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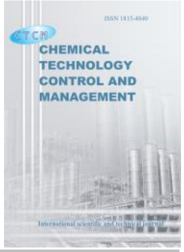

STEADY ALGORITHMS FOR SYNTHESIS OF MULTIDIMENSIONAL CONTROLLING SYSTEMS WITH INCOMPLETE INFORMATION ABOUT THE OBJECT STATE

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Abstract: *In this paper, questions of forming and creation of steady synthesis algorithms of multidimensional controlling systems at incomplete information on object status are considered. It is shown that the matrix of feedback coupling on a status at the complete information is not determined. The different algorithms of steady calculation of a matrix pseudo-inverse to the matrix of outputs using these or those decomposition of matrixes are analyzed. Algorithms of iterative specification of a required solution of "skeletal" decomposition are given. The given expressions allow synthesizing the simplified computing procedures of synthesis of multidimensional controlling systems at incomplete information on object status.*
Keywords: *object status, incomplete information, multidimensional controlling system, synthesis, steady algorithms.*

Аннотация: *Объект ҳолати ҳақида ахборотлар етарли бўлмаган шароитда кўп ўлчамли бошқариш тизимларини синтезлашнинг турғун алгоритмларини шакллантириш ва қуриш саволлари кўриб чиқилган. Ахборотлар етарли бўлмаган шароитда ҳолат бўйича тескари алоқа матрицаси тўлиқ аниқланмаган ҳисобланиши кўрсатилган. Турли хил матрица ажратишларидан фойдаланиб, чиқиш матрицасига мавҳум тескари матрицани турғун ҳисоблаш учун турли хил алгоритмлар таҳлил қилинган. Қидирилаётган скелетли ажратиш ечимини итерацион аниқлаш алгоритмлари келтирилган. Келтирилган алгоритмлар объект ҳолати ҳақида ахборотлар етарли бўлмаган шароитда кўп ўлчамли бошқариш тизимларини синтезлашнинг соддалашган ҳисоблаш амалларини синтезлашга имкон беради.*
Таянч сўзлар: *объект ҳолати, тўлиқ бўлмаган ахборот, кўп ўлчамли бошқариш тизими, синтез, турғун алгоритмлар.*

Аннотация: *Рассматриваются вопросы формирования и построения устойчивых алгоритмов синтеза многомерных систем управления при неполной информации о состоянии объекта. Показано, что матрица обратной связи по состоянию при полной информации является недоопределенной. Проанализированы различные алгоритмы устойчивого вычисления матрицы, псевдообратной к матрице выходов, использующие те или иные разложения матриц. Приводятся алгоритмы итерационного уточнения искомого решения «скелетного» разложения. Приведенные выражения позволяют синтезировать упрощенные вычислительные процедуры синтеза многомерных систем управления при неполной информации о состоянии объекта.*
Ключевые слова: *состояние объекта, неполная информация, многомерная система управления, синтез, устойчивые алгоритмы.*

Introduction

The object state vector, in most cases, is difficult or cannot be completely measured at all. Usually measurements are available only for some state variables. They form a vector of a signal output of an object of which the dimension r is less than the dimension n on the state vector. According to the results of output signal observation, it is often possible to restore the entire state vector and use it when the controller synthesis. However, to simplify the equipment, it is advisable (even at a is possible to measure the complete state vector) to ensure the desired quality of regulation by supplying only some state variables in chain to the feedback [1-6].

The problem of providing a given the set of own numbers (spectrum) of a multidimensional linear system at incomplete information on state vector is discussed in [2-5]. In [2,3], methods are proposed for controlling the spectrum by designing a feedback matrix of unit rank. The algorithm for shifting $\max(r,l)$ own numbers of the system (*r*, *l*-dimensions of the system's input and output vectors, respectively) is considered in [4], but the arrangement of the remaining *n*– max(*r*, *l*)-own numbers of the closed system is not defined here. The necessary and sufficient conditions of resolvability of an exact synthesis problem of systems with the set spectrum are discussed in [5].

The present work proposes a method of synthesis of systems with a given spectrum by designing a matrix of full-rank feedback, in which a closed system has all own numbers equal to the given ones.

Research Methods and the Received Results

Consider the linear stationary system described by the following matrix differential equation:

$$x_{k+1} = Ax + Bu\,,\ \ y = Hx\,, \tag{1}$$

where the x – n-dimensional state vector, u – r-dimensional vector of control, y – l-dimensional vector of an output, A, B, H – constant matrixes of the corresponding dimensions. The system (1) is supposed completely managed, i.e $rank[B, AB, ..., A^{n-1}B] = n$. It is also supposed that matrixes of B and H are full ranks.

It is necessary to introduce into the system (1) linear stationary feedback by the output

$$u = Ky = KHx \tag{2}$$

so that the closed-loop system has a predetermined spectrum $\lambda_1, \lambda_2, ..., \lambda_n$ (λ_i – is the eigenvalue of the matrix $A + BKA$, K – is a feedback matrix of dimension $r \times l$).

From controllability of a system (1), follows the existence of a nondegenerate matrix with N dimensions $n \times n$ and permutation matrixes M (the permutation matrix contains one unit element in every line (column), and all other elements are equal to zero) dimension $r \times r$ [2], such that conversion

$$\hat{x} = Nx\,,\ \hat{u} = M^{-1}u\,,\ \hat{y} = y$$

leads a system (1) to a view:

$$\hat{x}_{k+1} = A\hat{x} + B\hat{u}\,,\ \hat{y} = \hat{H}\hat{x}\,. \tag{3}$$

Here, the matrices $\hat{A}$, $\hat{B}$ and $\hat{H}$ have the following structure:

$$\hat{A} = NAN^{-1} = \begin{bmatrix} A_1 \\ \hdashline A_2 \end{bmatrix} = \underset{\begin{matrix} l_1 & \quad l_2 & \quad l_3 & \quad \cdots & \quad l_v \end{matrix}}{\begin{bmatrix} 0 & A_{1,2} & 0 & \cdots & 0 \\ 0 & 0 & A_{2,3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & A_{v-1,v} \\ \hdashline A_{v,1} & A_{v,2} & A_{v,3} & \cdots & A_{v,v} \end{bmatrix}} \begin{matrix} l_1 \\ l_2 \\ \vdots \\ l_{v-1} \\ l_v \end{matrix} \tag{4}$$

$$\hat{B} = NBM = \begin{bmatrix} 0 \\ \hdashline B_v \end{bmatrix}\begin{matrix} n-r \\ r \end{matrix}\,, \tag{5}$$

$$\hat{H} = HN^{-1}\,,$$

where

$$\hat{A}_{i,i+1} = \underset{l_{i+1}-l_i \quad l_i}{[0 \ \vdots \ I]}\, l_i\,,\ i = 1, 2, \ldots, v-1\,,$$

$$B_v = \begin{bmatrix} I & 0 & \cdots & 0 \\ \Delta & I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \Delta & \Delta & \cdots & I \end{bmatrix} \begin{matrix} l_v - l_{v-1} \\ \\ \vdots \\ l_1 \end{matrix},$$
$$l_v - l_{v-1} \quad \cdots \quad l_1$$

$A_{v,1}, A_{v,2}, \ldots, A_{v,v}$, Δ – are uncertain sub matrixes, I – is a unit matrix. Here the index of controllability v is equal to the smallest whole at which $rank[B, AB, ..., A^{v-1}B] = n$. The sequence l_i is defined from the following expressions:

$$l_i = rank[B, AB, ..., A^{v-i}B] - rank[-B, AB, ..., A^{v-i-1}B], \ (i = 1, 2, ..., v),$$

$$l_v = rankB = r, \ \sum_{i=1}^{v} l_i = n.$$

The control for the system (3) is sought in the form [2,5]:

$$\hat{u} = \hat{K}\hat{y} = \hat{K}\hat{H}\hat{x}, \tag{6}$$

where $\hat{K} = M^{-1}K$. Let the matrix S dimensions $(n-l)\times n$ become orthogonal addition of a matrix ($\hat{H}$ ($\hat{H}S^T = 0$, $rankS = n - l$). Let us assume that the matrix of the closed system $C = \hat{A} + \hat{B}\hat{K}\hat{H}$ has a following appearance:

$$C = \begin{bmatrix} A_1 \\ \text{--} \\ Z \end{bmatrix},$$

where ($(n-r)\times n$) -submatrix A_1 is determined from (4) ($r \times n$) -submatrix Z is set so that the matrix C had a desirable spectrum.

It can be shown [2,4,5] that if system (1) is controllable and for its canonical representation (3) the condition is satisfied

$$[Z - A_2]S^T = 0, \tag{7}$$

then the spectrum of system (1) can be arbitrarily set using feedback of a view (2), at that $K = MB_v^{-1}[Z - A_2]\hat{H}^+$ (a matrix $\hat{H}^+ = \hat{H}^T(\hat{H}\hat{H}^T)^{-1}$ – pseudoinverse [6-8] for a matrixes $\hat{H}$, A_2 and B_v are defined from expressions (4), (5)).

The existence of a canonical representation (3) [2] follows from controllability of a system (1). Then, considering a matrix $K^* = \hat{K}\hat{H}$ in (6) as a matrix of feedback coupling on a status at the complete information, it is possible to show [5] that

$$\hat{K}\hat{H} = B_v^{-1}[Z - A_2]. \tag{8}$$

Expression (8) can be rewritten as follows:

$$\hat{K}\hat{H} = \left[\hat{K} \ \vdots \ 0\right]\begin{bmatrix} \hat{H} \\ \text{--} \\ S \end{bmatrix} = B_v^{-1}[Z - A_2], \tag{9}$$

where 0^T – ($(n-l)\times r$) - zero submatrix. Given the non-degeneracy of the matrix $\begin{bmatrix} \hat{H} \\ \text{--} \\ S \end{bmatrix}$ and the relation $\begin{bmatrix} \hat{H} \\ \text{--} \\ S \end{bmatrix}^{-1} = \left[\hat{H}^+ \ \vdots \ S^+\right]$, from (9) we obtain

$$\left[\hat{K} \ \vdots \ 0\right] = B_v^{-1}[Z - A_2]\begin{bmatrix} \hat{H} \\ \text{--} \\ S \end{bmatrix}^{-1} = B_v^{-1}[Z - A_2]\left[\hat{H}^+ \ \vdots \ S^+\right]. \tag{10}$$

If condition (7) is satisfied, then the relation $B_v^{-1}[Z - A_2]S^+ = 0$ will also be satisfied. Then the feedback matrix is determined from (10)

$$\hat{K} = B_v^{-1}[Z - A_2]\hat{H}^+ .$$

Finally, in the original basis, the matrix K has the following form:

$$K = \hat{M}B_v^{-1}[Z - A_2]\hat{H}^+ . \tag{11}$$

This is caused by the fact that very often the system of equations (11) is, as a rule, not determined. For the purpose of stabilization of a required solution and giving of bigger numerical stability to the procedure of the pseudo-address in (10), (11), it is necessary to use regular methods [9-14].

Let us consider the algorithms of steady calculation $\hat{H}^+$ using these or those decomposition of a matrix $\hat{H}$ [9.10]. Taking into account that if $rank\hat{H} = p$ ($\hat{H} \in R^{p\times n}$ with $p \le n$), the pseudo-inverse of the matrix $\hat{H}$ is a matrix $\hat{H}^+$, which is determined by the second Gaussian transformation:

$$\hat{H}^+ = \hat{H}^T(\hat{H}\hat{H}^T)^{-1}. \tag{12}$$

Equity of expression (12) is due by the fact that any matrix $\hat{H} \in R^{p\times n}$ can be presented in the form of "skeletal" decomposition [6-8.15]:

$$H = U \cdot V$$

with the matrix $U \in R^{p\times r}$ and $V \in R^{r\times p}$, where $r = rankC \le \min(p, n)$.

We now set

$$\hat{H}^+ = V^+ \cdot U^+$$

where according to (12) and $\hat{H}^+ = (\hat{H}^T\hat{H})^{-1}\hat{H}^T$ at $p > n$ we can come to the relations.

$$V^+ = V^T(VV^T)^{-1},$$

$$U^+ = (U^TU)^{-1}U^T .$$

Then

$$\hat{H}\hat{H}^+\hat{H} = UVV^T(VV^T)^{-1}(U^TU)^{-1}U^TUV = UV = \hat{H} .$$

If the following is accepted $P = V^T(VV^T)^{-1}(U^TU)^{-1}V$ then it is possible to show that $P\hat{H}^T = V^+$.

Also fair is equality $\hat{H}^+ = \hat{H}^TG$ with $G = U(U^TU)^{-1}(VV^T)^{-1}(U^TU)^{-1}U$. When circulating matrix $\hat{H}$, you can also use the method based on the calculation of $Q = \hat{H}\hat{H}^T$ in expression (9). Taking into account that Q is a symmetric non-negatively defined matrix of the order $p \times p$ of rank $r < p$, then

Also fair is equality $\hat{H}^+ = \hat{H}^TG$ with $G = U(U^TU)^{-1}(VV^T)^{-1}(U^TU)^{-1}U$. When circulating matrix $\hat{H}$, based on the calculation of $Q = \hat{H}\hat{H}^T$ in expression (9). Taking into account that Q is a symmetric non-negatively defined matrix of the order $p \times p$ of rank $r < p$, then

$$Q^+ = T^T(TT^T)^{-2}T, \tag{13}$$

where the matrix $T_{(p\times r)}$ of rank r is determined from the decomposition

$$Q = T^TT. \tag{14}$$

Decomposition (14), in generally speaking, is not the only one [6,8,9]. However, the pseudo reverse matrix $Q^+ = T^T(TT^T)^{-2}T$ is determined unequivocally regardless of the decomposition method $Q = T^TT$. Thus, expression (12) taking into account (13) can be written down in the form:

$$\hat{H}^+ = \hat{H}^TQ^+ = \hat{H}^TT^T(TT^T)^{-2}T . \tag{15}$$

In case the matrix Q is poorly conditioned, then to increase the stability of the pseudoinversion procedure in (15), it is reasonable to use regular procedures [9,10,13] of the kind to :

$$\hat{H}^{+} = \hat{H}^{T} T^{T} (TT^{T} + \alpha I)^{-2} T ,$$

where $\alpha > 0$ – regularization parameter, I – a unit matrix.

Here it is reasonable to the determine regularization parameter α based on a way of the model examples [6, 9]. If the matrix $Q = \hat{H}\hat{H}^{T}$ nondegenerate, then $Q^{+} = Q^{-1}$ takes place expression (12).

Very effective at a solution of the equation (11), also are the algorithmic procedures connected with calculation $Z = (\hat{H}\hat{H}^{T})^{-1}$, $rank\hat{H} = p$, which is often referred to as not scaled covariation matrix.

Let's consider the following decomposition of a matrix [15]:

$$D\hat{H} = \left[R^{T} \;\vdots\; 0\right]^{T}, \tag{16}$$

$$\tilde{D}\hat{H}S = \left[\tilde{R}^{T} \;\vdots\; 0\right]^{T}. \tag{17}$$

In expressions (16), (17) $R_{p\times p}$ and $\tilde{R}_{p\times p}$ – upper triangular matrixes, D, $\tilde{D}$ – orthogonal matrixes of the corresponding dimensions, – S a permutation matrix.

It is possible to show [11,14], that for (16) and (17) at $rank\hat{H} = p$ respectively fair ratios:

$$Z = (\hat{H}\hat{H}^{T})^{-1} = (R^{-1})^{T} R^{-1}, \tag{18}$$

Then at the inverse triangular matrix 111 in (18) is turned, expressions can be used

$$t_{ii} = r_{ii}^{-1}, \quad i = 1, \ldots, p,$$

$$t_{ij} = -t_{jj} \sum_{l=i}^{j-1} t_{il} r_{lj}, \quad j = i+1, \ldots, p, \quad i = 1, \ldots, p-1.$$

For a case (17) it is also necessary to take into account the left and right multiplying operations by permutation matrices S and S^{T} respectively.

Conclusion

The above expressions allow synthesizing simplified computational procedures for synthesis of multidimensional control systems with incomplete information about the object state.

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