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Abstract: *Algorithms for increasing the roughness of the procedure for assessing the state vector of control objects to the influence of uncertainty factors are given. Expressions are obtained for extended state vectors and observations. Stable inversion algorithms are given for a nondegenerate block matrix with the allocation of its left and right zero divisors of maximum rank. The presented stable computational procedures allow us to regularize the problem of synthesis of algorithms for estimating the parameters of regulators in adaptive control systems with a customizable model and to improve the quality indicators of control processes under conditions of parametric uncertainty.*

Keywords: *state vector of objects, increasing the roughness of the evaluation procedure, uncertainty factors.*

I. Introduction

Currently, there are numerous approaches to valuation in the face of uncertainty. However, all of them are based on the use of certain system models and are tied to their specific implementations. In particular, the founder of random signal filtering, N. Wiener considered the possibility of creating an optimal filter based on the existing full implementation of a random process [1]. The well-known Kalman-Bucy filtering method [2] works already in real time, but its action is based on a priori information about the studied object.

For linear systems with a Gaussian (normal) input or noise, the Kalman filter is optimal [1-3]. Thus, a purely probabilistic approach, being a convenient mathematical formalization of a real situation, does not cover practically important cases of uncertainty. The task of estimating the vector of a linear system is much more complicated in statistically uncertain situations when there are only estimates of the statistical characteristics of noise and the initial state of the system.

II. Problem definition

Consider a linear model of an object with parametric uncertainty of the form:

$$x_{k+1} = \tilde{A}x_k + \tilde{B}u_k + w_k\,,\ x_{0|0} = x_0^0\,, \tag{1}$$

$$y_k = Hx_k + v_k\,, \tag{2}$$

in which $x \in R^n$, $y \in R^l$, $u \in R^m$ - vectors of object state, object output and control; $w_k \in R^n$, $v_k \in R^l$ - mutually independent white-noise Gaussian sequences in the equation of the object and in the equation of observation, for which

$$M\left[x_0^0\right] = \bar{x}_0^0\,,\ M\left[(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T\right] = P_0\,,$$

$$M\left[w_k\right] \equiv M\left[v_k\right] \equiv 0\,,\ M\left\{w_k w_\tau^T\right\} = Q_k \delta_{k-\tau}\,,\ M\left\{v_k v_\tau^T\right\} = R_k \delta_{k-\tau}\,,$$

in which Q_k, P_0 - positive semidefinite matrices; Is a positive definite matrix.

Assume that the parameters of the control object (1) $\tilde{A}$ and $\tilde{B}$ have the form $\tilde{A} = A_0 + \Delta A$, $\tilde{B} = B_0 + \Delta B$, in which A_0, B_0 - specified nominal values of the parameters where the object in the absence of parametric disturbances has the desired transient characteristics; ΔA, ΔB – quasistationary unknown matrices of perturbed object parameters.

Kalman score $\hat{x}_k$ for state vector x_k is unbiased and optimal in terms of the minimum mean square error [3-6]:

$$J_0 = M\left[(x_k - \hat{x}_k)^T (x_k - \hat{x}_k)\right] ,$$

in which $\hat{x}_k$ – optimal condition assessment.

For the nominal model, the Kalman estimate $\hat{x}_k$ calculated according to well-known formulas [4,6]:

$$\hat{x}_{k+1} = \tilde{A}\hat{x}_k + \tilde{B}u_k + K_k\left[y_k - H\hat{x}_k\right],\ \hat{x}_{0|0} = \bar{x}_0^0 , \tag{3}$$

$$K_k = P_k H^T R_k^{-1} , \tag{4}$$

$$P_{k+1} = \tilde{A}P_k + P_k\tilde{A}^T - P_k H^T R_k^{-1} H P_k + Q_k . \qquad P_{k,0} = P_k^0 . \tag{5}$$

We form using linear feedback as

$$u_k = u_{1,\mathrm{k}} - k_k^T x$$

internal control loop, the equations of which are of the form [5,6]:

$$x'_{k+1} = [A_0 - B_0 k_k^T]x'_k + B_0 u_{1,k} + w'_k ,\ x_{0|0} = x_0^0 , \tag{6}$$

$$y'_k = Hx'_k + v'_k , \tag{7}$$

The internal circuit state estimator is a Kalman-Bucy filter built for the internal circuit reference model [4-6]:

$$\hat{x}'_{k+1} = [A^M - \tilde{k}_k H]x'_k + \tilde{k}_k y_k + B^M u_{1,k} , \tag{8}$$

$$\hat{y}'_k = H\hat{x}'_k ,\ \tilde{k}_k = P_k H^T R_k^{-1} ,$$

$$P_{k+1} = A^M P_k + P_k (A^M)^T - P_k H^T R_k^{-1} H P_k + Q_k .$$

We introduce extended state and observation vectors $x_{p,k} = (x_k , x'_k)$, $y_{p,k} = (y_k , y'_k)$. The serial connection of the internal circuit and the Kalman-Bucy filter forms a dynamic system described by equations of the form:

$$x_{p,k+1} = A_p x_{p,k} + B_p u_{1,k} + w_k , \tag{9}$$

$$y_{p,k} = H_p x_{p,k} + v_k , \tag{10}$$

in which

$$A_p = \left[\begin{array}{c:c} A_0 - B_0 k_k^T & 0 \\ \hdashline \tilde{k}_k H & A^M - \tilde{k}_k H \end{array}\right],\ B_p = \left[\begin{array}{c} B_0 \\ \hdashline B_0 \end{array}\right],\ H_p = \left[\begin{array}{c:c} H & 0 \\ \hdashline 0 & H \end{array}\right]. \tag{11}$$

A parallel reference model is defined by equations of the form:

$$x_{k+1}^M = A_p^M x_k^M + B_p u_{1,k} ,$$

$$y_k^M = Hx_k^M ,$$

in which matrix A_p^M obtained by setting A^M in expression for A_p (11). It is easy to verify that for $B_{0,k} = B_0$ the conditions for the full adaptability of the internal circuit are fulfilled [6,7]. Note that in the general case, the reference model is non-stationary.

For the expanded nominal model (9), (10), one can obtain the following estimate:

$$\hat{x}_{p,k+1} = A_p \hat{x}_{p,k} + B_p u_{1,k} + K_k\left[y_k - H\hat{x}_{p,k}\right] , \tag{12}$$

$$K_k = L_k H^T R_k^{-1}, \tag{13}$$

$$L_{k+1} = A_p L_k + P_p L_p^T - L_k H^T R_k^{-1} H L_k Q\ ,\ L_0 = P_0 . \tag{14}$$

In [5-8] it is shown that the main minimization tool when using the extended object model is the mutual coordinate-wise compensation of vector components x_k and x'_k extended state vector $x_{p,k}$.

When using formulas (12) - (14), the optimal estimate of the initial state vector $\hat{x}_k$ – systems (1) at nominal values of matrices A_0, B_0, C_0 obtained after partial mutual compensation of the components of the assessment of the extended state vector $\hat{x}_{p,k}$:

$$\hat{x}_{p,k} = \hat{x}_k + \hat{x}'_k . \quad (15)$$

Assessment Vectors $\hat{x}_k$ and $\hat{x}'_k$ obtained as a result of vector decomposition $\hat{x}_{p,k}$ from (12). If system (1), (2) is considered with parametric uncertainty ($\Delta A, \Delta B, \Delta C$), then estimate (15) becomes biased, since the mutual compensation of vectors is violated $\hat{x}_k$ and $\hat{x}'_k$.

To get rid of this, an additional correction of the obtained estimate of $\hat{x}_{p,k}$ the extended state vector is needed [8]. As a new estimate, instead of (15) we will consider an expression of the form:

$$\tilde{x}_{p,k} = D\hat{x}_{p,k} = \begin{bmatrix} D_1 & D_2 \end{bmatrix} \cdot \begin{bmatrix} \hat{x}_k \\ \hat{x}'_k \end{bmatrix}, \quad (16)$$

in which $\tilde{x}_{p,k}$ – adjusted estimate vector for the state vector x_k, $D = \begin{bmatrix} D_1 & D_2 \end{bmatrix}$ – custom dimension parameters matrix $n \times 2n$.

Physically introducing matrix D means transforming x_k with the matrix D_1 and conversion of x'_k with the matrix D_2 in order to improve previous grades $\hat{x}_k$ in the presence of uncertainty. For optimal tuning, we consider the minimization problem with respect to D of the quadratic functional [1,3,8]:

$$J = M\{tr[y_k - HD\hat{x}_{p,k}] \cdot [y_k - HD\hat{x}_{p,k}]^T\} . \quad (17)$$

At each sampling step *k*, the output vector is measured y_k and the evaluation vector is computed $\hat{x}_{p,k}$ for an extended model of an object. Next, the true measured value $l \times 1$ of the vector y_k is compared with its analogue, obtained on the basis of the estimate of the expanded state vector (12) by the formula $\hat{y}_k = H\tilde{x}_{p,k} = HD\hat{x}_{p,k}$. Based on (17) we get:

$$J = tr\left\{M[y_k y_k^T] - 2HDx_{p,k}]M[y_k^T] + HDx_{p,k}x_{p,k}^T D^T H^T\right\}. \quad (18)$$

From here, using the matrix pseudoinverse operation, we obtain

$$D = (H^T H)^+ H^T M[y_k]\hat{x}_{p,k}^T[\hat{x}_{p,k}\hat{x}_{p,k}^T]^+ . \quad (19)$$

Formula (19) contains a matrix $(H^T H)$, which is most often not a full-rank matrix, since usually not all coordinates of the state vector are subject to observation. Therefore, it is possible that some rows of the matrix D consist of zero elements. Then an additional correction of the estimate of the state vector by formula (16) is possible not for all coordinates of the state vector, but only for those that correspond to nonzero rows of the matrix D.

This is a disadvantage of the proposed method in solving the estimation problem. Although in some cases such a solution is still possible even if there are zero rows in the matrix D.

III. Solution of the task

Expression (19) contains a pseudoinverse matrix $F^+ = (H^T H)^+$ to form tunable parameters *D* [9-12]. It is clear that the quality of control processes of the synthesized adaptive control system substantially depends on the accuracy of determining the parameters (19). In view of this circumstance, it becomes necessary to use efficient pseudoinverse algorithms for square matrices.

The matrix $F = (H^T H)$ in expression (19) is a degenerate square matrix, i.e.

$$F \in R^{n \times n}, \ \det F = 0, \ rankF = m < n . \quad (20)$$

Let us consider stable algorithms connecting the inverse operation of a nondegenerate block matrix composed of a given matrix and its left and right zero divisors of maximum rank. We form the block matrix [13-15]:

$$\begin{bmatrix} F^T & F_R \\ \hline F_L & 0 \end{bmatrix} = \begin{bmatrix} F & F_L^T \\ \hline F_R^T & 0 \end{bmatrix} \in R^{(2n-m)\times(2n-m)}, \tag{21}$$

In which F_L, F_R – left and right zero divisors of maximum rank, i.e. $F_L F = 0$,

$$rank F_L = n - m; \tag{22}$$

$$FF_R = 0,\ rank F_R = n - m. \tag{23}$$

Further, it is always assumed that the divisors F_L and F_R have maximal ranks in the sense of (20), (22), (23). In this case, the following rank identity is fulfilled [16]:

$$rank \begin{bmatrix} F^T & F_R \\ \hline F_L & 0 \end{bmatrix} = 2n - m. \tag{24}$$

Then the pseudoinverse matrix $F^+ \in R^{n\times n}$ has the form:

$$F^+ = TFT, \tag{25}$$

in which

$$T = (F + F_L^T \varphi^T F_R^T)^{-1} = (F + (F_R \varphi F_L)^T)^{-1}$$

- non-unique invertible matrix; Is an arbitrary square invertible matrix.

If the conditions of semi-orthogonality are satisfied

$$F_R^T F_R = F_L F_L^T = \varphi^T \varphi = I_{n-m},$$

then the formula for calculating the pseudoinverse matrix (25) takes the following form:

$$F^+ = (F + (F_R \varphi F_L)^T)^{-1} - F_R \varphi^T F_L. \tag{26}$$

For simplicity, excluding the matrix φ, from consideration, instead of (26) we write the equation

$$F^+ = (F + (F_R F_L)^T)^{-1} - F_R F_L.$$

IV. Conclusion

The presented stable computational procedures allow us to regularize the problem of synthesis of algorithms for estimating the parameters of regulators in adaptive control systems with a customizable model and to improve the quality indicators of control processes under conditions of parametric uncertainty.

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