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Cover Page Footnote

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ANALYTIC APPROACH TO GROUND STATE ENERGY OF CHARGED ANYON GAS IN STRONG MAGNETIC FIELD

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Abstract

We present analytic formulas for the ground state energy of two-dimensional ($2D$) anyon gas in strong magnetic field (Landau level filling factor $\nu_L \leq 1$). The formulas are obtained by applying harmonic potential regularization for the vanishing confinement to harmonically confined Coulomb anyon gas. In a case of absence of the Coulomb interaction our analytic result provides an exact solution. It contains a contribution of the anyon gauge field characterized by the anyon parameters ν and ν_L. In a case of presence of the Coulomb interaction we introduce a function depending on the parameters ν, ν_L and the density parameter r_s. The function is determined by fitting the Fano-Ortolani interpolation equation in the fractional quantum Hall effect regime for spin-polarized electrons and by consistence requirement with known results for the ground state energy of the $2D$ Coulomb Bose gas in strong magnetic field. We show that our formulas are valid not only for fermions ($\nu = 1$) but quite generally for anyons ($0 \leq \nu \leq 1$).

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1 Introduction

The $2D$ electron gas in a perpendicular magnetic field has become a subject of intense theoretical and experimental studies since the observation of the fractional quantum Hall effect (FQHE) [1]. In recently observed Spin Hall effect [2, 3] the role of the weak current induced magnetic field for the spin polarization at the edges of quasi $2D$ semiconductors remains unclear so far [4]. The FQHE and the Spin Hall effect are certainly particular cases in a list of interesting properties of the system of electrons in magnetic field. Another object, the $2D$ Coulomb Bose gas, may attract attention to explain the High-T_c superconductivity [5]. Among properties of the electron gas and Coulomb Bose gas in magnetic field the ground state energy and its dependence on system parameters are of principal interest.

The known calculations of the ground state energy of $2D$ electrons in magnetic field are mainly related with FQHE. In a pioneering work [6] Laughlin demonstrated that the energy of spin-polarized electrons in strong magnetic field can be lower than the energy of a charge-density wave state corresponding to the lowest Landau level (LLL). Later, Levesque *et al.* [7] obtained a more accurate result by using Monte Carlo simulations for the classical one-component $2D$ plasma. Fano and Ortolani [8] improved further this result in the range very close to $\nu_L \approx 1$ by taking into account the electron-hole symmetry. The quantum Monte Carlo calculations of the ground state energy were performed for fixed values $\nu_L < 1$ and density parameter r_s (the mean interparticle distance r_0 expressed in units of the Bohr radius $a_B = \hbar^2/(Me^2)$) in Refs. [9, 10]. With these numerical data an interpolation formula was found for the interval $0 \leq r_s \leq 100$ [10]. All these results provide information on the ground state energy of spin-polarized $2D$ electrons in strong magnetic field for discrete values of r_s. It would be desirable, however, to have an analytic formula for the ground state energy that covers the whole range of the system parameters.

In the present paper we derive analytic formula by applying a concept of anyons [11]. The anyons are quasiparticles whose origin is related with the fundamental group of the covering space over $2D$ space [12] and whose statistics varies continuously in the interval $0 \leq \nu \leq 1$, i.e., between bosons ($\nu = 0$) and fermions ($\nu = 1$). It is believed that these quasiparticles are responsible for FQHE [13] (see also Ref. [14]).

In general, due to mathematical complexity of the model, complete theory of anyons is not constructed yet. However, in a special case of the system of anyons in magnetic field without Coulomb interaction quite sufficient theoretical results have been obtained. Namely, the quantum mechanical two-anyon problem is exactly solved in Refs. [15], and some part of an exact spectrum for many-anyon system is given in Refs. [16]. The case of applied external magnetic field and including Coulomb interaction for two and three anyons in a harmonic potential is considered in Refs. [17, 18]. Ref. [19] provides a detailed review on all these studies.

It has been shown [20] that in the presence of strong magnetic field, i.e., for the LLL, the $2D$ anyon model reduces to $1D$ Calogero model [21], and the anyon statistics coincides exactly with the Haldane fractional exclusion statistics [22]. In Ref. [23] the critical analysis of the model parameters and conditions under which this reduction and correspondence to the Haldane statistics might occur is presented.

In a previous paper [24] we have obtained an approximate analytic formula for the ground state energy of the $2D$ Coulomb anyon gas in the absence of magnetic field. It was derived from the corresponding result for the harmonically confined Coulomb anyon gas [25] by applying a regularization procedure for vanishing confinement. In order to take into account the fractional statistics and the Coulomb interaction we introduced a function depending on both the statistics and density parameters (ν and r_s, respectively). The function is determined by fitting the ground state energies of the classical $2D$ electron crystal at very large r_s (the $2D$ Wigner crystal) and of the dense $2D$ Coulomb Bose and Fermi gases at very small r_s. Applied to the spin-polarized electron ($\nu = 1$) and charged boson ($\nu = 0$) cases, our analytic expression yields the ground state energies which are in reasonable agreement with known numerical and analytic results.

For harmonically confined anyons we have considered the effect of magnetic field [25]. In our current treatment, we apply the vanishing confinement regularization procedure to the results obtained earlier [25] by extending the calculation in Ref. [24] to the problem including magnetic field. The formula for the ground state energy of the confined system derived in [25] differs from that in a field-free case by the presence of an additional parameter which contains the ratio ω_0^2/ω_c^2, as outlined before in [26], where ω_0 and ω_c are the $2D$ oscillator and cyclotron ($\omega_c = |eH|/(Mc)$) frequencies, respectively. As we will show below the parameter ω_0^2/ω_c^2 reduces to $\nu_L = 2l_H^2/r_0^2$, where l_H is the magnetic length (see Ref. [10]). We obtain analytic expression for the ground state energy per particle for the cases with and without Coulomb interaction for the LLL ($\nu_L \leq 1$). As a function of three parameters ν, ν_L and r_s (in the case when the energy is expressed in Rydberg units, $Ry = Me^4/(2\hbar^2)$) or of ν, ν_L and l_H (when the energy is expressed in units e^2/l_H), our expression is exact in the absence of Coulomb interaction and still represents an approximate result when such interaction takes place. In the last case it contains a function determined by fitting to the interpolation formula of Fano-Ortolani for the FQHE states [8] for fermions ($\nu = 1$) and to the numerical result of Yoshioka [27] for the ground state energy of $2D$ Coulomb Bose gas in strong magnetic field for bosons ($\nu = 0$). We have obtained a generalized formula which includes ν dependence of the energy as well.

To make the paper self-contained we briefly outline the main ideas of the developed approach [25] and the harmonic potential regularization procedure [24]. We use the bosonic representation of anyons and a product ansatz for the N-body wave function in our treatment [25] of the N-anyon problem in the harmonic potential. The wave function is constructed from single-particle gaussians of variable shape. As it was obtained in earlier perturbative consideration for the ground state energy of anyons in the oscillator potential (see references in [25]), our variational calculation reveals the logarithmic divergence related with a cut-off parameter for the interparticle distance. We regularize equations for the energy by taking into account that for $\nu \neq 0$ this distance must have a finite value [28] and should be determined by fitting to existing exact values (analytical or numerical) for the various cases of the Coulomb interaction and magnetic fields. Following this way, we have also reproduced the asymptotic expression $E_0 \approx \hbar\omega_0\nu^{1/2}N^{3/2}$ of energy obtained by Chitra and Sen [29] in the Thomas-Fermi approximation for $N \to \infty$ and for the case of absence of Coulomb interaction and magnetic field.

We make use the Chitra and Sen formula to explain the main idea of harmonic potential regularization. For the case of the $2D$ ideal fermion ($\nu = 1$) gas with density $\rho = N/S = 1/\pi r_0^2$, where S is area of the gas, (in the absence of magnetic field) the ground state energy determined by the Pauli exclusion principle is given by the expression:

$$E_0(\rho) = \pi\hbar^2\rho N/M\,. \tag{1}$$

Comparing Eq. (1) with that obtained by Chitra and Sen for $\nu = 1$ one has the following relation

$$\omega_0(N) = \pi\hbar\rho/(MN^{1/2})\,. \tag{2}$$

In the thermodynamic limit, when N and S increase infinitely (meanwhile the density ρ remains finite), the Eq. (2) implies a vanishing harmonic confining potential.

We have generalized the harmonic potential regularization idea for the case of anyons with the Coulomb interaction in Ref. [24] and demonstrate that the energy can be expressed in the form

$$E_0(\rho,\nu) = \pi\hbar^2\rho N\phi(\rho,\nu)/M\,, \tag{3}$$

where the function $\phi(\rho,\nu)$ is determined by the harmonic frequency

$$\omega_0(N,\nu) = \pi\hbar\rho f(\nu,\rho)/(MN^{1/2})\,. \tag{4}$$

An explicit form of the unknown function $f(\nu,\rho)$ is derived by fitting procedure in Ref. [24] along the lines given above.

In Sec. 2 we describe the idea of the harmonic potential regularization procedure to obtain the analytic expression of the ground state energy per particle for noninteracting anyons in magnetic field. The corresponding expression for the energy for the anyon system with Coulomb interaction is derived in Sec. 3. We summarize and conclude the paper in the last section.

2 Harmonic potential regularization

The system of N spinless anyons of mass M and charge e confined to the $2D$ harmonic well in the presence of the external homogeneous magnetic field, $\vec{H} = H\vec{e}_z$, is described by the Hamiltonian

$$\hat{H} = \frac{1}{2M}\sum_{k=1}^{N}[(\vec{p}_k - (\vec{A}_\nu(\vec{r}_k) + e\vec{A}_{ext}(\vec{r}_k)/c))^2 + M^2\omega_0^2|\vec{r}_k|^2] \,. \tag{5}$$

Here $\vec{r}_k$ and $\vec{p}_k$ represent the position and momentum operators of the k-th anyon in $2D$ space dimensions, and

$$\vec{A}_\nu(\vec{r}_k) = \hbar\nu\sum_{j\neq k}^{N}\frac{\vec{e}_z \times \vec{r}_{kj}}{|\vec{r}_{kj}|^2} \tag{6}$$

is the anyon gauge vector potential [30], $\vec{r}_{kj} = \vec{r}_k - \vec{r}_j$, and $\vec{e}_z$ is the unit vector normal to the $2D$ plane. The external magnetic field is defined by the vector potential $\vec{A}_{ext}(\vec{r}_k) = \vec{H} \times \vec{r}_k/2$.

We have employed a variational method by minimizing the total energy

$$E = \frac{\int \Psi_T^*(\vec{R})\hat{H}\Psi_T(\vec{R})\, d\vec{R}}{\int \Psi_T^*(\vec{R})\Psi_T(\vec{R})\, d\vec{R}} \tag{7}$$

with a trial wave function $\Psi_T(\vec{R})$ depending on the configuration $\vec{R} = \{\vec{r}_1....\vec{r}_N\}$ of N anyons in order to find an analytic expression for the ground state energy as a function of $|\nu|, N$, ω_0 and ω_c/ω_0.

It is reasonable, in the bosonic representation of anyons, to take the normalized trial wave function as a product

$$\Psi_T(\vec{R}) = \left(\frac{\alpha}{\pi}\right)^{N/2}\prod_{k=1}^{N}\exp\left(-\alpha\frac{(x_k^2 + y_k^2)}{2}\right) \,, \tag{8}$$

where α is a variational parameter of the single particle wave function having gaussian form (we express all length quantities in units of the characteristic length $r_0 = (\hbar/M\omega_0)^{1/2}$ of the harmonic oscillator [25]).

To justify the form of the wave function $\Psi_T(\vec{R})$ we have applied the mean field approximation [25] to the gauge vector field

$$\overline{\vec{A}_\nu}(\vec{r}) = \frac{1}{2}\vec{B}_\nu \times \vec{r} \tag{9}$$

introduced by Fetter, Hanna and Laughlin [31]. So that, every particle interacts to the homogeneous mean "magnetic" field $\vec{B}_\nu = 2\pi\rho\hbar\nu\vec{e}_z$ created by all particles and determined by the carrier density ρ and anyonic factor ν. As it is known, the spectrum of the single particle in the external magnetic field has Landau form [32] with the gaussian shape for the ground state wave function.

Minimization procedure of the expectation value E, Eq. (7), with respect to the variational parameter α gives the energy [25]

$$E_0 = N\hbar\left(\omega_0^2 + \frac{\omega_c^2}{4}\right)^{1/2}\mathcal{N}^{1/2} - \frac{\nu\beta\hbar\omega_c}{4}N(N-1)\,, \tag{10}$$

which is the minimal energy of N anyons in the harmonic confinement. We have specified the expressions for $\mathcal{N}$ for weak and strong magnetic fields [25], for which the analytical N dependence of E_0 can be derived. This has been done after logarithmic divergence regularization for the nearest interparticle distance mentioned above. We have obtained $\mathcal{N} = 1 + |\nu|(N-1)$ for weak magnetic fields under the condition $0 \leq \omega_c \leq \omega_0/(K(K-1))^{1/2}$, where $K \geq 2$ is the number of closed shells of anyon quantum states in the harmonic potential. The expression $\mathcal{N} = (1 + \nu\beta(N-1)/2)^2$ has been derived for strong magnetic fields, for which the condition $\omega_c \geq \omega_0(N-2)/(N-1)^{1/2}$ is valid. Taking into account the fact that $\omega_0 \sim 1/N^{1/2}$ (see the Introduction) and $K \to \infty$ when $N \to \infty$ in the weak magnetic field condition, one can find that the interval for ω_c shrinks to zero in the thermodynamic limit and the magnetic field dependence of the ground state energy per particle, $\mathcal{E}_0 = E_0/N$, can not be obtained.

According to results in Ref. [33] the ground state energy should be invariant under simultaneous sign changes of the magnetic field direction, $\beta = eH/|eH|$, and the anyon parameter, $\nu = e\phi/(2\pi\phi_0)$, where ϕ is the magnetic flux of the anyon gauge field and $\phi_0 = e/hc$ is the elementary flux quantum. We consider the case when the vector of the external magnetic field is parallel to the anyon magnetic flux. Therefore, we may replace $\beta\nu$ by ν in the interval $0 \leq \nu \leq 1$ in the second term of Eq. (10) and in the expression for $\mathcal{N}$ for the strong magnetic field case. We apply the modified

Eq. (10) with $\mathcal{N} = (1 + \nu(N-1)/2)^2$ to obtain an approximate analytic expression for $\mathcal{E}_0$ in the thermodynamic limit for noninteracting anyons. To do this we use the harmonic potential regularization procedure [24].

For the strong magnetic field case, with $\omega_0 \sim 1/N^{1/2}$, the harmonic potential regularization leads to a lower limit of the cyclotron frequency (or magnetic field), which for $N \to \infty$ becomes independent of the particle number N. For this case, the ground state energy per particle takes the form

$$\mathcal{E}_0 = \hbar \left(\omega_0^2 + \frac{\omega_c^2}{4}\right)^{1/2} \left(1 + \frac{\nu(N-1)}{2}\right) - \frac{\nu\hbar\omega_c}{4}(N-1)\,. \tag{11}$$

This expression coincides with the energy $E_0 = N\mathcal{E}_0$ given by Eq. (38) in Ref. [25].

We will use an ansatz similar to Eq. (4) and introduce $\omega_0 = \hbar\tilde{f}/(Mr_0^2 N^{1/2})$, where $\tilde{f}$ is the unknown constant. Thus the strong magnetic field condition is defined by the relation $\hbar\omega_c \geq \hbar^2\tilde{f}/(Mr_0^2)$ in the $N \to \infty$ limit. Keeping in mind that $\hbar\omega_c = \hbar^2/(Ml_H^2)$, this relation reduces to $\tilde{f}l_H^2/r_0^2 \leq 1$. However, we already know that $\nu_L = 2l_H^2/r_0^2$ is the definition of the Landau level filling factor for electrons. Therefore, we may put $\tilde{f} = 2$ for the unknown constant $\tilde{f}$ and obtain $\nu_L \leq 1$ for the LLL existence condition of the spin-polarized fermions ($\nu = 1$). Hence, our strong magnetic field condition reduces to the LLL existence condition. It is remarkable to notice from this analysis that ν_L is defined only for fermions. We recall that the strong magnetic field condition has been obtained in our previous work [25] for fermions.

Expanding the first term of Eq. (11) in powers of $4\omega_0^2/\omega_c^2$, substituting ω_0 in it (with $\tilde{f} = 2$) and taking into account the relation $\nu_L = 2l_H^2/r_0^2$ we obtain the exact expression for the ground state energy per particle in the thermodynamic limit ($N \to \infty$)

$$\mathcal{E}_0(\nu, \nu_L, \omega_c) = (1 + \nu\nu_L^2)\frac{\hbar\omega_c}{2}\,. \tag{12}$$

The last equation can be expressed in Ry units

$$\mathcal{E}_0(\nu, \nu_L, r_s) = (1 + \nu\nu_L^2)\frac{2}{\nu_L r_s^2}\,. \tag{13}$$

As seen from these equations the ground state energy per particle of anyons in the LLL is a function of three parameters.

It should be noticed that the second term on the right hand side of Eqs. (12) and (13) vanishes with either decreasing the anyon parameter ν or increasing the magnetic field. This is a result of the thermodynamic limit calculation. To make clear the physical origin of the term $\nu\nu_L^2$ we compare it with the corresponding result on the ground state energy of anyons for the case without magnetic field and Coulomb interaction. For that purpose we use the expression for the energy in boson limit of anyons [34]

$$E_0(\rho,\nu) = \pi\hbar^2\rho N\nu/M\,. \tag{14}$$

If we put in this expression $\nu = 1$ then it will reduce to Eq. (1) for the ideal fermion gas. Therefore, one can apply the Eq. (14) for an arbitrary anyon parameter ν. In the thermodynamic limit ($N \to \infty$), with the notation $\mathcal{E}_0(\rho,\nu) = E_0(\rho,\nu)/N$ it can be rewritten as (in Ry units)

$$\mathcal{E}_0(\nu, r_s) = \nu/r_s^2\,. \tag{15}$$

For spin-polarized fermions ($\nu = 1$) this is the energy of a filled Seitz circle with radius $k_F = 2(\pi\rho)^{1/2} = (2/(r_s a_B))$ and the areal particle density $\rho = 1/(\pi r_s^2 a_B^2)$. Thus the Eq. (15) describes the ground state energy per particle of anyons, which interact only via the anyon gauge field. In the light of this result, we treat $\mathcal{E}_0(\nu,\nu_L,\omega_c)$ (Eq. (12)) as a ground state energy per anyon in the presence of the external magnetic field and the second term on the right hand side of Eq. (12) as a contribution due to the anyon gauge field, i.e., due to Pauli principle. To our best knowledge, this result has not been reported in the literature so far.

3 Coulomb interacting anyon gas

The Hamiltonian of the system of N anyons with the Coulomb interaction and in magnetic field is given by the expression:

$$\begin{aligned} \hat{H} &= \sum_{k=1}^{N}[(\vec{p}_k - (\vec{A}_\nu(\vec{r}_k) + e\vec{A}_{ext}(\vec{r}_k)/c))^2] \\ &+ \frac{1}{2}\sum_{k,j\neq k}^{N}\left(\frac{e^2}{|\vec{r}_k - \vec{r}_j|} + V(\vec{r}_k)\right)\,, \end{aligned} \tag{16}$$

where (for $N \to \infty$) $V(\vec{r}_k)$, the interaction energy of the k-th electron with the jellium background

$$V(\vec{r}_k) = -\rho\int_S \frac{e^2\ d^2r}{|\vec{r}_k - \vec{r}|} \tag{17}$$

plays a role of the confining potential considered in Ref. [24]. Expressions for the vector potentials $\vec{A}_\nu(\vec{r}_k)$ and $\vec{A}_{ext}(\vec{r}_k)$ are given in the previous section.

We include the harmonic confining potential into the Hamiltonian, Eq. (16), and calculate the expectation value E, Eq. (7), with the wave function given by Eq. (8).

The minimization of the energy functional E with respect to the variational parameter α gives the ground state energy

$$\begin{aligned}\mathcal{E}_0 = \frac{E_0}{N} &= \hbar\omega_0 c^{1/3}\left[\frac{\mathcal{N}c^{1/3}}{2\bar{X}_0^2} + \frac{\bar{X}_0^2}{2} + \frac{\mathcal{M}}{\bar{X}_0}\right] \\ &\quad - \frac{\nu\hbar\omega_c(N-1)}{4}, \end{aligned} \tag{18}$$

where $c = 1 + \omega_c^2/(4\omega_0^2)$ and

$$\bar{X}_0 = (\bar{A}+\bar{B})^{1/2} + [-(\bar{A}+\bar{B}) + 2(\bar{A}^2 - \bar{A}\bar{B} + \bar{B}^2)^{1/2}]^{1/2}\ , \tag{19}$$

$$\begin{aligned}\bar{A} &= \left[\mathcal{M}^2/128 + ((c\mathcal{N}/12)^3 + (\mathcal{M}^2/128)^2)^{1/2}\right]^{1/3}, \\ \bar{B} &= \left[\mathcal{M}^2/128 - ((c\mathcal{N}/12)^3 + (\mathcal{M}^2/128)^2)^{1/2}\right]^{1/3}, \end{aligned} \tag{20}$$

we use the notation from Ref. [24]

$$\mathcal{M} = -\frac{aN^{1/2}}{a_B}\left(\frac{\hbar}{M\omega_0}\right)^{1/2} . \tag{21}$$

Notice, that the Eq. (18) coincides with the Eq. (50) in Ref. [25] after the replacement $\nu\beta \to \nu$ (see Sec. 2).

The parameter a was introduced in Ref. [24] as a constant which is determined by fitting the energy to the energy of the classical Wigner crystal for $N \to \infty$. In our case, the parameter a becomes a function of three parameters. For the LLL case ($\nu_L \le 1$) we employ the expressions $\mathcal{N} = (1 + \nu(N-1)/2)^2$ and $\omega_0 = \hbar\tilde{f}/(Mr_0^2 N^{1/2})$ with $\tilde{f} = 2$ used in the previous section to reach the thermodynamic limit.

Using the explicit expressions for $\mathcal{M}$, ω_0 and $\mathcal{N}$ one can expand the right hand side of Eq. (18) in series on powers of the small parameter $t = (\mathcal{M}^2/128)^2/(c\mathcal{N}/12)^3)$ (in the limit $N \to \infty$). Retaining the first order terms in this expansion the expression for the energy $\mathcal{E}_0$ reads

$$\mathcal{E}_0 = \hbar\omega_0 c^{1/3}\left(\mathcal{N}^{1/2}c^{1/6} - \frac{|\mathcal{M}|}{\mathcal{N}^{1/4}c^{1/12}}\right) - \frac{\nu\hbar\omega_c(N-1)}{4}\,, \tag{22}$$

from which one obtains the expression for the correlation energy

$$\mathcal{E}_0(\nu,\nu_L,\omega_c) - (1+\nu\nu_L^2)\frac{\hbar\omega_c}{2} = -\frac{\hbar^{3/2}\omega_c^{1/2}a(\nu,\nu_L,\omega_c)}{M^{1/2}a_B\nu^{1/2}}\,, \tag{23}$$

which is given by the subtraction of the kinetic energy obtained in the previous section from the ground state energy per particle. The expression of the correlation energy in e^2/l_H units is

$$\mathcal{E}_0(\nu,\nu_L,l_H) - (1+\nu\nu_L^2)\frac{a_B}{2l_H} = -\frac{a(\nu,\nu_L)}{\nu^{1/2}}\,. \tag{24}$$

All higher order terms in series expansion of the right hand side of Eq. (18) are proportional to power degrees of $|\mathcal{M}|/(c^{1/4}\mathcal{N}^{3/4})$ and vanish in the limit $N \to \infty$. Therefore, Eqs. (23) and (24) provide an exact result in the thermodynamic limit. The negative correlation energy is the result of the Coulomb interaction. The positively valued function a depends on the same parameters as the energy $\mathcal{E}_0$ (except the case of $\mathcal{E}_0$ in e^2/l_H units (see Eq. (24))).

To determine the function a one can use the Fano-Ortolani interpolation formula [8] for the correlation energy of spin-polarized fermions for the different FQHE states (in e^2/l_H units)

$$\begin{aligned} E_c^{FO}(\nu_L) &= \nu_L^{1/2}[-0.782133(1-\nu_L)^{3/2} \\ &+ (0.683(1-\nu_L)^2 - (\pi/8)^{1/2})\nu_L^{1/2} \\ &- 0.806\nu_L(1-\nu_L)^{5/2}]\,. \end{aligned} \tag{25}$$

We extend this formula to the gas of anyons by applying the following physical arguments:

i) in the fermion case ($\nu = 1$), the correlation energy of anyons should be equal to the Fano-Ortolani interpolation formula;

ii) for the vanishing filling factor ($\nu_L \to 0$) it must be independent on the anyon parameter ν, since this limit corresponds to the classical $2D$ Wigner crystal;

iii) for the boson gas ($\nu = 0$), the correlation energy should reproduce the numerical results of Yoshioka (see Fig. 3(b) of Ref. [27]) for the $2D$ Coulomb Bose gas in the LLL.

It turns out, the results of Yoshioka (except for the cusps at fractional fillings) can be interpolated by modifying the Fano-Ortolani interpolation formula, Eq. (25). This is achieved by replacing the second term inside the square brackets in (25) by $(0.683\nu(1-\nu_L)^2 - (A(1-\nu)+\nu(\pi/8)^{1/2}))\nu_L^{1/2}$, where $A = 0.45$, and the third term by $0.806(\nu + (1-\nu)\nu_L)\nu_L(1-\nu_L)^{5/2}$. If we denote the modified expression of $E_c^{FO}(\nu_L)$ as $E_c(\nu, \nu_L)$ then, by equating the right hand side of the Eq. (24) to $E_c(\nu, \nu_L)$, one obtains

$$a(\nu, \nu_L) = -\nu^{1/2} E_c(\nu, \nu_L)\,. \tag{26}$$

In Fig. 1 we show the ν_L dependence of the correlation energy $E_c(\nu, \nu_L)$ for two limiting cases of charged anyon gases: spin-polarized fermions ($\nu = 1$), for which $E_c(\nu = 1, \nu_L) = E_c^{FO}(\nu_L)$, and bosons ($\nu = 0$), for which the correlation energy is $E_c(\nu = 0, \nu_L)$. The two curves represent approximate interpolations of the results for

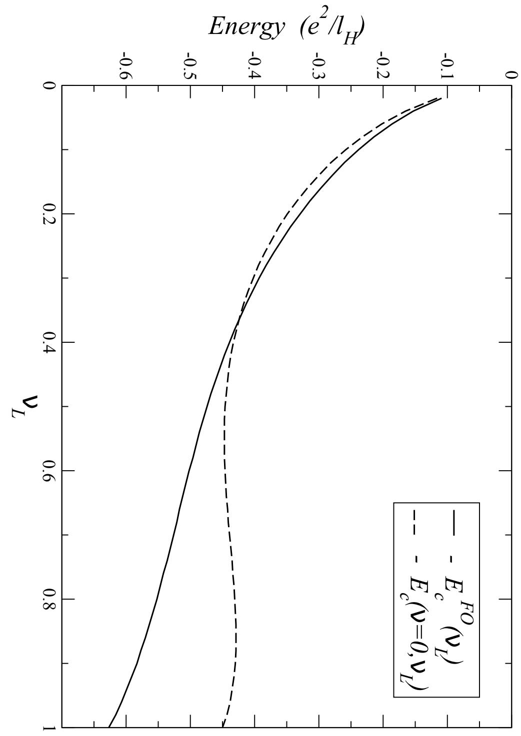

Figure 1: Correlation energy per particle in the LLL (expressed in e^2/l_H units) vs. ν_L for spin-polarized fermions, $E_c(\nu = 1, \nu_L) = E_c^{FO}(\nu_L)$, (solid line) and for bosons, $E_c(\nu = 0, \nu_L)$, (dashed line).

fermions and bosons in Ref. [27] which are valid in the whole range of filling factors $0 \leq \nu_L \leq 1$.

Thus the Eq. (24) together with Eq. (26) represents an analytic formula for the ground state energy of charged anyons in energy units of e^2/l_H in the quantum limit of an applied magnetic field.

4 Conclusions

We have derived analytic formulas for the ground state energy of the Coulomb anyon gas in strong magnetic field. Following the concept of our previous paper for the case without magnetic field, we have applied the harmonic potential regularization to the harmonically confined $2D$ Coulomb anyon gas in magnetic field. This concept is based on flattening out the confinement potential with simultaneously increasing the particle number to reach the thermodynamic limit. For the noninteracting anyon system we have obtained an analytic expression for the ground state energy per particle, which applies to strong magnetic field and represents an exact result in the thermodynamic limit. It contains a new contribution from the anyon gauge potential, which was not known before. For the interacting anyon system our analytic expression for the energy contains a function determined by fitting to the modified interpolation formula of Fano and Ortolani by taking into account the ground state energy of $2D$ Coulomb Bose gas in strong magnetic field.

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References

[1] Prange R.E. and Girvin S.M. The Quantum Hall Effect, Springer-Verlag, Berlin, (1987).

[2] Kato Y.K., Myers R.C., Gossard A.C., and Awschalom D.D. Observation of the Spin Hall Effect in Semiconductors. Science, Vol. 306, 1910–1913 (2004).

[3] Wunderlich J., Kaestner B., Sinova J. and Jungwirth T. Experimental Observation of the Spin-Hall Effect in a Two-Dimensional Spin-Orbit. Coupled Semiconductor System. Phys. Rev. Lett., Vol. 94, 047204 (2005).

[4] Abdullaev B., Choi M. and Park C.-H. Examination of Current-Induced Magnetic Field in the Slab Geometry: Possible Origin of Spin Hall Effect. J. Korean Phys. Soc., Vol. 49, S638–S641 (2006).

[5] Abdullaev B. Implicit Anyon or Single Particle Mechanism of HTCS and Pseudogap Regime. In: A.V. Ling (Ed.), Trends in Boson Research, Nova Science Publishers, N.Y., pp 139-161, (2006); Abdullaev B. and Park C.-H. Bosonization of 2D Fermions due to Spin and Statistical Magnetic Field Coupling and Possible Nature of Superconductivity and Pseudogap Phases Below E g. J. Korean Phys. Soc., Vol. 49, S642-S646 (2006).

[6] Laughlin R.B. Anomalous Quantum Hall Effect: An Incompressible Quantum Fluid with Fractionally Charged Excitations. Phys. Rev. Lett. Vol. 50, 1395 (1983).

[7] Levesque D., Weis J.J. and MacDonald A.H. Crystallization of the incompressible quantum-fluid state of a two-dimensional electron gas in a strong magnetic field. Phys. Rev. B., Vol. 30, 1056 (1984).

[8] Fano G. and Ortolani F. Interpolation formula for the energy of a two-dimensional electron gas in the lowest Landau level. Phys. Rev. B., Vol. 37, 8179 (1988).

[9] Ortiz G., Ceperley D.M. and Martin R.M. New stochastic method for systems with broken time-reversal symmetry: 2D fermions in a magnetic field. Phys. Rev. Lett., Vol. 71, 2777 (1993).

[10] Price R., Platzman P.M. and He S. Fractional quantum Hall liquid, Wigner solid phase boundary at finite density and magnetic field. Phys. Rev. Lett., Vol. 70, 339 (1993).

[11] Wilczek F. Magnetic Flux, Angular Momentum, and Statistics. Phys. Rev. Lett., Vol. 48, 1144–1147 (1982).

[12] Leinaas J.M. and Myrheim J. On the Theory of Identical Particles. Nuovo Cimento B., Vol. 37, 1–23 (1977).

[13] Halperin B. Statistics of Quasiparticles and the Hierarchy of Fractional Quantized Hall States. Phys. Rev. Lett., Vol. 52, 1583 (1984); Erratum, Phys. Rev. Lett., Vol. 52, 2390 (1984).

[14] Forte S. Quantum mechanics and field theory with fractional spin and statistics. Rev. Mod. Phys., Vol. 64, 193 (1992); Quantum Hall Effect, Ed. Stone M., World Scientific, Singapore, (1992).

[15] Johnson M.D. and Canright G.S. Anyons in a magnetic field. Phys. Rev. B., Vol. 41, 6870 (1990); Vercin A., Phys. Lett. B., Vol. 260, 120 (1991).

[16] Grundberg J., Hansson T.H., Karlhede A. and Westerberg E. Landau levels for anyons. Phys. Rev. B., Vol. 44, 8373 (1991).

[17] Roy B., Roy P. and Varshni Y.P. Two anyons in a constant magnetic field: a nonperturbative approach. Mod. Phys. Lett. B., Vol. 8, 159 (1994).

[18] Perez R. and Gonsales A. Few-anyon systems in a parabolic dot. Phys. Rev. B., Vol. 58, 7412 (1998).

[19] Khare A., Fractional Statistics and Quantum Theory, World Scientific, Singapore, (1997).

[20] Hansson T.H., Leinaas J.M. and Myrheim J. Dimensional reduction in anyon systems. Nucl. Phys. B., Vol. 384, 559–580 (1992).

[21] Calogero F. Solution of a Three-Body Problem in One Dimension. J. Math. Phys., Vol. 10, 2191–2197 (1969).

[22] Haldane F.D. Fractional statistics in arbitrary dimensions: A generalization of the Pauli principle Phys. Rev. Lett., Vol. 67, 937 (1991).

[23] Ouvry S. On the relation between the anyon and Calogero models. Phys. Lett. B., Vol. 510, 335–340 (2001).

[24] Abdullaev B., Rössler U. and Musakhanov M. An analytic approach to the ground state energy of charged anyon gases. Phys. Rev. B., Vol. 76, 075403(1–7) (2007).

[25] Abdullaev B., Ortiz G., Rössler U., Musakhanov M. and Nakamura A. Approximate ground state of a confined Coulomb anyon gas in an external magnetic field. Phys. Rev. B., Vol. 68, 165105(1–9) (2003).

[26] Wensauer A. and Rössler U. Exchange-correlation energy densities for two-dimensional systems from quantum dot ground states. Phys. Rev. B., Vol. 69, 155301 (2004).

[27] Yoshioka D. Ground state of the two-dimensional charged particles in a strong magnetic field and the fractional quantum Hall effect. Phys. Rev. B., Vol. 29, 6833 (1984).

[28] Lerda A. Anyons. Springer-Verlag, Berlin, (1992).

[29] Chitra R. and Sen D. Ground state of many anyons in a harmonic potential. Phys. Rev. B., Vol. 46, 10923 (1992).

[30] Wu Y.-S. Multiparticle Quantum Mechanics Obeying Fractional Statistics. Phys. Rev. Lett., Vol. 53, 111–115 (1984); Erratum, Phys. Rev. Lett., Vol. 53, 1028 (1984); Laughlin R.B. Superconducting Ground State of Noninteracting Particles Obeying Fractional Statistics. Phys. Rev. Lett., Vol. 60, 2677 (1988).

[31] Fetter A.L., Hanna C.B. and Laughlin R.B. Random-phase approximation in the fractional-statistics gas. Phys. Rev. B., Vol. 39, 9679 (1989).

[32] Galicki V.M., Karnakov B.M. and Kogan V.I. Problems in Quantum Mechanics, Nauka, Moscow, (1981) (in russian).

[33] Dasnieres de Veigy A. and Ouvry S. Equation of state of an anyon gas in a strong magnetic field. Phys. Rev. Lett., Vol. 72, 600 (1994).

[34] Sen D. and Chitra R. Anyons as perturbed bosons. Phys. Rev. B., Vol. 45, 881 (1991).