work.

**Keywords:** Unimodular matrix, power geometry, Newton polyhedra.

---

**Приложение степенных преобразований к алгебраическим уравнениям**

**Аннотация.** В данной работе рассматривается метод степенных преобразований в алгебраических уравнениях, для их упрощения (укорочения) соответствующего некоторой грани n-го порядка. Так же в работе приведён пример его использования для некоторого алгебраического уравнения.

**Ключевые слова:** Унимодулярная матрица, степенная геометрия, многогранник Ньютона.

**Darajali almashtirishlarning algebraik tenglamalarga qo’llash**

**Annotatsiya.** Ushbu maqolada algebraik tenglamalarda darajali almashtirishlar usuli ko'rib chilgan, ya'ni ularni n - darajasidagi ba'zi yuzlarga mos keladigan soddalaishtrish (qisqartmalarga) keltirish ko’rsatilgan. Uning bir algebraik tenglama uchun ishlatilishiga misol ham keltirilgan.

**Kalit so’zlар:** Unimodulyar matritsa, darajali almashtirishlar, Nyuton ko’pyog’i.

Consider a function that is analytic at a point. It decomposes into a Maclaurin power series $f(X)X = 0$

$$f(X) = \sum f_Q X^Q,$$

Where and or, converging in some neighborhood of the point and. -real or complex vector and $Q \in \mathbb{Z}_+^n f_Q \in C f_Q \in RU = \{X: |x_i| < \varepsilon, i = 1, \ldots, n\} X =
\[ f(0) = 0X = (x_1, x_2, ..., x_n)Q = (q_1, q_2, ..., q_n) \in \mathbb{R}^n \]

\[ X^Q \overset{\text{def}}{=} x_1^{q_1}x_2^{q_2}...x_n^{q_n} \]

If in this product put

\[ x_i = \tau^{p_i}, i = 1, ..., n \]

then, that is, a power function of is obtained, the exponent of which is the dot product

\[ X^Q = x_1^{q_1}x_2^{q_2}...x_n^{q_n} = \tau^{p_1q_1+...+p_nq_n} = \tau^{(P,Q)}\tau \]

Let be a discrete set in. Consider a sum with real or complex odds \( R^n \)

\[ f(X) = \sum f_Q X^Q \text{ by } Q \in S \]

assuming that similar terms have already been given, i.e. each vector degree corresponds to one and only one term \( c \), which is the carrier of the sum. \( Qf_QX^Qf_Q \neq 0S \)

Across \( R^3 \) we will denote a real three-dimensional space. Then each point in space can be written as - Cartesian coordinates and represented as a three-dimensional vector. Let the real numbers be fixed. Then the solution to the equation

\[ (q_1, q_2, q_3)Q = (q_1, q_2, q_3)p_1, p_2, p_3 \]

form in a plane perpendicular to the vector. \( R^3P = (p_1, p_2, p_3) \)

We will change the value of \( c \) and consider the planes (1) for different ones. All these planes are parallel to each other: When the plane passes through the origin. When the plane is shifted in the direction of the vector When, the plane is shifted in the opposite direction. Each plane divides the space into two half-spaces, positive, in which, and negative, in which If you introduce the scalar product, then the equation of the plane (1) will be written in the form.

\[ c > 0. \forall c < 0 R^3q_1p_1 + q_2p_2 + q_3p_3 \geq cq_1p_1 + q_2p_2 + q_3p_3 \leq c. \langle Q, P \rangle = q_1p_1 + q_2p_2 + q_3p_3 \langle Q, P \rangle = c \]

Let a set be given in space. The smallest convex set containing is called the convex hull set. If there is a finite set of points, then its convex hull is either a closed polygon or a closed polytope, whose vertices are points of the set. \( D\Delta DD.DD \)

Let the set and vector be fixed. Let for all. Then the equation defines the set in the reference plane by the vector \( D \subset R^3_{1}. P \in R^3_{2} \sup \langle Q, P \rangle Q \in D\langle Q, P \rangle = cR^3_{2}L_{p}DP. \)

The inequality defines the corresponding support half-space. Moreover, there are no points of the set in the half-space. If there is a finite point set, then the
upper bound is equal to the maximum and on the support plane there are points of the set $(Q, P) \leq cR_1^2 L_{p}^{-} \langle Q, P \rangle > cD.DL_{p}D$.

The intersection consists of all those points for which the dot product has the greatest value. We denote by the intersection of all support half-spaces of the sets, i.e., over all $D \cap L_{p} = D_{p}D, \Gamma D, \Gamma = \cap L_{p}^{-} P \in R^3_2$

The set is a closed convex set containing the convex hull of the set, i.e. $\Gamma - \Delta D$, $\Gamma \supset \Delta \supset D$

Conuses: A set is called a cone if, together with a point, it contains a ray with for all. Denote by the set of all vectors for which there is a support half-space to the set. Then it will be a convex cone. This cone is called the normal cone of the set.

Let it be a face of a closed set. The set of all vectors for which the reference plane intersects with the set exactly along the face is called the normal cone of the face.

$K \subset R^3PPc > 0, UP \in R^3_2 L_{p}^{-}.UD. \Gamma j^{(d)} \Gamma U_j^{(d)} P \in R^3_2 L_{p} \Gamma_j^{(d)} \Gamma_j^{(d)}$

Let the set consist of a finite number of points. Then the convex hull of the set is closed and coincides with. The set is a polyhedron. Its vertices are points. Other points of the set that are not vertices are located on edges, on faces, or inside a polyhedron. A polyhedron has a finite number of vertices, edges, and faces. The vertices are denoted by the letters of the edges, the faces through $DQ_1, Q_2, ..., Q_m \Delta D \Gamma \Gamma D, D, \Gamma, \Gamma \Gamma j^{(0)} \Gamma_j^{(1)} \Gamma_j^{(2)}$

Integer lattices. The collection of all integer vectors is called an integer lattice and is denoted $Q = (q_1, q_2, q_3)Z^3$.

The following statement is true that if $u$ lies in the first octant, then the number of boundary subsets corresponding to vectors is finite. The intersection of all supporting half-spaces of a set is called a Newton polytope $D \subset Z^3\{Q \geq 0\}, D_p, P \leq 0 \Gamma D$

Newton polytopes are used to find near zero all solutions of the equation $x_1 = x_2 = x_3 = 0$

$$\sum f_{q_1 q_2 q_3} x_1^{q_1} x_2^{q_2} x_3^{q_3} = 0$$

where the integer exponents are non-negative $q_1, q_2, q_3$

Consider the following problem: Let a set of three-dimensional vectors be given for each vector (or) it is necessary to select such a boundary subset that $D, P \neq 0P < 0D_p$

$$\langle Q_i, P \rangle = \langle Q_j, P \rangle Q_i, Q_j \in D_p$$

$$\langle Q_k, P \rangle < \langle Q_j, P \rangle Q_k \in D \setminus D_p$$

Consider a linear transformation of coordinates into $R^3_2$

$$P_1' = \alpha_{11}p_1 + \alpha_{12}p_2 + \alpha_{13}p_3$$
with no special matrix

\[ \alpha = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} \]

i.e. This transformation in vector form is \( \text{det} \alpha \neq 0 \) (4.1) \( P' = \alpha P \).

scalar product for, and will be preserved after transformation (2), if we simultaneously make a linear transformation of coordinates in space \( \langle Q, P \rangle \)

\[ Q' = \alpha^{-1} Q \] (4.2)

where is the transposed matrix. \( \alpha^* \alpha \)

Then, by the property of the scalar product, we have \( \langle \beta Q, P \rangle = \langle Q, \beta^* P \rangle \)

\[ \langle Q', P' \rangle = \langle \alpha^{-1} Q, \alpha P \rangle = \langle Q, \alpha^{-1} \alpha P \rangle = \langle Q, P \rangle \]

Thus, under the transformations (2), (3) in the spaces and, accordingly, the scalar product between the vectors of these spaces is preserved. Transformation (2) is a one-to-one mapping of space onto itself. In this case, straight lines turn into straight lines, planes turn into planes. All linear inequalities are preserved.

Let a set be given in space and the sets constructed earlier correspond to it. \( R_1^3 R_2^3 R_1^3 D \Gamma, U, \Gamma_j^{(d)}, D_j^{(d)}, U_j^{(d)} \),

Let be the set obtained from after transformation (3) and there correspond the sets \( D' D \)

\[ \Gamma', U', \Gamma_j^{(d)'} D_j^{(d)'} U_j^{(d)'} \ldots \]

Then

\[ \Gamma' = \alpha^*-1 \Gamma, \ldots, \Gamma_j^{(d)'} = \alpha^*-1 \Gamma_j^{(d)}, D_j^{(d)'} = \alpha^*-1 D_j^{(d)} U' = \alpha U_j^{(d)'} = U_j^{(d)} \]

A task. Let some face be required to make such a transformation (3) so that the face \( \Gamma_j^{(2)} \) —

was parallel to one of the coordinate planes. A matrix in which all elements are integer and is called unimodular. For a unimodular matrix, the inverse matrix is also unimodular. The main property of transformation (2) with unimodular is that it is one-to-one on the integer lattice \( \Gamma_j^{(2)'} = \alpha^{-1} \Gamma_j^{(2)} \) \( \alpha, \alpha_{ij} \text{det} \alpha \neq 0 \alpha \alpha^{-1} \alpha Z^3 \).

Let us solve the problem in the class of unimodular transformations (2), assuming that the set. In this case, all vertices of the polytope are integers. So, there is a face with integer ends, it is required, using the unimodular
transformation (2), to translate it into which will be parallel to one of the coordinate planes. Let the unit vector on the face, i.e. difference between adjacent integer points of the plane. Then the vector is perpendicular to the plane, and the integers are coprime. It is necessary to find a unimodular matrix such that the vector $D \subset \mathbb{Z}^3 \Gamma_j^{(2)} \Gamma_j^{(2)} R = (r_1, r_2, r_3) \Gamma_j^{(2)} \Gamma_j^{(2)} r_1, r_2 \quad \text{and} \quad r_3 \alpha$, one of the coordinates was canceled. To simplify further calculations, we assume that we indicate such a sequence of unimodular coordinate changes, as a result of which we obtain the required transformation. $P > 0$.

Consider a nonlinear system of polynomial equations (1). Suppose that for each polynomial Newton polytopes $M_i$ are constructed and their corresponding elements are highlighted. The normal cones of the truncated system (2) are constructed from the normal cones of the faces (see [3]). The dimensions of the cones of truncated systems will be zero-dimensional, one-dimensional and two-dimensional, etc.

Consider a method for finding the matrix of a power transformation.

To bring it to some standard form, we use power transformations. Recently, power transformation has been widely used to solve various problems [1]. By means of power transformations, nonlinear in, but linear in, it is possible to simplify the sum in the function. Thus, the nonlinear problem is reduced to several quasilinear problems. It is remarkable that in this case nonlinear problems are closely related to some linear geometry in the "space of exponents" $f(X) X \ln X f(X)$.

Let be a square matrix of size with real elements and. Transformation $\alpha = \left(\alpha_{ij}\right) n \alpha_{ij} \det \alpha \neq 0$

$$y_i = x_1^{\alpha_{i1}} \ldots x_n^{\alpha_{in}}, i = 1, \ldots, n$$

one (one)
is called a power transformation with a matrix. Reverse transformation $\alpha$

$$x_i = y_1^{\beta_{i1}} \ldots y_n^{\beta_{in}}, i = 1, \ldots, n$$

is also a power transformation with a matrix. Indeed, transformations (1) and (2) are linear with respect to the logarithms of the coordinates. If we introduce vectors and, then power transformations (1) and (2) are linear transformations of these vectors: and $\beta = \left(\beta_{ij}\right) = \alpha^{-1} \ln X = (\ln x_1, \ldots, \ln x_n) \ln Y =$
\((\ln y_1, \ldots, \ln y_n) \ln Y = a \ln X \ln X = \beta \ln Y\)

For, the unimodular matrix of the power transformation can be constructed using continued fractions (see [2,3]). For or more dimensions, it is also possible to construct a unimodular matrix of a power transformation using generalized continued fractions. But in this case, the construction of these unimodular matrices leads to a lot of computation. Here we have proposed a method for constructing unimodular matrices for \(n = 5\) using the following example. \(n = 2\alpha n = 3\alpha \alpha\)

**Example.** Let \(R5\) be given an algebraic equation of the form
\[
f(X) = x_1^3x_2x_3^2x_4 + x_1^2x_2^3x_3x_5 + x_2^2x_3x_5^2 + x_1^3x_3^3x_4^4x_5 + x_1x_2x_4^2x_5 = 0 \quad (3)
\]
Here the exponents of the monomials: \(Q_1 = (3,1,2,1,0), Q_2 = (2,3,1,0,1), Q_3 = (0,2,1,0,2), Q_4 = (3,0,3,1,4), Q_5 = (1,1,0,2,1)\)

Find vectors, \(m = 1,2,3,4, Q_m\)

\[
k_1Q_1 = Q_2 - Q_1 = (-1, 2, -1, -1, 1), \quad k_1 = 1
\]
\[
k_2Q_2 = Q_3 - Q_1 = (-3, 1, -1, -1, 2), \quad k_2 = 1
\]
\[
k_3Q_3 = Q_4 - Q_1 = (0, -1, 1, 0, 4), \quad k_3 = 1
\]
\[
k_4Q_4 = Q_4 - Q_1 = (-2, 0, -2, 1, 1), \quad k_4 = 1
\]

Let's compose the matrix \(n4\) from vectors, \(m = 1,2,3,4, Q_m\)

\[
A = \begin{pmatrix}
-1 & 2 & -1 & -1 & 1 \\
-3 & 1 & -1 & -1 & 2 \\
0 & -1 & 1 & 0 & 4 \\
-2 & 0 & -2 & 1 & 1
\end{pmatrix}
\]

Using the Maple computer algebra system, with ready-made commands LinearAlgebra, SubMatrix, Rank, Determinant, all values for and were calculated. The rank of the matrix is \(4. M_{ij}^r l, i, j = 1,2,3,4, 5r = 1,2,3,4A\)

\[
M_{12}^1 = 15, M_{13}^1 = -7, M_{14}^1 = 12, M_{15}^1 = -2, M_{24}^1 = -15, M_{25}^1 = -5,
\]
\[
M_{12}^2 = 14, M_{13}^2 = -10, M_{14}^2 = 19, M_{15}^2 = -1, M_{24}^2 = 1, M_{25}^2 = -3,
\]
\[
M_{12}^3 = 3, M_{13}^3 = -4, M_{14}^3 = 5, M_{15}^3 = -3, M_{24}^3 = 2, M_{25}^3 = 2,
\]
\[
M_{12}^4 = 1, M_{13}^4 = -3, M_{14}^4 = 6, M_{15}^4 = 1, M_{24}^4 = -9, M_{25}^4 = 2,
\]
\[
M_1 = 13, M_2 = 25, M_3 = -29, M_4 = 33, M_5 = 1.
\]

Substituting all these values into the matrix, we getα
\[ \alpha = \begin{pmatrix} -15 & 14 & -3 & -1 & 13 \\ 30 & -28 & 6 & 2 & -25 \\ 34 & -32 & 7 & 2 & -29 \\ 39 & -37 & 8 & 3 & -33 \\ -1 & 1 & 0 & 0 & 1 \end{pmatrix} \]

\[ \det \alpha = 1 \ldots \] After cancellation by and a power transformation with a unimodular matrix, equation (3) is reduced to the equation

\[ x_1^3 x_2 x_3^2 x_4 \alpha g(Y) = 1 + y_1 + y_2 + y_3 + y_4 = 0 \]

References

