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Erratum

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ALGORITHMS FOR REGULAR SYNTHESIS OF ADAPTIVE SYSTEMS MANAGEMENT OF TECHNOLOGICAL OBJECTS BASED ON THE CONCEPTS OF IDENTIFICATION APPROACH

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Abstract: *Stable algorithms of adaptive control, estimation of controller parameters, synthesis of adaptive control of dynamic objects by the criterion of minimum dispersion, and synthesis of suboptimal adaptive-local control of dynamic objects based on predictive models are presented. Algorithms for the stable identification of dynamic control objects based on regularization and pseudoinversion methods based on a singular decomposition are presented. Based on the use of approximations in the form of a finite sum of Gaussian distributions, recurrent identification algorithms have been developed using multiple models and adapting their parameters. Stable algorithms are proposed for identifying the parameters of an object and a controller in a closed-loop control system based on the principle of iterative regularization using the method of variational inequalities, ensuring the convergence of the desired estimates of the parameters of the object and the controller almost certainly to true values. Stable algorithms for generating control actions in locally optimal adaptive control systems for dynamic objects based on non-orthogonal factorizations and pseudoinversions of ill-conditioned or degenerate square matrices are proposed that enhance the accuracy of forming control actions in a closed control loop.*

Key words: *closed-loop control system, identification, iterative regularization principle, variational inequality method, incorrectly posed problem, locally optimal adaptive control system, control action, regularized algorithms.*

Аннотация: *Адаптив бошқариш, ростлагич параметрларини баҳолаш, динамик объектларни адаптив бошқариш жараёнларини дисперсиянинг минимумлиги мезони бўйича синтезлаш, башоратловчи моделлар асосида динамик объектларни субоптимал адаптив-локал бошқаришни синтезлашнинг турғун алгоритмлари келтирилган. Сингуляр ажратишга асосланган мунтазамлаштириш ва мавҳум тескари тартибда ёзиш усуллари асосида динамик бошқариш объектларини идентификациялашнинг турғун алгоритмлари келтирилган. Тўпламли моделлар ва уларнинг параметрларини гаусс тақсимотининг чекли йиғиндиси кўринишида силлиқлантиришдан фойдаланиш асосида мослаштириш ёрдамида реккурент идентификациялаш алгоритмлари ишлаб чиқилган. Объект ва ростлагич параметрларининг изланаётган баҳоларини деярли ҳақиқий қийматларга мувофиқлигини таъминловчи вариацион тенгсизликлар усулидан фойдаланиб, итератив мунтазамлаштириш тамойили асосида берк бошқариш системасидаги объект ва ростлагич параметрларини идентификациялашнинг турғун алгоритмлари ишлаб чиқилган. Алгоритмлар берк бошқариш контурида бошқариш таъсирини шакллантириш аниқлигини оширишга имкон берувчи ортогонал бўлмаган омиллаштириш ва ёмон шартланган ёки хос квадрат матрицаларни мавҳум тескари тартибда ёзиш асосида динамик объектларни локал-оптимал адаптив бошқариш системаларида бошқариш таъсирларини шакллантиришнинг турғун алгоритмлари таклиф этилган.*

Таянч сўзлар: *берк бошқариш системаси, идентификация, адаптив ростлагич, итератив мунтазамлаштириш тамойили, вариацион тенгсизлик усули, нокорректк қўйилган масала, локал-оптимал адаптив бошқариш тизими, бошқарувчи таъсир, мунтазамлаштириш алгоритмлари.*

Аннотация: *Приводятся устойчивые алгоритмы адаптивного управления, оценивания параметров регулятора, синтеза процедуры адаптивного управления динамическими объектами по критерию минимума дисперсии и синтеза субоптимального адаптивно-локального управления динамическими объектами на основе прогнозирующих моделей. Приведены алгоритмы устойчивой идентификации динамических объектов управления на основе методов регуляризации и псевдообращения, основанные на сингулярном разложении. На основе использования аппроксимаций в виде конечной суммы гауссовских распределений разработаны алгоритмы* *рекуррентной идентификации с помощью множественных моделей и адаптации их параметров. Предложены устойчивые алгоритмы идентификация параметров объекта и регулятора в замкнутой системе управления на основе принципа итеративной регуляризации с привлечением метода вариационных неравенств, обеспечивающие сходимость искомых оценок параметров объекта и регулятора почти наверное к истинным значениям. Предложены устойчивые алгоритмы формирования управляющих воздействий в локально-оптимальных адаптивных системах управления динамическими объектами на основе неортогональных факторизациях и псевдообращениях плохообусловленных или вырожденных квадратных матриц, способствующие повышению точности формирования управляющих воздействий в замкнутом контуре управления.*

Ключевые слова: *замкнутая система управления, идентификация, принцип итеративной регуляризации, метод вариационных неравенств, некорректно поставленная задача, локально-оптимальная адаптивная система управления, управляющее воздействие, регуляризованные алгоритм.*

Currently, there is no universal approach to the identification of closed systems. In each case, different models, methods, techniques and algorithms are used. Techniques for ensuring identifiability by changing the algorithm of the functioning of the control system include: full or partial opening of feedback loops; alternate inclusion of several regulators with various settings; the introduction of delay in the feedback loop; use of non-linear or non-stationary regulators. Depending on the purposes of identification, the mode of operation of the system, the availability of a priori information, the methods and techniques used, there are several definitions of the concept of identifiability of closed systems: deterministic, stochastic, systemic, structural, parametric, in probability, by distribution, active, passive, for analysis, for management etc. It is known that the existence of feedback leads to the appearance of significant features of solving identification problems. This is due to problems of non-identifiability of closed systems and problems of convergence of identification methods developed for open systems. The problem of convergence of estimates in a closed loop arises due to correlations of perturbations at the output of the object and in the control signal. The convergence problem is solved by modifying identification methods to account for this correlation. However, there are such control systems in which certain structural restrictions are imposed on the driving influences, which lead to the fundamental impossibility of obtaining a unique solution to the identification problem [1-6].

Consider a linear system described by the following model:

$$A(q^{-1})\,y(t) = B(q^{-1})u(t) + \xi(t)\,, \qquad (1)$$

where $A(q^{-1}) = I - \sum_{j=1}^{p} A_j (q^{-1})^j$, $B(q^{-1}) = \sum_{j=1}^{q} B_j (q^{-1})^j$, $u(t)$ - $(m \times 1)$ - dimensional input, $y(t)$ represents $(l \times 1)$-dimensional output, $\xi(t)$ – $(l \times 1)$-dimensional random sequence, I - identity matrix [6,7].

Following [1,8], it can be shown that the optimal steady-state predictor $\hat{z}(t)$ for $z(t) = \zeta(q)\,y(t)$ satisfies an equation of the form:

$$\overline{C}(q^{-1})\hat{z}(t) = \overline{\beta}(q^{-1})u(t) + \overline{\alpha}(q^{-1})y(t)\,,$$

where

$$\overline{C}(q^{-1}) = I + \overline{C}_1 q^{-1} + \dots + \overline{C}_n q^{-n}\,,\ \ \overline{\beta}(q^{-1}) = \overline{\beta}_0 + \overline{\beta}_1 q^{-1} + \dots + \overline{\beta}_l q^{-l}\,,\ \overline{\beta}_0 = V\,,$$

$$\overline{\alpha}(q^{-1}) = \overline{\alpha}_0 + \dots + \overline{\alpha}_r q^{-r}\,.$$

The controller that provides the minimum covariance of regulation $\zeta(q)\,y(t)$ relative to $\zeta(q)\,y^*(t)$ is determined by the equation:

$$\overline{\beta}(q^{-1})u(t) + \overline{\alpha}(q^{-1})y(t) = \overline{C}(q^{-1})\zeta(q)y^*(t)\,,$$

where $\{y^*(t)\}$ represents the preassigned desired path, while the tracking error at the output $y(t) - y^*(t)$ satisfies the equation:

$$\overline{C}(q^{-1})\zeta(q)\left[y(t) - y^*(t)\right] = \overline{C}(q^{-1})\,\mathrm{F}(q)\xi(t)\,.$$

Model (1) can be considered as a one-step predictor with a lead error of

$$\xi(t) = y(t) - \hat{y}(t),$$

where

$$\hat{y}(t) = E\{y(t) \mid F_{t-1}\}.$$

The lead on d steps can be obtained using equation (1), using the identity:

$$I = \tilde{F}(q^{-1})A(q^{-1}) + q^{-d}\tilde{G}(q^{-1}), \tag{2}$$

where

$$\tilde{F}(q^{-1}) = \tilde{F}_0 + \tilde{F}_1 q^{-1} + \ldots \tilde{F}_{d-1}q^{-d+1}, \quad \tilde{G}(q^{-1}) = \tilde{G}_0 + \tilde{G}_1 q^{-1} + \ldots .$$

Multiplying (1) by $\tilde{F}(q^{-1})$ and using identity (2), we obtain the equation:

$$E\{y(t+d) \mid F_t\} = \tilde{G}(q^{-1})y(t) + \Gamma_2(q^{-1})\xi(t) + \tilde{\beta}(q)u(t),$$

where

$$\tilde{\beta}(q) = q^d \tilde{F}(q^{-1})B(q^{-1}) = \tilde{\beta}_0 + \tilde{\beta}_1 q^1 + \ldots \tilde{\beta}_{d-1}q^{d-1} + \beta'(q^{-1}),$$

$$\beta'(q^{-1}) = \beta'_1 q^{-1} + \beta'_2 q^{-2} \ \ldots \ , \quad \Gamma_2(q^{-1}) = \Gamma_{20} + \Gamma_{21}q^{-1} + \ldots .$$

The control law, leading $E\{y(t+d) \mid F_t\}$ to zero and having the lowest energy, it is advisable to determine the expression

$$u(t+j) = -\tilde{L}_j\left[\tilde{G}(q^{-1})y(t) + \Gamma_2(q^{-1})\zeta(t)\right]; \quad \mathrm{j} = 0, \ldots, d-1, \tag{3}$$

where

$$\tilde{L}_j = \left[\tilde{\beta}_j\right]^T \left[\sum_{k=0}^{d-1} \tilde{\beta}_k \tilde{\beta}_k^T + \alpha I\right]^{-1},$$

$\alpha > 0$ - regularization parameter.

The suboptimal control law (3) can be turned into adaptive if one uses an algorithm for estimating the matrices $A(q^{-1})$ and $B(q^{-1})$ included in model (1).

In the theory of optimal adaptive systems, methods are developed for controlling dynamic objects under conditions when a number of essential parameters and factors determining their behavior are unknown. In such cases, it is advisable to use locally optimal control algorithms [9-11]. Local-optimal control of a dynamic object described by equations of the form

$$x_{t+1} = Ax_t + Bu_t + \xi_{t+1},$$

$$y_t = H^T x_t, \quad t = 0, 1, \ldots,$$

where $x_t \in R^n$ is a state vector; $u_t \in R^m$ - management; $y_t \in R^l$ - observed yield; A, B, H - matrices of the corresponding dimensions; $\xi_1, \xi_2, \ldots$ - a sequence of random vectors, in the sense of criterion $V(x) = x^T Cx, \ C^T = C \geq 0$, is determined by the expression [11]:

$$u_t^* = \arg\min_{u_t} E[\Delta V(x_t, u_t) / x_0^t, u_0^{t-1}],$$

where $\Delta V(x_t, u_t) = V(Ax_t, Bu_t) - V(x_t)$.

Management u_t^* under conditions $E(\xi_{t+1} / x_0^t, u_0^t) = 0$, $E(\xi_{t+1}\xi_{t+1}^T / x_0^t, u_0^t) = \mathrm{const}$, is determined from the equation:

$$B^T CB u_t^* = -B^T CAx_t, \tag{4}$$

with $\Delta V(x_t, u_t^*) = -x_t^T Q x_t$, $Q = C - A^T CA + A^T CB(B^T CB)^+ B^T CA$.

Thus, the control u_t^* based on (4) has the form:

$$u_t^* = \theta^T x_t,$$

where θ^T is determined from the expression

$$B^T CB\theta^T = -B^T CA. \tag{5}$$

The systems of equations (4) or (5) can be poorly conditioned. Poorly conditioned and degenerate systems can be indistinguishable within a given accuracy. We present a regular algorithm for estimating the control law based on equation (5) [12-14]. Take $B^T CB = D, \quad -B^T CA = S$.

Then

$$D\theta^T = S. \tag{6}$$

We introduce the following approximation conditions:

$$\left\| D - \overline{D} \right\| \le h, \left\| s_j - \bar{s}_j \right\| \le \delta,$$

where $\overline{D}$, $\bar{s}_j$ are the exact values of the matrix *D* and the *j*-th column of the matrix *S*, *j* = 1,2, ..., *n*.

We will construct approximations to the pseudo-solution $\theta_j = D^+ s_j$ of equation (6) in the form

$$\theta_{j,r} = g_r(D)s_j, \tag{7}$$

$$\bar{\theta}_{j,r} = g_r(D)D\theta_{j,r}, \tag{8}$$

where θ_j - is the *j*-th column of the matrix θ^T, $j = 1,2,...,n$; s_j - is the *j*-th column of the matrix *S*.

Taking into account that the matrix operator is self-adjoint, we will use the method of M.M. Lavrentiev [13, 14] to regularize the solution of equation (7). The method of M.M. Lavrentiev corresponds to functions

$$g_r(\lambda) = (\alpha + \lambda)^{-1}, \ \alpha = r^{-1}, \ 0 \le \lambda < \infty.$$

Then approximations (7), (8) take the form

$$\theta_{j,\alpha} = (D + \alpha I)^{-1} s_j, \ \bar{\theta}_{j,\alpha} = (D + \alpha I)^{-1} D\theta_{j,\alpha}.$$

Passing to an iterated version of the method under consideration, we can write [14]

$$\theta_0 = 0, \ \theta_j^{(l)} = \theta_j^{(l-1)} - \mathrm{B}_\alpha \left(D\theta_j^{(l-1)} - s_j \right), \ l = 1,..,r,$$

$$\bar{\theta}_0 = 0, \ \bar{\theta}_j^{(l)} = \bar{\theta}_j^{(l-1)} - \overline{\mathrm{B}}_\alpha \left(D\bar{\theta}_j^{(l-1)} - D\theta_{j,r} \right), \ l = 1,..,r.$$

$$\mathrm{B}_\alpha = g(D), \ \overline{\mathrm{B}}_\alpha = \mathrm{B}_\alpha D \mathrm{B}_\alpha.$$

The choice of parameter r is advisable to carry out on the basis of a relationship of the form

$$\left\| D\theta_{j,r} - s_j \right\| = b\left(\delta + \left\| \theta_{j,r} \right\| h \right), \ b > 1.$$

Let the control object in a closed system be described by an *n*-th order difference equation [15,16]:

$$x[k] = \sum_{i=1}^{p} a_i x[k-i] + \sum_{j=1}^{q} b_j \eta[k-j],$$

$$y[k] = x[k] + v[k]; \ \eta[k] = u[k] + w[k],$$

where $\eta[k]$ and $x[k]$ are the input and output of the object; $w[k]$ and $v[k]$ are the perturbing effect and the error in measuring the output, respectively; $u[k]$ and $y[k]$ - the control action and the result of measuring the output, used as input and output data in the identification procedure; a_i, b_j - unknown parameters of the object. The sequence $v[k]$ is not correlated with the disturbance $w[k]$, the background of signals $u[k]$ and $y[k]$.

The difference equation of the controller has the form:

$$u[k] = \sum_{i=1}^{l1} c_i u[k-i] + \sum_{j=0}^{l2} d_j \varepsilon[k-j],$$

$$\varepsilon[k] = y^*[k] - y[k],$$

where $y^*[k]$ is the current value of the task; $\varepsilon[k]$ - mismatch signal; c_i, d_j – known parameters of the controller.

When solving the identification problem under consideration, it becomes necessary to determine the parameter vector of the object θ based on equation

$$H \cdot \theta = Y,$$

where the matrix H and vector Y are formed on the basis of realizations of the control action $u[k]$ and output $y[k]$, $\theta = [a_1 \cdots a_p \mid b_1 \cdots b_p]^T$.

The solution to this problem is based on the regularization method of A.N.Tikhonov [12] and effective pseudoinversion based on the singular decomposition of matrices [17]. The regularization principle leads to an estimate of the desired solution $\hat{\theta}$ based on equation

$$\hat{\theta}_\alpha = (H^T H + \alpha I)^{-1} H^T Y,$$

where $\alpha > 0$ is the regularization parameter, I – is the identity matrix.

If the error levels of the initial data

$$h = \left\| H - \overline{H} \right\|, \quad \delta = \left\| Y - \overline{Y} \right\|,$$

are known, then the regularization parameter α can be determined on the basis of the principle of generalized residual [18,19], where $\overline{H}$ and $\overline{Y}$ are the exact values of the matrix operator H and the vector of the right-hand side Y. If the numbers h and δ are unknown, then it is advisable to determine the regularization parameter α based on the quasi-optimal method:

$$\left\| \hat{\theta}_{\alpha_{i+1}} - \hat{\theta}_{\alpha_i} \right\| = \min, \quad \alpha_{i+1} = \kappa \alpha_i, \quad i = 0, 1, 2, \ldots, \quad 0 < \kappa < 1.$$

It is shown that if the rank of the matrix is H $r = p$, then the estimation of the parameter vector $\hat{\theta}$ should be determined on the basis of effective pseudoinverse matrices [20]:

$$\hat{\theta}_\tau = V S_\tau U^T Y = \sum_{i=1}^{r'} \frac{1}{\mu_i} v_i \cdot u_i^T,$$

where S_τ^+ is the effective pseudo-matrix $S = diag(s_{1\tau}^+, \ldots, s_{n\tau}^+)$; $r' < r$, $s_{i\tau}^+ = 1/\mu_i$ if $\mu_i > \tau$, and $s_{i\tau}^+ = 0$ if $\mu_i = 0$.

To analyze the quality of management processes, it is necessary to analyze the conditions of convergence. Consider a control object with input $u(t)$ and output $y(t)$, described by the equation [6,7]:

$$A(d) y(t) = B(d) u(t) + \xi(t),$$

where A and B are polynomials with the backward shift operator d:

$$(1 + a_1 d + \ldots + a_n d^m) y(t) = (b_1 d + \ldots + b_m d) u(t) + \xi(t),$$

$$y(t) = -a_1 y(t-1) - \ldots - a_n y(t-n) + b_1 u(t-1) + \ldots + b_m u(t-m) + \xi(t),$$

or

$$y(t) = \theta^T \varphi(t) + \xi(t),$$

where

$$\theta = [a_1 \ldots a_n \vdots b_1 \ldots b_m]^T, \quad \varphi(t) = [-y(t-1) \ldots - y(t-n) \vdots u(t-1) \ldots u(t-m)]^T.$$

It is assumed that stationary white noise $\xi(t)$ has zero mathematical expectation and dispersion σ_ξ^2. An identifiable system with changing parameters can be represented as the following dynamic model in the state space:

$$\left. \begin{aligned} \theta(t-1) &= \theta(t) + w(t) \\ y(t) &= \varphi^T(t) \hat{\theta}(t) + \xi(t) \end{aligned} \right\},$$

where $\xi(t)$ and $w(t)$ are perturbations, and $w(t)$ models the process of changing system parameters.

If the process of changing the parameters is associated with the use of a Gaussian distribution with a changing covariance matrix, then the classical Kalman filter can be used [1,21]. It can be shown that in the case when $w(t)$ is not Gaussian, then the Kalman filter does not provide an optimal estimate. In this case, it is advisable to use an algorithm of the form [5,21]:

$$P_i(t) = P_i(t-1) - \frac{P_i(t-1)\varphi(t)\varphi^T(t)P_i(t-1)}{R_2 + \varphi^T(t)P_i(t-1)\varphi(t)}, \ i = 1, 2, ..., M ,$$

$$\varepsilon_i(t) = y(t) - \varphi^T(t)\bar{\theta}_i(t-1),$$

$$\bar{\theta}_i(t) = \bar{\theta}_i(t-1) + \frac{1}{R_2} P_i(t)\varphi(t)\varepsilon_i(t),$$

where R_2 – is the noise variance.

An estimate $\hat{\theta}(t)$ of the parameter vector $\theta(t)$ is determined by the relation

$$\hat{\theta}(t) = \sum_{i=1}^{M} \beta_i(t)\bar{\theta}_i(t) \cdot$$

Consider the class of models with noise in the object and control device, which is widely used in practical problems. This class of control systems with noise in the object and the control device can be described by equations of the form [22,23]:

$$y_n = \sum_{i=1}^{p} a_i y_{n-i} + \sum_{i=1}^{q} b_i u_{n-i} + \sigma_1 v_{1n},$$

$$u_n = \sum_{i=1}^{\mu} c_i y_{n-i} + \sum_{i=1}^{\upsilon} d_i u_{n-i} + \sigma_2 v_{2n},$$

where $\{u_n\}$, $\{y_n\}$ – the observed sequences at the input and output of the object, respectively; $\{v_{1n}\}$, $\{v_{2n}\}$ – Gaussian sequences with zero expectation and unit variance, the joint distribution is Gaussian. At the same time,

$$M[v_{1k}\, v_{1j}] = M[v_{2k}\, v_{2j}] = \delta_{kj}, \quad M[v_{1n}\, y_{n-j}] = M[v_{2n}\, y_{n-j}] = 0,\ M[v_{1n}\, u_{n-j}] = M[v_{2n}\, u_{n-j}] = 0,$$

$$M[v_{1n}\, u_{2m}] = \rho\delta_{nm} \quad (j \geq 1), \quad 1 > \rho > -1,\ \delta_{nm} - \text{ is the symbol of Kronecker.}$$

To determine the desired parameters of the object and the control device, it is necessary to solve the following system of equations:

$$S \cdot \theta = y, \tag{9}$$

or in expanded form:

$$S = \begin{bmatrix} \alpha_{11}S_1^T S_1 & \alpha_{11}S_1^T S_2 & \alpha_{12}S_1^T S_1 & \alpha_{12}S_1^T S_3 \\ \alpha_{11}S_2^T S_1 & \alpha_{11}S_2^T S_2 & \alpha_{12}S_2^T S_1 & \alpha_{12}S_2^T S_3 \\ \alpha_{12}S_3^T S_1 & \alpha_{12}S_3^T S_2 & \alpha_{22}S_3^T S_1 & \alpha_{22}S_3^T S_3 \\ \alpha_{12}S_1^T S_1 & \alpha_{12}S_1^T S_2 & \alpha_{22}S_1^T S_1 & \alpha_{22}S_1^T S_3 \end{bmatrix}, \ \theta = \begin{bmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \\ \hat{\theta}_3 \\ \hat{\theta}_4 \end{bmatrix}, \ y = \begin{bmatrix} \alpha_{11}S_1^T Y_N + \alpha_{12}S_1^T S_N \\ \alpha_{11}S_2^T Y_N + \alpha_{12}S_2^T S_N \\ \alpha_{12}S_3^T Y_N + \alpha_{22}S_3^T S_N \\ \alpha_{12}S_1^T Y_N + \alpha_{22}S_1^T S_N \end{bmatrix},$$

where vectors Y_N, S_N and matrices S_1, S_2 are formed based on implementations of input and output signals $\{u_n\}$, $\{y_n\}$, $n = 0, 1,\dots, N$; $\alpha_{11} = \sigma_2^2, \alpha_{12} = -\rho\sigma_1\sigma_2$, $\alpha_{22} = \sigma_1^2$; parameters θ_1, θ_2, θ_3, θ_4 are defined by the following expressions:

$$\theta_1^T = (a_1, \dots, a_k, b_1, \dots, b_I),\ \theta_2^T = (a_{k+1}, \dots, a_p, b_{I+1}, \dots, b_q),$$

$$\theta_3^T = (c_1, \dots, c_k, d_1, \dots, d_I),\ \theta_4^T = (c_{k+1}, \dots, c_\mu, d_{I+1}, \dots, d_\upsilon).$$

It is known that the solution of an operator matrix equation of type (9) can be reduced to the problem of minimizing the residual functional, that is, finding a quasisolution of the equation on a closed convex set Q in the sense of V.K. Ivanov [13,14]. As a residual, we can take a quadratic functional of the form:

$$\Phi(\theta) = \|S\theta - y\|_H{}^2 .$$

Then the iterative sequence for $\hat{\theta}_r$ can be written in the following form [24]:

$$\hat{\theta}_{r+1} = P_Q(\hat{\theta}_r - \alpha_r(F_\delta(\hat{\theta}_r) + \varepsilon_r\hat{\theta}_r)),\ r = 0,1,\dots,$$

where P_Q – is a metric projector; $\partial\Phi = F$, r – iteration number; the regularization parameters $\alpha_r > 0,\ \varepsilon_r > 0$ are determined by the expressions:

$$\alpha_r = (1+r)^{-1/2}, \qquad \varepsilon_r = (1+r)^{-p},\ \ 0 < p < 1/2 .$$

The iterative process under consideration can be stopped based on relations of the form [14]:

$$\lim_{\delta\to 0} \delta / \varepsilon_{r(\delta)} = 0,\ \lim_{\delta\to 0} \delta^{1/2} / \varepsilon^2_{r(\delta)} = 0,$$

$$\rho(F_\delta - F) \le \delta,\ \delta \ge 0,\ F_\delta \in \mathscr{F} .$$

Moreover,

$$\lim_{\varepsilon\to 0} \|\theta_\varepsilon - \theta^*\| = 0,$$

where $\theta^* \in \Xi$ – is the only solution to the variational inequality $(\theta, \theta - d) \le 0,\ \forall d \in \Xi$, Ξ – is the set of solutions (9).

Quite often, in the synthesis of a controller in closed systems, the methods of locally optimal adaptive control are used [25-28]. Consider the control object defined in the form

$$A(z^{-1})y_{t+1} = B(z^{-1})u_t + w_{t+1},$$

where $y_t \in R^l$ - the measured outputs, $u_t \in R^m$ - is the control.

Then, taking into account that

$$u_t = \theta^T(\Omega_t)\eta_t,\ \ \eta_t^T = (y^T_{t-n+1}, \dots, y^T_t,\ u^T_{t-n+1}, \dots, u^T_t),$$

$$\Omega^T = (-A^{(n)}, \dots, -A^{(1)}, B^{(n-1)}, \dots, B^{(0)}),\ \ \Phi_t^T = (y^T_{t-n+1}, \dots, y^T_t,\ u^T_{t-n+1}, \dots, u^T_t),$$

the locally optimal control law will take the form [5,27]:

$$\begin{aligned} u_t = (H^T B^{(0)})^{-1}[(A^{(1)} + S^{(1)})y_t + \dots + (A^{(n)} + S^{(n)})y_{t-n+1} + \\ + (-B^{(1)} + D^{(1)})u_{t-1} + \dots + (-B^{(n-1)} + D^{(n-1)})u_{t-n+1}], \end{aligned} \tag{10}$$

where $S^{(i)}, i = \overline{1,n}$ and $D^{(j)},\ j = \overline{1,n-1}$ – arbitrary matrices of dimension $l \times l$ and $l \times m$, respectively, H is a matrix of $m \times l$ such that $\det H^T B^{(0)} \ne 0$.

In expression (10), a square matrix of the form $G^{(0)} = H^T B^{(0)}$ is inverted. This matrix may be poorly conditioned. Below is an algorithm for stable estimation of the inverse matrix $G^{(0)^{-1}}$ in (10).

The approximation for a pseudo-inverse matrix can be determined using the relation [29,30]:

$$G_\varepsilon^+ = R_k^+ U_k^+ , \tag{11}$$

where R_k, U_k - are determined using the expressions:

$$G_\varepsilon = U_k R_k,\ U_k^T = \left[U_1^T \vdots U_2^T\right],\quad R_k = \left[R_1 \vdots R_2\right],\ U_2 = G_{21}^{(k)},$$

$$R_2 = G_{12}^{(k)},\quad U_1 = \begin{bmatrix} 1 & 0 & \dots & 0 \\ g_{21}^{(k)} & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ g_{k1}^{(k)} & g_{k2}^{(k)} & \dots & 1 \end{bmatrix},\ R_1 = \begin{bmatrix} g_{11}^{(k)} & g_{12}^{(k)} & \dots & g_{1k}^{(k)} \\ 0 & g_{22}^{(k)} & \dots & g_{2k}^{(k)} \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & g_{kk}^{(k)} \end{bmatrix}.$$

The calculation of R^+ and U^+ in (11), when R and U^T, respectively, are upper trapezoidal matrices, is effectively carried out by orthogonal factorization $R = SP$ using the Givens or

Householder transformations [17,29], where S – is the lower triangular square, P – is the orthogonal matrix. Then $R^{+} = P^{T} S^{-1}$.

The given regularized algorithms make it possible to stabilize the procedure for the formation and development of control actions in locally optimal adaptive control systems for dynamic objects under conditions of poor conditioning of matrices G and $G^{(0)}$.

References:

1. Krasovsky A.A. 1987. The directory of the automatic control theory. Moscow: Science.
2. Tsykunov A.M. Adaptive and robust control of dynamic objects by output. Publisher: Fizmatlit, 2009. - 268 p.
3. Igamberdiyev H., Yusupbekov A., Zaripov O., Sevinov J. Algorithms of adaptive identification of uncertain operated objects in dynamical models. Procedia Computer Science. Volume 120, 2017, Pages 854-861.
4. Igamberdiyev X.Z., Sevinov J.U., Zaripov O.O. Regulyarniye metodi i algoritmi sinteza adaptivnix system upravleniya s nastraivayemimi modelyami. -T.: TashGTU, 2014. -160 p.
5. Zybin E.Yu. On the identifiability of linear dynamic systems in a closed loop in the normal operation mode. Izvestiya SFedU. Technical science. №4 (165), 2015. -PP. 160-170.
6. Ljung L. 1991. Systems identification, p.432. The theory for the user: Translated from English. Moscow: Science.
7. Kashyap R.A., Rao A.R. Building dynamic models from experimental data. Per. from English -M.: Nauka, 1983. - 384 p.
8. Puzyrev V.A. Management of technological processes for the production of microelectronic devices. M.: Radio and communications, 1984. -160 p.
9. Kelmans G.K., Poznyak A.S., Chernitser A.V. Algorithms of “local” optimization in problems of asymptotic control of nonlinear dynamic objects // Autom. 1977. No. 11. -FROM. 73-88.
10. Neymark Yu.I., Kogan M.M., Savelyev V.P. Dynamic models of control theory. -M.: Science, 1985.
11. Igamberdiyev, H.Z; Sevinov, J.U; Yusupbekov, A.N; and Zaripov, O.O (2018) "Sustainable algorithms for synthesis of local-optimal adaptive dynamic object management systems.," Chemical Technology, Control and Management: Vol. 2018: Iss. 3, Article 14. Available at: https://uzjournals.edu.uz/ijctcm/vol2018/iss3/14.
12. Tikhonov, A., Arsenin, V. (1979) Methods of Ill-conditioned Problems Solution, p. 285. Nauka Publishers, Moscow.
13. Vainikko G.M., Veretennikov A.Yu. Iterative procedures in incorrect tasks. - M.: Nauka, 1986. - 178 p.
14. Bakushinsky A.B., Kokurin M.Yu. Iterative methods for solving irregular equations. - M.: Lenand, 2006. - 214 p.
15. Vorchik B.G. Identifiability of linear parametric stochastic systems. II. Identifiability conditions. /Automation and Remote Control, 1985, 46, 867–878.
16. Sevinov J.U. Algorithms of Sustainable Identification of Dynamic Objects in the Closed Control Line // International Journal of Innovative Technology and Exploring Engineering (IJITEE) ISSN: 2278-3075, Volume-9 Issue-2, December 2019. –PP.54-56. DOI: 10.35940/ijitee.A4842.129219.
17. Lawson C., Henson R. Numerical solution of the least squares method problems / Per. from English –M.: Science. Ch. ed. Phys.-Math. lit., 1986. –232 p.
18. Tikhonov A.N., Goncharsky A.V., Stepanov V.V., Yagola A.G. Numerical methods for solving ill-posed problems, Moscow: Nauka, 1990.
19. 19.Morozov V.A. 1987. Regular methods of the decision it is incorrect tasks in view. Moscow: Science.
20. Demmel J. Computational linear algebra. Theory and applications: Per. from English –M.: Mir, 2001 –430 s.
21. Derevitsky D.P., Fradkov A.A. Applied theory of discrete adaptive control systems. -M.: Science, 1981. - 216 p.
22. Vorchik B.G., Fetisov V.N., Shteinberg Sh.Ye. The identification of a stochastic feedback system / Autom. Remote Control, 34:7 (1973), 1069–1081.
23. Vorchik B.G. Plant identification in a stochastic closed model / Autom. Remote Control, 36:4 (1975), 550–565.
24. 24.Bakushinskij A.B., Goncharsky A.V. 1989. Iterative approach of ill-conditioner problems solution, p.119. Moscow: Science.
25. Bodyanskiy E.V., Boryachok M.D. Locally-optimal pseudodual control of objects with unknown parameters // Autom. 1992. № 2. - PP.90-97.
26. Darkhovsky B.S. On the roughness conditions of a locally optimal stabilization system // Automation and Automation, 1988, No. 5. –C.41-50.
27. Kogan M.M., Neymark Yu.I. Functional capabilities of adaptive locally optimal control // Autom. 1994. -№6. -FROM. 94-105.
28. Degtyarev G.L., Rizaev I.S. Synthesis of locally optimal aircraft control algorithms. -M.: Mechanical Engineering, 1991. - 304 p.
29. Gantmakher F.R. Matrix theory. - M.: Nauka, 2010. - 560 p.
30. Meleshko V.I. On regularized nonorthogonal factorizations and pseudoinversions of perturbed matrices // Journal of Computational Mathematics and Mathematical Physics. Volume 26, 1986, No. 4. –C.485-498.