Bulletin of National University of Uzbekistan: Mathematics and Natural Sciences

Volume 3 | Issue 3 Article 2

9-15-2020

To the education of operational thinking in the higher mathematics courses of the higher technical school

Valentin Zharov
Russian State University for the Humanities, valcon@mail.ru

Arslan Mardanov
Tashkent State Technical University, apardayevich@mail.ru

TO THE EDUCATION OF OPERATIONAL THINKING IN THE HIGHER MATHEMATICS COURSES OF THE HIGHER TECHNICAL SCHOOL

Zharov V.K.[1] and Mardanov A.P.[2]
[1]*Russian State University for the Humanities, Moscow, Russia*
[2]*Tashkent State Technical University named after I.A.Karimov, Tashkent, Uzbekistan*
e-mail: valcon@mail.ru, apardayevich@mail.ru

Abstract

This article is devoted to the tasks of educating Operational thinking in the courses of higher mathematics of a higher technical educational institution. In particular, it comprehensively discusses the definitions of mathematical and operational thinking. It substantiates the need for a differentiated and structural approach to teaching mathematics in higher education and higher technical school in particular.

Keywords: *Mathematical thinking, operational thinking, rational thinking, engineering thinking.*

Mathematics Subject Classification (2010): *97A40, 97A80, 97B10, 97B30, 97B40, 97C30, 97D30.*

1 Introduction

Let's ask a non-territorial question, is it necessary for a modern person to be able to solve problems? In fact, many studies bring us closer to artificial intelligence, and, therefore, set a task and let the "intellect" to work for itself! Note, firstly you need to be able to formulate at least a task, we are not talking about the problem, and secondly you need to understand is the task solved correctly or not. It is clear that there are third, and fourth, etc. here; finally you still need to create this artificial intelligence that will require an unartificial brother! Therefore it is obvious that its creation will require very high skills and knowledge. Therefore, you need to answer the question how to solve the problem, for example, for a future engineer? Humanity has been looking for the answer to this question while hundreds of years. Each historical period of its development gave the humanity special answer. It is clear, however, that the state of the art corrected this answer. In fact, the inventor went from a living imitation and model to an increasingly abstract concept of a mechanism, product, or mathematical model of various types. In this article a short excursion to the history of problem solving methods is sufficient for our goals.

It is known that the ability to teach rational thinking and the necessary system of knowledge, as well as skills passed from generation to generation always and recently especially has been valued in society, and therefore it was a task for the pedagogical professional community. Therefore, historically the education of thinking is an urgent problem, however the methods of education, as mentioned above, differ from era to era. The education of thinking, as a task posed in psychology at the beginning of the twentieth century, has become extremely more significant in the age of information technology, Big Data and artificial intelligence. There is a problem of definition, typology of thinking.

According to G. Weil, one of the outstanding mathematicians of the twentieth century, it is known that "by the mathematical way of thinking I mean firstly a special form of reasoning through which mathematics penetrates the sciences of the external world - physics, chemistry, biology, economics, etc., and even in our reflections on everyday affairs and worries, and secondly the form of reasoning that the mathematician resorts to in his own area, being left to himself. In the process of thinking we are trying to comprehend the truth with the mind; our mind seeks to enlighten itself basing on our experience" [1].

Another outstanding mathematician W. Clifford in his work "Common Sense in the Exact Sciences" equated common sense and mathematics. Mathematics is a helpful tool for penetration into any area of human knowledge and thereby turning them into scientific knowledge [2]. In this case we can easily recognize the famous statements of ancient and medieval intellectuals about the meaning of mathematics in science.

And if to remember "A Brief Book on Completion and Opposition" by Al Khorezmi (mathematical treatise by Muhammad ibn Musa al-Khorezmi (IX century)), which has rules for working with algebraic equations necessary to obtain all the results needed for life. They are important not only to abstract and applied mathematics but also to modern algorithmic systems. Other his works are mentioned in the preface to the modern editions [3, 6].

B.M. Kedrov points out that “the sciences should be arranged in a sequential row and be connected in a certain way not because it seems convenient to us, but because the objects themselves, the very forms of motion of matter, studied and reflected by the corresponding sciences, are so interconnected, that is, in the sequence in which they themselves objectively, historically arise and develop one after another - the highest from the lowest, the complex from the simple; the principle of development. One of the most important sources on the history of natural science and philosophical thought, in particular the teachings on the classification of sciences, is the encyclopedic work "Mafatih al-ulum" ("Keys of Sciences") - a fundamental work of one of the major intellectuals of early medieval Central Asia Abu Abdullah al-Khorezmi" [3, 12]. It is clear that the system of sciences in the works of prominent representatives of the East differs from the system of worldview of Western cultures, but preserved and significantly developed them for all mankind [4].

Finally, for completeness of the picture about the importance of mathematics in human activity it is worth to remember K.E. Tsiolkovsky who speaks of mathematics as the Absolute [5].

From the above quotes of outstanding intellectuals it is not difficult to notice, in relation to the educational environment, the following conclusions (tasks). First, mathematics as a fundamental way of reasoning is universal in all environments and gives a methodological apparatus for studying knowledge; second, therefore, mathematical thinking is identified with common sense, adaptable to the corresponding knowledge, which has a universal language "species-determined" to the knowledge being studied; third, the ways of representing knowledge by mathematical language are implemented using the rules of the "grammar" of the corresponding specific knowledge, and the experience of applying knowledge in combination with the psychology of the culture of assimilating knowledge gives a variety of teaching methods, both for specific knowledge and for mathematics itself: and finally, various techniques, methods, and representations of mathematics in specific subject areas give rise to equally diverse teaching methods in various professional environments [7]–[11].

From the above text, the following conclusion can be drawn. Due to the natural circumstances of mathematics, philosophers have been thinking about the education and pedagogy of thinking from time immemorial. The book "Psychology and Pedagogy of Thinking" [6] was written by John Dewey. Problems are not limited to the education of mathematical thinking. But as it follows from it, the education of abstract thinking is one of the most difficult psychological problems. Note that the education of thinking, first of all, should be cultivated in the environment and in the professional school and demanded by the state.

2 On mathematical thinking according to G. Weil

In our works, we accept and slightly modify the definition of mathematical thinking given by G. Weil. It is quite clear from the definition that the invariant part of this thinking is a functional property. The development of a functional element to the level of real models creating of any type is the basis of engineering thinking.

In other words, mathematical thinking according to G. Weil, that is realized not in abstract fields of knowledge, but in professional concrete environments, for example: mechanical (lever, mechanisms, etc.), electrical (electricity, magnets, etc.), electronics (trigger, logical , relay circuits, etc.) environments we call engineering thinking [7, 8].

It is generally accepted that Engineering Thinking (ET) is a special type of thinking that is formed and manifested in solving engineering problems that allows quick, accurate and in an original way to solve the assigned tasks aimed to the technical needs for knowledge, methods, techniques, in order to create technical means. It is clear that this definition is fully described by our definition of engineering thinking. It is more accurate, since the concept of an engineering problem narrows the range of applications for engineering creativity [13, 14].

A more short and, in our opinion, also not an impeccable definition, is the following one - Engineering thinking is a special type of professional thinking that is formed and manifested in the ability to independently navigate new technologies, in their rationalization, modernization and implementation into production. Unfortunately, in this definition there is a problem with logic, at the beginning independence, is mentioned but in what? In technologies, improvement and modernization while introducing them into production, but modern engineer, as we understand it, is not an artisan and not only an innovator! So this definition is not entirely strict and makes possible the representation of modern engineer creative work . Let us recall that in the old days in Russia, an engineer was called "rozmysl", i.e. a person reflecting on a specific technical situation. It becomes clear that for the development of engineering thinking it is necessary to form the necessary set of theoretical tasks for the beginning, and only then tasks of a practical nature, but at the same time satisfying the passport of the specialty of the future profession. Thus, an engineer must possess the skills of mathematical thinking, but in a specific professional environment [13]. And, consequently, the methodology of teaching mathematics in a higher technical school should be oriented towards the student's future specialty, but at the same time possess the property of being fundamental, i.e. strictly scientific, satisfying the principles of continuity and sufficiency. But do not forget the formulated tasks by students must satisfy the principles:

- integrity (the object is considered as something whole);
- complexity (the requirement to take into account all interactions of the object with the environment and internal factors);
- organization (the requirement to take into account the structural ordering of the object);
- hierarchy (the requirement to consider relationships not only between elements of the same level, but also between different levels of the concept system).

The structure of a system or object is the internal structure of a given system, characterized by the presence of stable connections between its elements (parts) satisfying the investigated relations. Otherwise, the environment studied by the student is presented structurally, and the tasks acquire the status of constructive objects, hence the system itself is a text that can be both narrative and interrogative, and for the strongest students is a way of formulating theorems and statements regardless of the subject area.

3 Operational thinking concept

Mathematical thinking as a means of creating various type models, while implementing requires the use and development of operational properties in professional activity.

By operational thinking as a part of mathematical thinking, therefore engineering, we mean the first degree of abstraction in a specific subject area of an individual's activity. The concept of operational thinking is naturally the main property of functional thinking that underlies mathematical thinking [1, 10, 14].

In psychology the operational nature of thinking is understood as: analysis, synthesis and grouping. It is obvious that all these operations constitute a property of functional thinking - the main property of mathematical thinking. Hence, the study of classifications of sets with specific properties is a natural task for the disciplines of the mathematical group of higher and general education schools. The main feature of this group is logical and systematic (regular) construction of modally related knowledge of subject areas. Consequently, having a basic school, a future student accustomed to asking questions and looking for answers to them in various areas of professional environments gains experience in solving problems. Mathematics acts as a safe training ground for training in thinking. Therefore, the following statement is true:

4 Research for new type of differently methodically organized manual

It is necessary to create a new type of differently methodically organized manuals based on the principle of continuity, accessibility and development of professional skills required by the passport of the future specialty.

In fact, structurally, these manuals should be, so called, with the "public key", i.e. in addition to classical tasks, there should be tasks that are formulated in the language of the future specialty. For example, in the textual and knowledge environment of electronics, or biology, chemistry, automated control systems, servicing of urban housing, etc. In other words, structurally formulated tasks for a future professional, where mathematics acts as a language, grammar and text for modeling a professional problem.

Let's give an example. Suppose a student solves a problem, and he does not know from which area. He knows the process described in the problem and the only requirement is to optimize the result of the process, which is expressed in some condition. What will the individual do? Obviously, he should start asking questions. What kind? One can assume: from what area of knowledge or his profession did he read, hear or see similar tasks? What is the conceptual apparatus of the process? What's the point of optimization? Is it possible to determine the objective function, or can it be easier to describe the process by differential or other equations, etc. In fact, this set of questions can be divided into three classes of question types:

The first is the type, let's call it active;

Let's call the second type passive;

The third type will be called retrospective.

How do they differ from each other and where do the questions lead? In the first case, questions are asked to determine the structure, ideas embedded in the process (organized, constructive thinking); the second - "it seems that we did not study such processes, we were not told about it, when they tell, then I will think about it" (passive, often disorganized thinking); the third - "have we solved such problems before, it seems there were similar problems" (stereotyped thinking).

One of the authors had to observe the following reactions last semester. Third-year students specializing in applied mathematics.

Example 1. Two cylinders are given of the same diameter, the same thickness, but made of different materials, welded together along the bases. A heat source is located. The characteristics of the source and its location, the height of the common cylinder, the density of materials, etc. set all the necessary parameters and characteristics yourself as initial and boundary conditions. Describe the heat propagation in the cylinder with an equation.

Note that this problem was given in the middle of the course Equations of Mathematical Physics, while the course in Physics will be presented in the next semester, in addition, the solution of this problem is necessary to obtain credit and is admission to the exam. Finally, only limitation is the time of the session. So, we started solving the problem three weeks before the test. Questions: can I use material from other universitiy courses? And what about the articles? Can I join groups? The students received positive answers to all these questions, but with one caveat that each student defends his decision on his own! All emerging issues were discussed in practical classes. The solutions could be using application packages. What were the difficulties? The main difficulty was to describe the initial and boundary conditions of the problem, imagine this problem really. Try to understand what are the structural connections in the condition, then, according to preliminary reflections, write down the equation of thermal conductivity of parts of a common cylinder, and only after that carry out a computer experiment with changing conditions and build graphical representations of the experiment on heating an inhomogeneous cylinder and give possible interpretations. Being asked the question what was the most difficult thing in this problem, the students answered: “to start solving, it is not clear what the problem is, what conditions? Everything is simple in the book, it is clear to which section the task belongs, all the conditions are given there, work carefully and “communicate” with the Internet, but here it is not clear what to do! ”. This type of problem belongs to problems with vague conditions and therefore it is real.

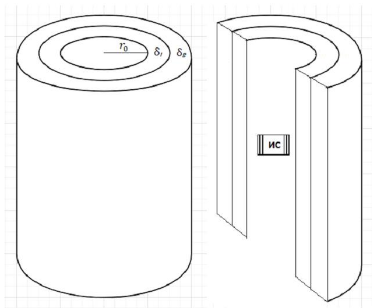
To solve the problem, you must first define the experiment itself, i.e. to accurately represent it, enter the necessary parameters and characteristics of objects, then select the location of the source either inside the cylinder, on the inner surface, or inside of cylinder walls, or on the outer surface, and even specify on which part of the common cylinder. In other words, the task was really difficult due to its reality, and it is clear that not all possible conditions are listed above, it is also clear that "physicists", "engineers" in thinking would solve the problem differently than "mathematicians", and "algorithms" turned to search engines. Therefore, comparing the manifestations of mental activity among students of the same group, we can observe who is who in their thinking. Here are two works that we refer to as different types of thinking. The first of the group of search, "algorithmic type", the second of the second or first type, the so-called "physicists" or "engineers".

5 The work of a student Vladislav Meneyev

Example 2.

Given

Suppose we have a two-layer thick-walled cylinder of finite dimensions, placed in a homogeneous medium with a constant temperature. The temperature in the cylinder cavity is constant, different from the initial temperature distribution inside the cylinder. At some time, a constant power heat source begins to operate.



Picture 1

IC - heat source

r_0 - inner radius of the cylinder;

δ_i - thickness of the i-th layer of the cylinder;

$\delta=\delta_1+\delta_2$ - cylinder thickness (total);

To find

Equation of heat propagation in a given cylinder.

The problem solution

Steps of solving the problem: it is necessary to find the temperature distribution in an unlimited cylinder and an unlimited plate. Further, using the superposition method, a general solution is found.

The differential equation of heat conduction in cylindrical coordinates at the first stage is written as follows: $a_i\left(\frac{\partial^2 T_i(\tau,r)}{\partial r^2}+\frac{1}{r}\frac{\partial T_i(\tau,r)}{\partial \mathrm{r}}\right)=\frac{\partial T_i(\tau,r)}{\partial \tau}$, where:

$i=1,2$ - cylinder layer number;

a_i - thermal diffusivity of the i-th layer;

$T_i\left(\tau,r\right)$ - temperature in the i-th layer;

r- radius;

τ - time.

Let us introduce the following boundary conditions: $T\left(0,r\right)=T_0$;

$T\left(\tau,r_0\right)=T_v$;

$T_2\left(\tau,r_0+\delta\right)=T_{rp}$;

$\lambda_1\frac{\partial T_1}{\partial r}|_{r_0+\delta_1}=\lambda_2\frac{\partial T_2}{\partial r}|_{r_0+\delta_2}$;

$T_1|_{r_0+\delta_1}=T_2|_{r_0+\delta_2}$.

where:

T_0- initial cylinder temperature;

T_v-temperature of the medium in the cylinder cavity; T_{rp}- temperature of the medium on the surface of the cylinder;

λ_i- coefficient of thermal conductivity of the i-th layer.

The solution is sought in the form of the sum of a particular solution of an inhomogeneous DE $a_i\left(\frac{\partial^2 T_i(\tau,r)}{\partial r^2}+\frac{1}{r}\frac{\partial T_i(\tau,r)}{\partial \mathrm{r}}\right)=\frac{\partial T_i(\tau,r)}{\partial \tau}$ and a general solution of a homogeneous DE of the form $a_i\left(\frac{\partial^2 T_i(\tau,r)}{\partial r^2}+\frac{1}{r}\frac{\partial T_i(\tau,r)}{\partial \mathrm{r}}\right)=0$.

To find a particular solution we use the method of separation of variables and represent the required function in the form $N_i\left(\tau,r\right)=Z_i\left(\tau\right)*X_i(r)$. Here:

$$X_i\left(r\right)=\sum_{\mathrm{j=1}}^{\infty}\left[A_{i,j}J_0\left(\mu_j r\right)+B_{i,j}Y_0\left(\mu_j r\right)\right],$$

$$Z_i\left(\tau\right)=\sum_{\mathrm{j=1}}^{\infty}exp(-a_i\mu_j^2\tau),$$

where:

$J_0\left(\mu_j r\right)$, $Y_0\left(\mu_j r\right)$- Bessel functions of the first kind of zero order;

$A_{i,j}$, $B_{i,j}$, μ_j - characteristic numbers of the problem.

Substituting in $N_i\left(\tau,r\right)=Z_i\left(\tau\right)*X_i(r)$ into the boundary conditions indicated earlier, and also taking into account that, $J_0'\left(z\right)=-J_1\left(z\right),Y_0'\left(z\right)=-Y_1\left(z\right)$, we obtain a determinant for finding the eigenvalues of the characteristic equation, that has the form:

$$\begin{vmatrix} J_0(\mu r_0) & Y_0(\mu r_0) & 0 & 0 \\ J_0(\mu(r_0+\delta_1)) & Y_0(\mu(r_0+\delta_1)) & -J_0(\mu(r_0+\delta_1)) & -Y_0(\mu(r_0+\delta_1)) \\ \lambda_1 J_1(\mu(r_0+\delta_1)) & \lambda_1 Y_1(\mu(r_0+\delta_1)) & -\lambda_1 J_1(\mu(r_0+\delta_1)) & -\lambda_1 Y_1(\mu(r_0+\delta_1)) \\ 0 & 0 & J_0(\mu(r_0+\delta)) & Y_0(\mu(r_0+\delta)) \end{vmatrix}$$

The system has a nontrivial solution when the determinant is equal to zero.

Let us determine the eigenvalues of the problem by solving the determinant with respect to μ. Find A_j and B_j from $X_i\left(r\right)=\sum_{\mathrm{j=1}}^{\infty}\left[A_{i,j}J_0\left(\mu_j r\right)+B_{i,j}Y_0\left(\mu_j r\right)\right]$. Let us express B_j through A_j and use the orthogonality condition of the function. We get:

$$A_{i,j}=\frac{\int_{r_0}^{r+\delta} r\left(T\left(0,r\right)-T_i\left(r\right)\right)\left[\ J_0\left(\mu_i r\right)-Y_0\left(\mu_i r\right)\frac{J_0(\mu_i r_0)}{Y_0(\mu_i r_0)}\right]dr}{\int_{r_0}^{r+\delta} r\left(J_0\left(\mu_i r\right)-Y_0\left(\mu_i r\right)\frac{J_0(\mu_i r_0)}{Y_0(\mu_i r_0)}\right)^2 dr}$$

Here $T_i\left(r\right)$ is the temperature distribution in the stationary mode. Integration is carried out numerically. To determine $T_i\left(r\right)$, we solve $a_i\left(\frac{\partial^2 T_i(\tau,r)}{\partial r^2}+\frac{1}{r}\frac{\partial T_i(\tau,r)}{\partial \mathrm{r}}\right)=0$ by presenting the solution in the form $T_i\left(r\right)=A_i\ln r\ +B_i$. Substituting the solution into the boundary conditions, we get the following:

$$A_2=\frac{T_v-T_{rp}}{\ln\left(\frac{r_0+\delta_1}{r_0}\right)\frac{\lambda_2}{\lambda_1}-\ln\left(\frac{r_0+\delta_1}{r_0+\delta}\right)},$$

$$A_1=\frac{\lambda_2}{\lambda_1},$$

$$A_2B_1 = T_v - A_1\ln(r_0)\ ,$$
$$B_2 = T_{rp} - A_2\ln(r_0+\delta)\ .$$

The temperature distribution in an unbounded cylinder at a constant temperature in the cavity is written as:

$$T_\mu(\tau,r) = \begin{cases} T_1(r) + N_1(\tau,r),\ if\ r_0 \le r \le (r_0+\delta_1) \\ T_2(r) + N_2(\tau,r),\ if\ (r_0+\delta_1) \le r \le (r_0+\delta) \end{cases}$$

At the second stage, solution the problem of thermal conductivity for an unbounded cylinder has the form:

$$-a_i\left(\frac{\partial^2 T_i(\tau,r)}{\partial r^2} + \frac{1}{r}\frac{\partial T_i(\tau,r)}{\partial \mathrm{r}}\right) + \frac{\omega}{c_i\rho_i} = \frac{\partial T_i(\tau,r)}{\tau}$$

Here:

ω - power of the heat source;

c_i - specific heat capacity of the i-th layer;

ρ_i - is the density of the i-th layer.

The boundary conditions are as follows:

$$T(0,r) = T_\mu(\tau,r);-\lambda_1\frac{\partial T_1}{\partial \mathrm{r}}|_{r_0} + \alpha(T_v - T_i(\tau,r_0)) = 0; T_2(\tau,r_0+\delta) = T_{\mathrm{rp}};$$

$$\lambda_1\frac{\partial T_1}{\partial \mathrm{r}}|_{r_0+\delta_1} = \lambda_2\frac{\partial T_2}{\partial \mathrm{r}}|_{r_0+\delta_1}; T_1|_{r_0+\delta_1} = T_2|_{r_0+\delta_1}.$$

With free convection on a vertical wall, the heat transfer coefficient is $\alpha(\tau) = 1,66\cdot\sqrt[3]{(T(\tau) - T(\tau,r_0))}$

Let's go back to the beginning of the second stage of solving DE and transfer the free term to the right side of the equation. Then the solution of this equation will be written as the sum of the general solution of the equation and the particular equation for the given boundary conditions:

$$T_i(r) = A_i\ln r\ + B_i.$$

where

$A_1 = -\frac{(\alpha(\tau))^4}{1{,}66^3} * \frac{r_0}{\lambda_1}$; $A_2 = \frac{\lambda_1}{\lambda_2}A_1$; $F = 2\pi r_0 H$;

F is the area of the inner surface of the cylinder;

H is the height of the cylinder.

Finding the coefficients B_i is similar to the first stage.

A particular solution to the equation is sought in the form $P(\tau,r) = H(\tau,r) + G(r)$, where:

$H(\tau,r)$ - a particular solution of the DE at the first stage under the boundary conditions of the second stage;

$G(r)$ - a particular solution to an equation of the form $a_i\left(\frac{\partial^2 T_i(\tau,r)}{\partial r^2} + \frac{1}{r}\frac{\partial T_i(\tau,r)}{\partial \mathrm{r}}\right) = \frac{\omega}{c_i\rho_i}$.

The solution is sought in the form $R_i(r) = A_i\ln r + B_i + C_i r^2$, where $C_i = \frac{\omega}{4a_i c_i \rho_i} = \frac{\omega}{4\lambda_i}$.

Substituting the solution of a particular solution into the boundary conditions, we obtain the following:

$$A_1 = \frac{(\alpha(\tau))^4}{1,66^3} * \frac{r_0}{\lambda_1} - 2C_1 r_0^2$$

$$A_2 = \frac{\lambda_1}{\lambda_2}\left(A_1 + 2C_1(r+\delta_1)^2\right) - 2C_2(r+\delta_1)^2$$

$$B_2 = T_{rp} - A_2 \ln(r_0+\delta) - C_2(r+\delta_1)^2$$

$$B_1 = B_2 + A_2 \ln(r_0+\delta_1) + C_2(r+\delta_1)^2 - A_1 \ln(r_0+\delta_1) - C_1(r+\delta_1)^2$$

A particular solution to the equation is:

$$G(r) = \begin{cases} R_1(r), \ if \ r_0 \le r \le (r_0+\delta_1) \\ R_2(r), \ if \ (r_0+\delta_1) \le r \le (r_0+\delta) \end{cases}$$

When finding a solution $H(\tau, r)$, the determinant is written in the form

$$\begin{vmatrix} J_1(\mu r_0) & Y_1(\mu r_0) & 0 & 0 \\ J_0(\mu(r_0+\delta_1)) & Y_0(\mu(r_0+\delta_1)) & -J_0(\mu(r_0+\delta_1)) & -Y_0(\mu(r_0+\delta_1)) \\ \lambda_1 J_1(\mu(r_0+\delta_1)) & \lambda_1 Y_1(\mu(r_0+\delta_1)) & -\lambda_1 J_1(\mu(r_0+\delta_1)) & -\lambda_1 Y_1(\mu(r_0+\delta_1)) \\ 0 & 0 & J_0(\mu(r_0+\delta)) & Y_0(\mu(r_0+\delta)) \end{vmatrix}.$$

The odds $A_{i,j}$ are:

$$A_{i,j} = \frac{\int_{r_0}^{r+\delta} r\left(T_\mu(\tau, r) - T_i(r)\right)\left[J_0(\mu_i r) - Y_0(\mu_i r)\frac{J_1(\mu_i r_0)}{Y_1(\mu_i r_0)}\right] dr}{\int_{r_0}^{r+\delta} r\left(J_0(\mu_i r) - Y_0(\mu_i r)\frac{J_1(\mu_i r_0)}{Y_1(\mu_i r_0)}\right)^2 dr}$$

Then $H_i(\tau, r) = \sum_{j=1}^{\infty} A_{i,j}\left[J_0(\mu_j r) - Y_0(\mu_j r)\frac{J_0(\mu_j(r_0+\delta))}{Y_0(\mu_j(r_0+\delta))}\right]\exp(-\mu_j^2 a_i \tau)$.

The general solution will be written as follows:

$$T_i(\tau, r) = H_i(\tau, r) + G(r) + T_i(r)$$

Knowing the heat transfer coefficient on the inner wall of the cylinder, one can find the wall temperature at any time, from which the air temperature is expressed using $\alpha(\tau) = 1,66 \cdot \sqrt[3]{(T_v(\tau) - T(\tau, r_0))}$.

Thus, an analytical expression was obtained for the temperature field of a two-layer hollow cylinder of finite dimensions, inside which a constant power source acts.

Anna Temyasheva's solution

The problem of heat distribution from a source in a thick-walled cylinder.

A thick-walled cylinder of height h is given, consisting of two cylinders of height. A heat source M (Q) is located inside the cylinder cavity. Derive equations that will describe the distribution of heat in the cylinders.

The heat distribution equation in a cylindrical coordinate system is written as follows:

$$\frac{\partial u}{\partial t}=a^2\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}+\frac{\partial^2 u}{\partial z^2}\right) \tag{1}$$

Initial condition: $u|_{t=0}=\varphi(r,\ \theta,\ z)=\pi r_0 h$

Border conditions: $u|_{z=h_1}=u|_{z=h_2}=0;\quad u|_{r=r_1}=u|_{r=r_2}=0.$

Applying the Fourier method and defining the introduced constants through the boundary conditions, we obtained particular solutions of equation (1), that looks as follows:

$$e^{-a^2\left(\lambda^2+\frac{m^2\pi^2}{h_i^2}\right)t}J_n\left(\lambda r\right)sin\frac{m\pi}{h_i}\left(z+h_i\right)\left(Acosn\theta+Bsinn\theta\right), \tag{2}$$

$i=1,2,m$ - positive integers.

The constant λ is related to the roots of the equation $J_n(\mu)=0$ by equality $\lambda=\frac{\mu}{r_i}$.

n is an integer since the temperature is represented as a periodic function of an angle θ with a period of 2π.

The solution to the problem will be presented as a sum of solutions (2) for all $n=0,1,2,\dots$, $m=1,2,3,...$ and for all positive roots $\mu_{n1},\ \mu_{n2},\ \mu_{n3},...$:

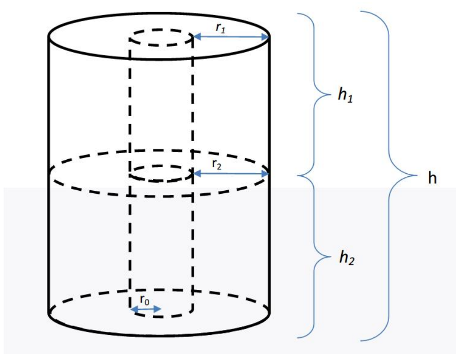
$$u=\sum_{k=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}e^{-a^2\left(\frac{\mu_{nk}^2}{r_i^2}+\frac{m^2\pi^2}{h_i^2}\right)t}J_n\left(\frac{\mu_{nk}r}{r_i}\right)\cdot\left(sin\frac{m\pi}{h_i}\left(z+h_i\right)\right)\cdot$$
$$\cdot\left(A_{kmn}cosn\theta+B_{km}sinn\theta\right)+\frac{c_i\omega}{V_i\rho_i},$$

where c_i– heat source power, ρ_i – specific heat of material, V_i – cylinder volume.

Need to define A_{kmn} and B_{kmn}.

Let $t=0$, then, taking the initial condition, we get:

$$\varphi(r,\theta,z)=\sum_{k=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}J_n(\frac{\mu_{nk}r}{r_i})sin\frac{m\pi}{h_i}(z+h_i)\cdot(A_{kmn}cosn\theta+B_{kmn}sinn\theta). \tag{3}$$



Picture 2

The right side of equality (3) is the expansion of the function $\varphi(r,\ \theta,\ z)$ in a Fourier series in sin and cos, then the coefficients can be expanded by the formulas:

$$\frac{1}{\pi}\int_0^{2\pi}\varphi\left(r,\theta,z\right)cos\ n\theta\ d\theta=\sum_{k=1}^{\infty}(\sum_{m=1}^{\infty}A_{kmn}sin\ \frac{m\pi}{h_i}(z+h_i))\ J_n(\frac{\mu_{nk}r}{r_i}),$$

$$\frac{1}{\pi}\int_0^{2\pi}\varphi\left(r,\theta,z\right)\ sin\ n\theta\ d\theta=\sum_{k=1}^{\infty}(\sum_{m=1}^{\infty}B_{kmn}sin\frac{m\pi}{h_i}(z+h_i))J_n(\frac{\mu_{nk}r}{r_i}).$$

These equalities represent the expansion of the function of r in a series in terms of the Bessel function. Then the coefficients will be calculated as follows:

$$\sum_{m=1}^{\infty}A_{kmn}sin\ \frac{m\pi}{h_i}\ (z+h_i)=\frac{2}{\pi r_i^2J_{n+1}^2(\mu_{nk})}\int_0^{r_i}\int_0^{2\pi}r\varphi(r,\ \theta,\ z)J_n(\frac{\mu_{nk}r}{r_i})cos\ n\theta drd\theta,$$

$$\sum_{m=1}^{\infty}B_{kmn}sin\frac{m\pi}{h_i}\left(z+h_i\right)=\frac{2}{\pi h_ir_i^2J_{n+1}^2\left(\mu_{nk}\right)}\int_0^{r_i}\int_0^{2\pi}r\varphi\left(r,\theta,z\right)J_n\left(\frac{\mu_{nk}r}{r_i}\right)sinn\theta drd\theta$$

The functions form $sin\frac{m\pi}{h_i}\left(z+h_i\right)$ an orthogonal system of functions on the segment [0, h], then the coefficients will be determined by the following formulas:

$$A_{kmn}=\frac{2}{\pi h_ir_i^2J_{n+1}^2(\mu_{nk})}\int_0^{r_i}\int_0^{2\pi}\int_0^{h_i}r(2\pi r_0(r_0+h)J_n(\frac{\mu_{nk}r}{r_i})cos\ n\theta\cdot$$

$$\cdot sin\ \frac{m\pi}{h_i}(z+h_i)drd\theta dz$$

$$B_{kmn}=\frac{2}{\pi h_ir_i^2J_{n+1}^2(\mu_{nk})}\int_0^{r_i}\int_0^{2\pi}\int_0^{h_i}r(2\pi r_0(r_0+h)J_n(\frac{\mu_{nk}r}{r_i})sin\ n\theta\cdot$$

$$\cdot sin\ \frac{m\pi}{h_i}(z+h_i)drd\theta dz$$

The general solution would look like this:

$$u\left(r,\theta,z\right)=\sum_{k=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}e^{-a^2\left(\frac{\mu_{nk}^2}{r_i^2}+\frac{m^2\pi^2}{h_i^2}\right)t}J_n\left(\frac{\mu_{nk}r}{r_i}\right)\cdot\left(sin\frac{m\pi}{h_i}\left(z+h_i\right)\right)\cdot$$

$$\left(\left[\frac{2}{\pi h_ir_i^2J_{n+1}^2\left(\mu_{nk}\right)}\int_0^{r_i}\int_0^{2\pi}\int_0^{h_i}r\left(\pi r_0h\right)J_n\left(\frac{\mu_{nk}r}{r_i}\right)\cos n\theta\cdot sin\ \frac{m\pi}{h_i}\left(z+h_i\right)drd\theta dz\right]\cdot\right.$$

$$\cdot cosn\theta+$$

$$\left[\frac{2}{\pi h_ir_i^2J_{n+1}^2\left(\mu_{nk}\right)}\int_0^{r_i}\int_0^{2\pi}\int_0^{h_i}r\left(\pi r_0h\right)J_n\left(\frac{\mu_{nk}r}{r_i}\right)\sin n\theta\cdot sin\ \frac{m\pi}{h_i}\left(z+h_i\right)drd\theta dz\right]\cdot$$

$$\left.\cdot \sin n\theta\right) + \frac{c_i \omega}{V_i \rho_i}.$$

Examples of using different parameters, graphic interpretations:

The works presented as an example refer to the first iteration, after that there were two more iterations. In the second work, the analysis of the response depending on the parameters was made and the graphs of solutions were presented depending on the change in the initial and boundary conditions. During the public discussion of the work, some shortcomings were corrected, but the main thing was a lively discussion of the work by all members of the study group. The scope of this article does not allow presenting the solution to the problem by "mathematicians", but its purpose is different - to draw attention to the possibilities of developing operational thinking in a technical university. Students relied on the popular Internet resources in the form of packages: Mathematica (any version), MatLab, WOLFRAM MATHEMATICA and the involvement of some graphics systems. The recommended list of references was as follows [11], [13]–[21].

However, in higher education we have a goal set a priori by the state - "at the exit" there should be a specialist who meets the specialty passport, but a specialist is not only a specialist with special knowledge, but above all a thinker in a special specific environment, not a functionary in the worst understanding of this word, but an activist with well-mannered thinking.

Therefore, in our opinion, modern education in higher education often follows the modern conditions of the development of a society with good and not very good qualities, for example, clip thinking and consciousness, with the absence of a predominance of critical thinking, in particular the engineering environment, should be as limited as possible from modern clip culture.

In order for its harmful effect to be minimized on the educational environment, it is enough to correct the mental activity of students, especially after the first year, with feasible tasks of the future specialty, requiring, for example, mathematical modeling or conducting experiments, i.e. creating an environment for the application of operational activities with concepts of various degrees of abstraction.

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