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Recommended Citation
DOI: https://doi.org/10.51346/tstu-01.20.2-77-0063
Available at: https://uzjournals.edu.uz/btstu/vol2020/iss2/8

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INVESTIGATION OF AN ELECTROMAGNETIC TWO-STROKE VIBRATING ACTIVATOR IN OSCILLATORY MODE

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Abstract. The article considers the results of the study of electromagnetic vibration exciter with sequentially included capacitor in the electrical circuit, consisting of mechanical and electrical subsystems. It is shown that by means of the Lagrangian-Maxwell equation the interconnection between mechanical and electric subsystems can be realized. The relations describing processes of establishment of amplitudes and phases of oscillations both in mechanical and in electric subsystems are deduced. The equations connecting the output (amplitude) of vibration of the vibrating exciter with its input (voltage) of the network are presented. As a result, the formulas allowing making corrections at the solution of the system describing operation of the electromagnetic vibrating exciter in two-stroke mode are presented.

Key words: vibrating exciter, vibrating system, Lagrangian-Maxwell equations, electromagnetic processes, inductance, equations linking amplitude and phase parameters, exponential function, integration, one-act operation mode, and capacitor.

Electromagnetic vibrators have been increasingly employed in the instrument in recent years-and in mechanical engineering. An elastic mechanical circuit system often has to be set up far enough away from the resonance, depending on the specific requirements of the vibration control system. Therefore a method suitable for calculating these vibrators needs to be developed.

Lagrangian-Maxwell equations (1) describe the interlinked electromagnetic and mechanical processes within the electromagnetic vibrator which is an electromechanical system.

\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{x}} + \frac{\partial H}{\partial x} - \frac{\partial L}{\partial x} = f \tag{1}
\]

\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} + \frac{\partial R}{\partial q} - \frac{\partial L}{\partial q} = e
\]

Where the variables q x- charge and travel respectively, q=i and x =ϑ their speeds, e=external E.E.S., f external mechanical force, R and H are the so-called electrical and mechanical dissipation functions of the Relay respectively

\[
R = \frac{1}{2} \sum_k r_k i_k^2, \quad H = \frac{1}{2} \sum_n h_n \dot{\varphi}_n^2
\]

Where variables q x-function Lagrange-L=(T+W)-(Π+V), T=\(\frac{1}{2} m \dot{q}^2\) is kinematic energy of mechanical moving part of vibrator; m-mass, Π=\(\frac{1}{2} c x^2\) - potential energy, c-hardness, W=\(\frac{1}{2} \omega^2 l^2\) - magnetic energy of electric part of vibrator, \(\Phi = \omega x i / R_M\), \(\omega = \Psi, \omega\) - number of turns, \(\Phi\) - magnetic flux, \(R_M\) - magnetic resistance of the flux path, \(R_M = R_c + 2(x_0+x) / \mu_0 S\), t.e. consists of the magnetic resistance of steel and gap, V-electric energy of the capacitor. Let the electromagnetic vibrator two-pin, does not contain in the electrical circuit of the capacitor, therefore, V = 0; it is not affected by external mechanical force, f = 0; the electrical circuit
operates alternating DC voltage, the sign of which changes with the frequency of the vibrator moving part. In this way the auto oscillation mode is investigated.

So, the Lagrange function of this vibrator

\[ L = \frac{1}{2} m \ddot{x} + \frac{1}{2} \left( \frac{2(x_0 + x)}{\mu_0 S} + R_c \right) \dot{\varphi}^2 - \frac{1}{2} c x^2 \]  

(2)

By putting it and its dissipative functions in the system, we will get

\[
\begin{align*}
    m \ddot{x} + h \dot{x} + c x &= -\frac{\psi^2}{\omega^2 \mu_0 S} \\
 \frac{d\psi}{dt} + \frac{r}{\omega^2} \left[ R_c + \frac{2(x_0 + x)}{\mu_0 S} \right] \psi &= \frac{4E}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{3n-1}}{2n-1} \cos(2n-1) \omega t \\
\end{align*}
\]

(3)

Although the circuit of the electromagnet is alternating DC voltage with high harmonics, in the threading clutch \( \Psi \) the corresponding high harmonics are strongly attenuated. Indeed, if we assume \( r=0 \) for simplicity, then we obtain the following:

\[
\Psi = \frac{4E}{\pi} \int \left( \cos \omega t - \frac{1}{3} \cos 3\omega t + \frac{1}{5} \cos 5\omega t - \ldots \right) dt = \frac{4E}{\pi \omega} \left( \sin \omega t - \frac{1}{9} \sin 3\omega t + \frac{1}{25} \sin 5\omega t - \ldots \right)
\]

i.e. the amplitude of the third one closest to the main harmonic is almost an order of magnitude smaller than the first one. Therefore, in the first approximation, we will not consider their influence. Therefore, we can assume that the voltage is acting \( e = \frac{4E}{\pi} \cos \omega t \).

In the theory of nonlinear mechanics we add the normal substitution of variables to solve the system of equations (1).

\[
x = A(t) \cos [\omega t + \psi(t)], \quad \frac{dx}{dt} = -A(t) \omega \sin \left[ \omega t + \psi(t) \right],
\]

(4)

\[
\Psi = B(t) \cos [\omega t + \phi(t)], \quad \frac{d\psi}{dt} = -B(t) \omega \sin \left[ \omega t + \phi(t) \right],
\]

(5)

By means of which the system of equations (3) for instantaneous variables can be replaced by a system of equations for amplitudes \( A(t), B(t) \) and phases \( \Psi(t), \phi(t) \), and solved with respect to their derivatives.

The square of the threading clutch is shown over the period graph in Fig. 1. But in the two-stroke vibrator in one half-period in one core works curve \( \approx\alpha\approx \) graphics, in the other half-period in the other core - curve \( \approx\delta\approx \). So the last curve can be flipped (curve \( \approx\alpha\approx \)). For the period \( T \) the curve \( \approx\alpha+\approx\approx \) can be described with the function

\[
\psi^2 = B_2^2 \frac{\pi}{4} \left[ \cos(\omega t + \phi) + \frac{1}{3} \cos 3(\omega t + \phi) + \frac{1}{5} \cos 5(\omega t + \phi) + \ldots \right]
\]

\[= \frac{\pi}{4} B_2^2 \sum_{n=1}^{\infty} \frac{1}{2n-1} \cos (2n-1)(\omega t + \phi)
\]

(6)

We differentiate the second expressions (4) and (5).
\[
\frac{d^2x}{dt^2} = -\frac{dA}{dt}\omega \sin(\omega t + \psi) - (\omega + \frac{d\varphi}{dt})A\omega \cos(\omega t + \psi) \\
(7)
\]

\[
\frac{d^2\psi}{dt^2} = -\frac{dB}{dt}\omega \sin(\omega t + \varphi) - (\omega + \frac{d\varphi}{dt})B\omega \cos(\omega t + \varphi) \\
(8)
\]

Fig.1. Slotting curve of the two-stroke vibrating activator.

Let's substitute the expressions \( x, \frac{dx}{dt}, \frac{d^2x}{dt^2}, \psi, \frac{d\psi}{dt}, \frac{d^2\psi}{dt^2} \), in a system

\[
(3) \frac{dA}{dt}\sin(\omega t + \psi) + \frac{d\varphi}{dt}A\omega \cos(\omega t + \psi) = \frac{\pi B^2}{2\mu_0 S} \frac{1}{\omega^2} \sum_{n=1}^{\infty} \cos((2n-1)(\omega t + \varphi)) - \frac{A}{\mu_0} \sin(\omega t + \psi), \\
(9)
\]

\[
\frac{dB}{dt}\sin(\omega t + \varphi) + \frac{d\varphi}{dt}B\omega \cos(\omega t + \varphi) = \frac{2B}{\mu_0 S} \left\{ \frac{2}{\mu_0 S} \frac{x_0 + A\cos(\omega t + \psi)}{\omega} \right\} + R_c - \frac{R}{\omega^2}B\cos(\omega t + \varphi) - \frac{2}{\mu_0 S}A\sin(\omega t + \psi), \\
\]

(10)

Differentiating the first expression and comparing it with the second one both in (4) and (5), we get

\[
\frac{dA}{dt}\cos(\omega t + \psi) - \frac{d\varphi}{dt}A\sin(\omega t + \psi) = 0 \\
(11)
\]

\[
\frac{dB}{dt}\cos(\omega t + \varphi) - \frac{d\varphi}{dt}B\sin(\omega t + \varphi) = 0 \\
(12)
\]

These equations should be considered as constraints imposed respectively on functions \( A(t), \psi(t), \) and \( B(t), \varphi(t) \) in the sense that \( \frac{dA}{dt} \ll A\omega, \frac{d\varphi}{dt} \ll \omega, \text{t.e. } \Delta A \ll A, \Delta \varphi \ll 2\pi \) for the period \( T = \frac{2\pi}{\omega} \) and also, \( \frac{dB}{dt} \ll B\omega, \frac{d\varphi}{dt} \ll \omega, \text{t.e. } \Delta B \ll B, \Delta \varphi \ll 2\pi \) for the period \( T = \frac{2\pi}{\omega} \).

Solving in pairs (9), (11) and (10), (12), we find equations in standard form presented with respect to derivative parameters of mechanical and electrical oscillations.

\[
\frac{dA}{dt} = -\left\{ A \frac{h}{m} \sin(\omega t + \psi) + \frac{\pi B^2}{2\mu_0 S} \frac{1}{\omega^2} \sum_{n=1}^{\infty} \cos((2n-1)(\omega t + \varphi)) \right\} \sin(\omega t + \psi) \\
\]

(13)
\[
\frac{dB}{dt} = \left( \frac{\pi E}{4} \sin \omega t - B \omega \cos(\omega t + \varphi) - \frac{R}{\omega^2} B \sin(\omega t + \varphi) \right) \left\{ \frac{2}{\mu_0 h} \cdot [x_0 + A \cos(\omega t + \psi)] + R_c \right\} - \frac{2R}{\omega^2 \mu_0 h} B \cos(\omega t + \varphi) A \sin(\omega t + \psi) \sin(\omega t + \varphi)
\]

\[
\frac{d\varphi}{dt} = \left( \frac{4E}{\pi B} \sin \omega t - \omega \cos(\omega t + \varphi) - \frac{R}{\omega^2} \sin(\omega t + \varphi) \right) \left\{ \frac{2}{\mu_0 h} \cdot [x_0 + A \cos(\omega t + \psi)] + R_c \right\} + \frac{2R}{\omega^2 \mu_0 h} B \cos(\omega t + \varphi) A \sin(\omega t + \psi) \cos(\omega t + \varphi) \quad (14)
\]

Proceeding from the above limitations and considering the oscillation parameters as
invariable over the period, the right parts of equations (13) and (14) assiduously integrate them
assiduously integrate them over the period T. As a result, in the first approximation we will get

\[
\frac{dA_1}{dt} = -\frac{hA_1}{2m} + \frac{\pi B_1^2}{8 \omega^2 \mu_0 m \omega} \sin(\varphi_1 - \psi_1), \quad \frac{dB_1}{dt} = -\frac{R}{2 \omega^2 \mu_0 h} B_1 + \frac{4}{\mu_0} E \cos \varphi_1, \quad \frac{d\varphi_1}{dt} = -\frac{\omega}{2 \pi B_1} E \sin \varphi_1 \quad (16)
\]

Using these equations one can investigate both the established mode, \(d\psi\), and the
processes of amplitude and phase setting of oscillations. In the established mode \(\frac{dA_1}{dt} = \frac{dB_1}{dt} = \frac{d\varphi_1}{dt} = 0\) therefore \(\cos(\varphi_1 - \psi_1) = 0\), hence \(\varphi_1 - \psi_1 = \frac{\pi}{2}\) or \(\varphi_1 - \psi_1 = \frac{3\pi}{2}\). But the
last ratio for phases does not satisfy the first equation (15).

\[
A_1 = \frac{\pi B_1^2}{4 \omega^2 \mu_0 m \omega h}; \quad (\varphi_1 - \psi_1) = \frac{\pi}{2}
\]

(17)

From the system (16), we find

\[
A_1 = \frac{\pi E}{4 \sqrt{\omega^2 + \frac{R^2}{2 \omega^2 \mu_0 h} + R_c}} = \frac{\pi E L_0}{\sqrt{\omega^2 L_0^2 + R^2}} \quad (18)
\]

\[
\varphi_1 = \arctg \frac{-\omega L_0}{R}
\]

Where \(L_0 = \frac{\omega^2}{\frac{2x_0}{\mu_0 h} + R_c}\) - static vibrator electromagnet inductance

Now the solution (18) by substituting (17) we get

\[
A_1 = \frac{4E L_0^2}{\pi \omega^2 \mu_0 m \omega h (\omega^2 L_0^2 + R^2)} \quad (19)
\]

Applied to the standard form of equations (13) and (14), the perturbation theory
method[2] makes it possible to find high approximations of the necessary vibration parameters
A(t), B(t), \(\psi(t)\) \(\varphi(t)\). Following the process, we assume that, as in the preceding average, these
oscillation parameters in the right parts of equations (13) and (14) are invariable and identical to
those found above, and we decompose these right parts into Fourier rows and define the
summation variables.
\[
\begin{align*}
\frac{dA_2}{dt}(t) &= \frac{A_2}{2m} \cos 2(\omega t + \psi_1) - \frac{\pi B_1^2}{8p} \sin(2\omega t + \varphi_1 + \psi_1) + \frac{\pi B_1^2}{24 p^*} \sin(2\omega t + 3\varphi_1 - \psi_1) - \cdots, \\
\frac{d\psi_2(t)}{dt} &= -\frac{h}{2m} \sin 2(\omega t + \psi_1) - \frac{\pi B_1^2}{8p A_1} \cos(2\omega t + \varphi_1 + \psi_1) - \cdots, \\
\frac{dB_2(t)}{dt} &= -\frac{2E}{\pi} \cos(2\omega t + \varphi_1) - \frac{\omega B_1}{2} \sin(\omega t + \varphi_1) + \frac{R}{2l_0} B_1 \cos 2(\omega t + \varphi_1) - \cdots, \\
\frac{d\varphi_2(t)}{dt} &= \frac{2E}{\pi B_1} \sin(2\omega t + \varphi_1) - \frac{\omega}{2} \cos(\omega t + \varphi_1) - \frac{R}{2l_0} \sin 2(\omega t + \varphi_1) - \cdots
\end{align*}
\]

(20)

Where: \( \rho = \omega \mu_0 \omega^2 mS, \ k=\omega^2 \mu_0 S. \)

By integrating them, we get:

\[
\begin{align*}
A(t) &= A_1 + A_2(t) = A_1 + \frac{Ah}{4\omega m} \sin 2(\omega t + \psi_1) + \frac{\pi B_1^2}{16 p \omega} \cos(2\omega t + \varphi_1 + \psi_1) - \frac{\pi B_1^2}{48 p \omega} \sin(2\omega t + 3\varphi_1 + +\psi_1) + \cdots, \\
\psi(t) &= \psi_1 + \psi_2(t) = \varphi_1 - \frac{\pi}{2} + \frac{h}{4\omega m} \cos 2(\omega t + \psi_1) - \frac{\pi B_1^2}{16 p A_1 \omega} \sin(2\omega t + \varphi_1 + \psi_1) - \cdots, \\
B(t) &= B_1 + B_2(t) = B_1 - \frac{E}{\pi \omega} \sin(2\omega t + \varphi_1) + \frac{B_1 Z}{2} \sin[2(\omega t + \varphi_1) + \xi] - \cdots, \\
\phi(t) &= \phi_1 + \phi_2(t) = \varphi_1 - \frac{E}{\pi B_1 \omega} \cos(2\omega t + \varphi_1) + \frac{Z}{2} \cos[2(\omega t + \varphi_1) + \xi v] - \cdots
\end{align*}
\]

(21)

Where:

\[
Z^2 = \omega^2 + \frac{R^2}{L_0^2}; \ \xi = \arctan \frac{\omega \beta_0}{R}.
\]

Having substituted them in formal expressions of solutions (4) and (5), we find it finally:

\[
\begin{align*}
x &= A_1 \cos(\omega t + \psi_1) + \frac{Ah}{8 m \omega} \sin(\omega t + \psi_1) + \frac{\pi B_1^2}{2 + 16 p \omega} \cos(\omega t + \varphi_1) - \cdots, \\
\Psi &= B_1 \cos(\omega t + \varphi_1) - \frac{E}{2 \pi \omega} \sin \omega t + \frac{B_1 Z}{2 \omega^2} \sin(\omega t + \varphi_1 + \xi) + \cdots
\end{align*}
\]

(22)
Based on these refined solutions, it is now possible to explain first approximations of the amplitudes of the fundamental harmonics (17) and (18)

Result of the work:

1. Throughout the recorded works devoted to the study of electromagnetic vibrators, represented by species equations (3), the latter were solved artificially (the second equation is separate from the first): either it was assumed in the second equation \( r = 0 \), or it was transformed into an independent parametric equation.

   The general dependence of the oscillation half-span on the parameters was obtained for the first time because of the joint solution of the equations: external voltage, excitation frequency, friction, electrical circuit resistances.

2. The theoretical and practical correlation (19) shows a sharp frequency limit from above in the field of application of vibrators, which is imposed on the oscillation amplitude. As can be seen from (19), if the active resistance of the electromagnet is significant in comparison with the inductive resistance, then half a span, \( A \), we can consider that it is inversely proportional to the excitation frequency in the first degree; if the inductive resistance of the electromagnet is much greater than the active resistance, then semi-span \( A \) is inversely proportional to the excitation frequency in the cube, i.e. for such vibrators at high frequencies of excitation is impossible to obtain a sufficiently large oscillation amplitude.

3. The correlation (19) can be of great practical importance because it helps to find friction in this vibrator design, which previously caused great difficulties to be identified, based on the data on the frequency of excitation and oscillation amplitude known from the experience.

4. The system of perturbation theory has obtained clarifications of the solutions. Specifications demonstrated complete interconnection of electromagnetic and mechanical processes, and the effect of not only the electrical component on the mechanical part.

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