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THE PROBLEM OF RAMCHUNDRA FOR A PROBLEM OF -CAPTURE

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l-TUTISH MASALASI UCHUN RAMCHANDR MASALASI

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Annatatsiya. *Biz bu maqolada Ramchandr masalasini l-tutish differensial o`yini yordamida o`rgandik. Ramchandr masalasini geometrik chegaralangan hol uchun o`rgandik. Qayiqning strategiyasi faqat geometrik chegaralanishga bog`liq holda qurilgan. Bu maqolada biz qayiqning kemaga yaqinlashish strategiyasini qurdik.*

***Kalit so`zlar:*** *Ramchadr, $l$-tutish, geometrik chegaralanish, differensial tenglamalar, differensial o`yin*

ЗАДАЧА РАМЧАНДРА ДЛЯ ЗАДАЧЫ l-ПОИМКИ l

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Аннотация. *В статье, мы изучаем задачу Рамчандра для l-поимки дифференциальной игры. Мы изучаем проблему Рамчандра с геометрическим ограничеением. Мы выстроили стратегию подхода корабля к лодке*

Ключевые слова: *Рамчандра, l-поимки, дифференциальной игры, геометрическим ограничеением, диффереціальная уравнения.*

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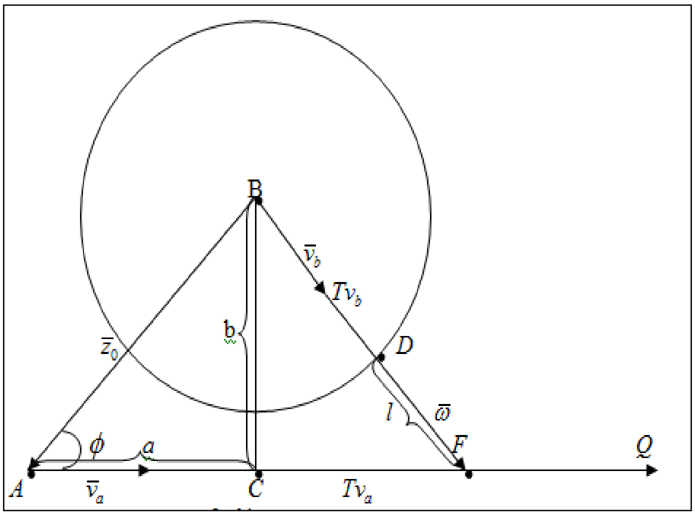
Abstract. *We study the problem for a differential game of l- capture. We study the problem of Ramchundra with geometric constraint. A strategy of the boat is constructed depending only with geometric constraint. In this article, we constructed a strategy for approaching the ship to the boat.*

Key words: *Ramchundra, l-capture, geometric constraint, differential equations, differential game*

One of the problems [2] discussed by Ramchundra (Indian mathematician Nesudas Ramchundra (1821-1880)) is the intercept problem for a slow pursuer versus a fast target. He wrote: Supposing a ship to sail from given place A , in a given place, at the same time that a boat from another place B sets out in order (if possible) to come up with her and supposing the rate at which eachvessel progresses to be given it is required to find in what direction the letter must proceed, so that if a cannot up with the former, it may however approach it as near as possible.



picture1

We learn the problem of Ramchundra in the case l –interception . Let in the space R^n the controlled object B (the boat), intercepts another object A (the ship). Suppose x and y are the locations of the boat and the ship respectively, and x_0, y_0 $(x_0 \neq y_0)$ are their initial locations. The motions of the objects are described by the equations

$$B:\ \dot{y}(t) = v_b\ ,\ y(0) = y_0;\quad A:\ \dot{x}(t) = v_a,\ x(0) = x_0\ . \tag{1}$$

where $x,\ y,\ v_b,\ v_a \in R^n$, $n \geq 1$,

$|v_a| \leq \beta$ is the control functions of the object A (the ship) and $|v_b| \leq \alpha$ is that of the object B (the boat), α and β are given positive numbers.

If we say $z(t) = x(t) - y(t)$,then from (1) we will get the equation:

$$\dot{z} = v_a - v_b\ ,\quad z_0 = z(0) \tag{2}$$

where $z_0 = x_0 - y_0$,

The goal of the boat B is achievement of the inequalities $|x(\bar{t}) - y(\bar{t})| \leq l$ if at some moment $\bar{t} > 0$. And according to the equation (2) we have $|z(\bar{t})| < l$ We know constructing strategy for boat (see [1], [3-4]). According to the figure we have

$$Tv_b + w = Tv_a - z_0 \tag{3}$$

Squaring both sides (2) of these equalities and we obtain a quadratic equation with respect to T

$$T^2\left(\beta^2-\alpha^2\right)-2T\left(l\alpha+\langle v_\beta,z_0\rangle\right)+|z_0|^2-l^2=0$$

$$T_{1,2}=\frac{1}{\beta^2-\alpha^2}\left[l\alpha+\langle v_\beta,z_0\rangle\pm\sqrt{\left(l\alpha+\langle v_\beta,z_0\rangle\right)^2-\left(\beta^2-\alpha^2\right)\left(|z_0|^2-l^2\right)}\right]=$$

$$=\frac{1}{\beta^2\left(1-\left(\frac{\alpha}{\beta}\right)^2\right)}\left[|z_0|\cdot\beta\left(\frac{l}{|z_0|}\cdot\frac{\alpha}{\beta}\right)+\langle v_\beta,z_0\rangle\pm\sqrt{\left(|z_0|\cdot\beta\cdot\frac{l}{|z_0|}\cdot\frac{\alpha}{\beta}+\langle v_\beta,z_0\rangle\right)^2-\beta^2\cdot z_0{}^2\cdot\left(1-\left(\frac{\alpha}{\beta}\right)^2\right)\left(1-\left(\frac{l}{z_0}\right)^2\right)}\right]=$$

$$=\frac{1}{\beta^2\left(1-\left(\frac{\alpha}{\beta}\right)^2\right)}\left[|z_0|\cdot\beta\left(\frac{l}{|z_0|}\cdot\frac{\alpha}{\beta}+\langle\hat v_\beta,\hat z_0\rangle\right)\pm|z_0|\cdot\beta\sqrt{\left(\frac{l}{|z_0|}\cdot\frac{\alpha}{\beta}+\langle\hat v_\beta,\hat z_0\rangle\right)^2-\left(1-\left(\frac{\alpha}{\beta}\right)^2\right)\left(1-\left(\frac{l}{z_0}\right)^2\right)}\right]=$$

$$=\frac{|z_0|\cdot\beta}{\beta^2\left(1-\left(\frac{\alpha}{\beta}\right)^2\right)}\left[\frac{l}{|z_0|}\cdot\frac{\alpha}{\beta}+\langle\hat v_\beta,\hat z_0\rangle\pm\sqrt{\left(\frac{l}{|z_0|}\cdot\frac{\alpha}{\beta}+\langle\hat v_\beta,\hat z_0\rangle\right)^2-\left(1-\left(\frac{\alpha}{\beta}\right)^2\right)\left(1-\left(\frac{l}{z_0}\right)^2\right)}\right]=$$

$$=\frac{|z_0|}{\beta\left(1-k^2\right)}\left[mk+\xi\pm\sqrt{(mk+\xi)^2-\left(1-k^2\right)\left(1-m^2\right)}\right]$$

and we have

$$T(\xi)=\frac{|z_0|}{\beta\left(1-k^2\right)}\left[mk+\xi-\sqrt{(mk+\xi)^2-\left(1-k^2\right)\left(1-m^2\right)}\right],$$

where $\xi=\langle\hat v_a,\hat z_0\rangle,\ k=\dfrac{\alpha}{\beta},\ m=\dfrac{l}{|z_0|},\ \hat v_a=\dfrac{v_a}{|v_a|},\ \hat z_0=\dfrac{z_0}{|z_0|}.$

Lemma. If $1\geq\xi\geq -mk+\sqrt{\left(1-k^2\right)\left(1-m^2\right)}$ then the function is $T(\xi)>0$.
It is not difficult to show that this function is positive.

We now substituting $T(\xi)$ into relation (3) find the strategy for boat in form

$$v_b=v_a-F(\xi)\left(mv_a+\alpha\hat z_0\right)\ (4)$$

where

$$F(\xi)=\frac{mk+\xi+\sqrt{(mk+\xi)^2-\left(1-k^2\right)\left(1-m^2\right)}}{\xi m+k+m\sqrt{(mk+\xi)^2-\left(1-k^2\right)\left(1-m^2\right)}}$$

Theorem. If $-mk+\sqrt{\left(1-k^2\right)\left(1-m^2\right)}\leq\xi\leq 1$ then the boat using of the strategy (3) realizes l –interception with the ship.

Proof:

Obviously

$$z(t)=z_0+\int_0^t(v_a-v_b)d\tau$$

Squaring both sides of this equality and we find

$$|z(t)|^2 = |z_0|^2 - 2\left\langle z_0, \int_0^t F(\xi(\tau))(mv_b(\tau)+\alpha\hat{z}_0)d\tau \right\rangle + \left|\int_0^t F(\xi(\tau))(mv_b(\tau)+\alpha\hat{z}_0)d\tau\right|^2 \le$$

$$\le |z_0|^2 - 2\int_0^t F\xi(\tau)\langle z_0, mv_b(\tau)+\alpha\hat{z}_0\rangle d\tau + \left(\int_0^t F\xi(\tau)|mv_b(\tau)+\alpha\hat{z}_0|d\tau\right)^2 =$$

$$= |z_0|^2 - 2\beta|z_0|\int_0^t N(\xi(\tau))d\tau + \beta^2\left(\int_0^t M(\xi(\tau))d\tau\right)^2,$$

where

$$N(\xi) = F(\xi)(m\xi+k), \quad M(\xi) = F(\xi)\sqrt{m^2+2km\xi+k^2}$$

And hence

$N(p) \le N(\xi) \le N(1),\ M(p) \le M(\xi) \le M(1)$,where $p = -mk + \sqrt{(1-k^2)(1-m^2)}$

Thus

$$|z(t)|^2 - l^2 \le |z_0|^2 - l^2 - 2\beta|z_0|\int_0^t N(\xi(\tau))d\tau + \beta^2\left(\int_0^t M(\xi(\tau))d\tau\right)^2 =$$

$$= |z_0|^2(1-m^2) - 2\beta|z_0|\sqrt{1-m^2}\int_0^t \frac{N(\xi(\tau))}{\sqrt{1-m^2}}d\tau + \beta^2\left(\int_0^t \frac{N(\xi(\tau))}{\sqrt{1-m^2}}d\tau\right)^2 +$$

$$+\beta^2\left(\int_0^t M(\xi(\tau))\right)^2 - \beta^2\left(\int_0^t \frac{N(\xi(\tau))}{\sqrt{1-m^2}}d\tau\right)^2 =$$

$$= \left[|z_0|\sqrt{1-m^2} - \beta\left(\int_0^t \frac{N(\xi(\tau))}{\sqrt{1-m^2}}d\tau\right)\right]^2 - \beta^2\left[\left(\int_0^t \frac{N(\xi(\tau))}{\sqrt{1-m^2}}d\tau\right)^2 - \left(\int_0^t M(\xi(\tau))d\tau\right)^2\right]$$

We show that

$$N(\xi) \ge \sqrt{1-m^2}M(\xi)$$

$$F(\xi)(m\xi+k) \ge \sqrt{1-m^2}\cdot F(\xi)\sqrt{m^2+2km\xi+k^2}$$

$$m^2\xi^2 + 2km\xi + k^2 \ge (1-m^2)(m^2+2km\xi+k^2)$$

$$\xi^2 \ge 1 - (m^2\xi^2 + 2km\xi + k^2)$$

$$\xi^2 + 2km\xi - 1 + k^2 + m^2 \ge 0,$$

From here we have

$$|z(t)|^2 - l^2 \le \left(|z_0|\sqrt{1-m^2} - \beta\int_0^t \frac{N(\xi(\tau))}{\sqrt{1-m^2}}d\tau\right)^2$$

The last function $N(\xi)$ is limited below with $\sqrt{(1-k^2)(1-m^2)}$

From here, we can say $|z(\bar{t})| < l$ if at some time $\bar{t} < \infty$

References

1. Isaacs R. Differential games. John Wiley and Sons, New York, 1965 .

2. Nahin P.J. Chases and Escapes: The Mathematics of Pursuit and Evasion. Princeton University Press, Princeton, 2012 .

3. Azamov A.A., Samatov B.T. The П-Strategy: Analogies and Applications. The Fourth International Conference Game Theory and Management , St. Petersburg, Russia: 2010, p. 33-47.
4. Samatov B.T. The Pursuit- Evasion Problem under Integral-Geometric constraints on Pursuer controls. Automation and Remote Control, Pleiades Publishing, Ltd. New York: 2013, 74(7), p. 1072-1081.