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Olim Suvonov
"Bulletin of TUIT: Management and Communication Technologies", olimsuvonov54@umail.uz

Elmira Nazirova
Bulletin of TUIT: Management and Communication Technologies, elmira_nazirova@mail.ru

MATHEMATICAL MODELING OF THE FUNCTIONING OF A HYDRODYNAMIC SYSTEM WITH DISTRIBUTED PARAMETERS

Suvonov Olim Omonovich
Associate Professor of the Department of Teaching Methods of Informatics, Navoi State Pedagogical Institute, Candidate of Technical Sciences, 201100, Navoi, Uzbekistan. e-mail: olimsuvonov54@umail.uz
Nazirova Elmira Shodmonovna
Tashkent University of Information Technology, DSc, Uzbekistan, e-mail: elmira_nazirova@mail.ru

Annotation: The work examines the process of functioning of the hydrodynamic system "reservoir-well" taking into account the dynamic process of shutting down and starting a group of production wells. A mathematical model for determining system states, an algorithm for solving an applied problem and the results of computational experiments in the form of graphs are given.

Keywords: system, distributed parameter, hydrodynamic process, mathematical model, filtration process, pressure, water saturation. killing, start-up, production well, computational experiment, iteration, approximation, difference schemes.

1. Introduction.

The mathematical theory of control of objects with distributed parameters has been greatly developed. With a significant number of scientific works devoted to this area, it is necessary to note the works of Russian scientists A.G. Butkovsky, A.I. Egorov, V.P. Zhivoglyadov, M.V. Meerov, K.A.Lurie, Yu.I. Samoilenko , T.K. Sirazetdinov, A.A. Feldbaum and Uzbek scientists Kabulov V.K., Satimov N.S., Kamilov M.M., Bekmuratov T.F., Zhuraev T.Zh., Salokhiddinov M.S. Abutaliev F.B., Sadullaev R.S., Rasulov A.S. and a number of others. An important contribution has also been made by foreign experts, among whom the names of RKS Wong, V. Malgrange, J.L. Lyons, V. Fleming, A. Friedman are known [1,2].

During the development of fields, for various reasons, wells are stopped for a while [3]. To kill them, drilling fluids are used, which, due to the formation of hydrostatic pressure, can penetrate deep into the formation. The penetration of filtrate into the reservoir in most cases worsens the permeability of the bottomhole zone of wells. Consequently, their productivity changes. The study of the processes occurring during killing and putting wells into operation allows a quantitative and qualitative assessment of the change in well productivity and contributes to the normal functioning of the "P-S" system as a whole.

Formulation of the problem.

The process of functioning of the P-S system, taking into account the dynamic process of stopping and starting a group of production wells, is divided into three stages: before well shutdown;

1) during their stop;
2) after putting the wells into operation.
3) after putting the wells into operation

For mathematical modeling, we will accept the following assumptions: the flow regime is isothermal, for each of the phases the generalized Darcy's law is valid; filtration occurs in a horizontal formation and can be represented as radial flows; gas and flushing liquid are mutually insoluble; their viscosity is constant; the compressibility of a liquid in comparison with the compressibility of a gas can be neglected; clean gas is produced before the well is shut down.

In the first case, which is the normal mode of operation, the state of the system is described by a partial differential equation, for $S=0,$ those. clean gas is produced [4,5]:

$$\frac{1}{r^{\alpha}}\frac{\partial}{\partial r}(r^{\alpha}\frac{\partial P^2}{\partial r})=\frac{2m\mu_2}{k}\frac{\partial P}{\partial t},\alpha R_c<r<R_k,t\in(0,T). \quad (1)$$

At the second stage, a dynamic process of non-piston displacement takes place in the formation structure, which is associated with artificial impact on the “P-S” system. Therefore, the dynamic process of well killing is described by the system of equations

$$\begin{cases}\frac{1}{r^{\alpha}}\frac{\partial}{\partial r}(r^{\alpha}f_2(S)\frac{\partial P^2}{\partial r})=-\frac{2m\mu_2}{k}P\frac{\partial S}{\partial t}+\frac{2m(1-S)}{k}\mu_2\frac{\partial P}{\partial t},\\ \frac{1}{r^{\alpha}}\frac{\partial}{\partial r}(r^{\alpha}f_1(S)\frac{\partial P}{\partial r})=\frac{m\mu_1}{k}\frac{\partial S}{\partial t},\alpha R_C<r<R_k,t\in(T_1,T_2).\end{cases} \quad (2)$$

There T_1 - start, and T_2 - well shut-in end.

At the third stage, a reverse dynamic process occurs in the formation structure, i.e. The drilling fluid is displaced by gas, the pressure of which is the internal energy of the formation. This process is also described by the system of equations (2), and $t \in [T_2, T]$. There T - total time of the process research. Equations (1) and (2) are solved under the condition at the boundaries of the reservoir

$$\alpha_1 (r^\alpha \frac{\partial P^2}{\partial r} - cq) + (1-\alpha_1)(P - P_c) = 0,$$

$$r = \alpha R_C; \qquad (3)$$

$$r^\alpha \frac{\partial P^2}{\partial r} = 0, r = R_k, \frac{\partial S}{\partial r}(\alpha R_c, t) = 0, t \in (T_1, T_2);$$

(4)

$$S(\alpha R_c, t) = 1 - S_*, S(R_k, t) = 0, t \in [T_1, T].$$

(5)

At the first stage of functioning

$$\begin{cases} P(r,0) = P_H, \\ S(r,0) = S_H, \end{cases}$$

(6)

$$P_c = \begin{cases} P_{1C}, t \in (T_1, T_2), \\ P_{2C}, t \in (T_2, T], \end{cases}$$

(7)

where $1 - S_*$ - residual gas saturation;

r - distance from origin to specified point;

μ_1, μ_2 - respectively, the viscosity of the gas and filtrate of the flushing liquid;

$f_1(S), f_2(S)$ - phase permeabilities for gas and filtrate of flushing liquid, respectively;

$P = P(r,t), S = S(r,t)$ - respectively, the pressure and saturation of the filtrate of the flushing liquid at a point at a time t;

R_c - central bore radius;

R_k - feed loop radius;

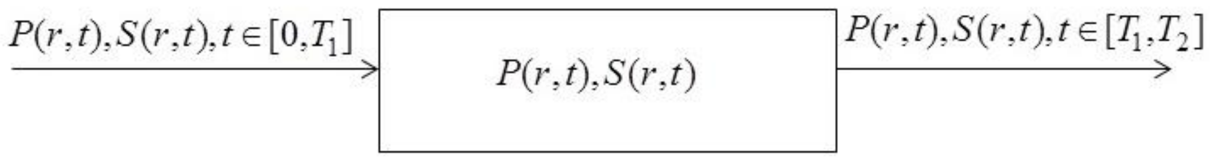
P_c - well pressure;

P_H - initial reservoir pressure;

α_1 - parameter defining a given well condition.

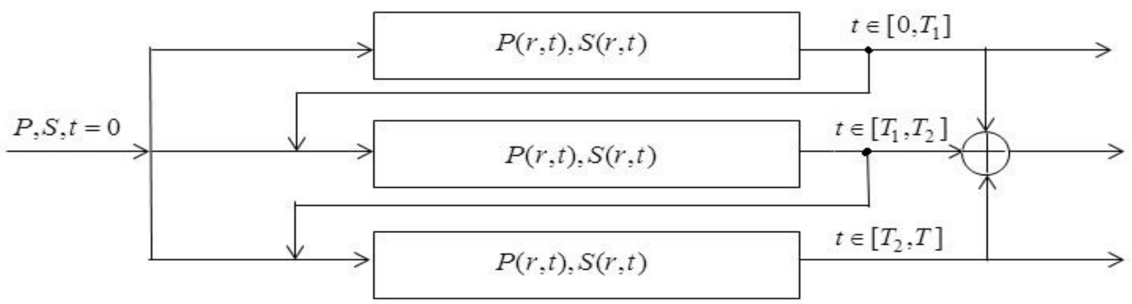
The mathematical model shows that the system under study, taking into account the dynamic process, shut-in and start-up of wells, is closed, non-linear, flexible, and the structure of functioning shows that it is multichannel.

With a graphical representation of a nonlinear block characterizing nonlinear phenomena in the "P-S" system with distributed parameters, in the structural diagram of the system inside a rectangle corresponding to this nonlinear block, we write the sought functions of the differential equations describing the state of the system (pic. 1).



Pic.1 "P-S" system with distributed parameters

The connection of nonlinear blocks of states of the "P-S" system, in which the output of the previous block is the input of the next block, can be called a series connection of state blocks (Pic. 2). In this case, the condition of consistency of the series-connected blocks must be met.



Pic.2. Block-structural diagram of the "P-S" system functioning

<u>3. Algoritm for solving the problem.</u> It should be noted that the class of equations that can be approximately solved by numerical methods is much wider than the class of equations available for analytical research. In addition, solving problems using numerical methods is usually more complete than analytical research.

For the numerical implementation of the mathematical model for analyzing the state of the system under study in equations (1), (2) and boundary conditions (3) - (7), we turn to dimensionless variables. The transition to dimensionless variables is carried out according to the formulas given in [4,5].

We leave the designations of the variables unchanged, then equation (1) takes the form

$$\varphi \frac{\partial^2 P^2}{\partial r^2} = \frac{\partial P}{\partial t},$$

$0 < r < 1, t \in [0, T_1],$ (8)

and the system of equations (2) after simple transformations will be represented in the form

$$\varphi \frac{\partial}{\partial r}(f_1 \frac{\partial P}{\partial r}) = \frac{1}{\mu_0} \frac{\partial S}{\partial t},$$

(9)

$$\varphi \frac{\partial}{\partial r}\left[(f_2 + \frac{\mu_2}{\mu_1} f_1) \frac{\partial P^2}{\partial r}\right] = 2\mu_0 f_1 (\frac{\partial P}{\partial r})^2 + (1-S)\frac{\partial P}{\partial t}.$$

(10)

The boundary and initial conditions are as follows:

$$\begin{cases} \alpha\left[\frac{\partial P^2}{\partial r} - q(\ln R_k / R_c)^\alpha\right] + (1-\alpha_1)(P - P_c), r = 0; \\ \frac{\partial P^2}{\partial r} = 0, r = 1; \end{cases}$$

(11)

$$\begin{cases} S(0,t) = 1 - S_*, t \in (T_1, T_2), \\ \frac{\partial S(0,t)}{\partial r} = 0, t \in [T_2, T]; \end{cases}$$

(12)

$$P(r,0) = 1,$$

(13)

$$S(r,0) = 0.$$

(14)

There

$$\varphi = \varphi(r) = \left[\ln(R_k / R_c) \cdot (R_c / R_k)^{1-r}\right]^{-2\alpha}.$$

Based on the foregoing, we construct a non-uniformly space-time grid covering the region of integration of the system of equations (8) - (14):

$$D = \{(r,t) : 0 < r < 1, 0 < t < T\}$$

$$D_{h,\tau} = \{(r_i, t_j) : r_i = r_{i-1} + h_i, i = \overline{1,n}, r_0 = 0, r_N = 1,$$

$$t_j = t_{j-1} + \tau_j, j = \overline{1,m}, t_0 = 0, t_{m_1} = T_1, t_{m_2} = T_2, t_m = T\}.$$

Using the integro-interpolation method for constructing difference schemes of problem (8) - (14), we obtain a system of algebraic equations for the values of the sought functions [6]. For $j = \overline{1, m_1}$ we have

$$a_i U_{i+1} - b_i U_i + c_i U_{i-1} = -d_i, \; i = \overline{1, n-1},$$

(15)

$$\gamma_{11} U_1 - \gamma_{12} U_2 - \gamma_{13} U_0 = \gamma_{14},$$

(16)

$$\gamma_{21} U_n - \gamma_{22} U_{n-1} - \gamma_{23} U_{n-2} = 0;$$

(17)

there

$$a_i = \varphi_i / \hbar_i h_{i+1}, \; c_i = \varphi_i / \hbar_i h_i,$$

$$b_i = a_i + c_i + \frac{1}{2\sqrt{U_i}\tau_j},$$

$$d_i = \overline{U}_i / 2\sqrt{U_i}\tau_j, \;\; U_i = P_i^2, \; \overline{U}_i = \overline{P}_i^2,$$

$$\gamma_{11} = c_1\hbar_1 / h_1h_2, \gamma_{12} = h_1 / \hbar_1h_2, \gamma_{13} = \frac{h_2 + \hbar_1}{h_1h_2} + c_1 - 1,$$

$$\gamma_{14} = (c_1 - 1)P_c^2 + q(\ln(R_k / R_c))^{\alpha} c_1,$$

$$\hbar_1 = h_1 + h_2,$$

$$\gamma_{21} = \frac{h_2 + \hbar_n}{h_n\hbar_n}, \ \gamma_{22} = \frac{\hbar_n}{h_nh_{n-1}}, \ \gamma_{23} = \frac{h_n}{h_{n-1}\hbar_n},$$

$$\hbar_n = h_n + h_{n+1}.$$

The system of nonlinear algebraic equations (15) - (17) is solved by a simple iteration method using the sweep method. The solution to this problem is the initial pressure distribution for the next problem.

To integrate the system of equations (9) - (10) with boundary conditions (11) - (13) at $t > t_m$ we approximate it as follows. Equation (10) and boundary condition (11) are approximated by equations (15) - (17). The coefficients of these equations are determined by the formulas:

$$\begin{cases} a_i = F_{i+0,5}\varphi_i / \hbar_i, c_i = F_{i-0,5}\varphi / \hbar_i h_i, \\ b_i = a_i + c_i + (1 - S_i)\dfrac{1}{2\sqrt{U_i}\tau_k}, \\ d_i = \dfrac{\overline{U_i}}{2\sqrt{U_i}\tau_k} - 2\mu_0 f_{1i}(\dfrac{P_{i+1} - P_{i-1}}{2\hbar_i})^2, \\ F_i = f_{2i} + \mu_0 f_{1i}. \end{cases}$$

(18)

To approximate equation (9), the following schemes can be used:

$$S_i = \overline{S}_{i+1} - \overline{S}_{i-1} / 2 + \mu_0\tau_k\Phi_i,$$

(19)

$$S_i = h_i\overline{S}_i + \left[1 - (h_{i+1} + h_i)\right]S_i + h_{i+1}S_{i-1} + 2\mu_0\Phi_i,$$

(20)

where

$$\Phi_i = \frac{\varphi_i}{\hbar_i}\left[\frac{P_{i+1} - P_i}{h_{i+1}} f_{1i+0,5} - \frac{P_i - P_{i-1}}{\hbar_i} f_{1i-0,5}\right]$$

To solve the system of algebraic equations (15) - (17), (19) (or (20)) for $t_{m_1} < t < t_{m_2}$ и (15) - (17) при $t > t_{m_2}$ various iteration schemes were tested.

As a result of the analysis of numerous computational experiments, it is shown that the proposed algorithm can be used to calculate the saturation and pressure fields in a plane-parallel flow of gas and flushing fluid in the formation. In this case, the above features of the numerical solution and the conditions $\mu_{\Phi}^{i} \to 0$ impose special requirements on the choice of steps for spatial and temporal grids. An unfortunate choice of these parameters leads to an unstable result in time. At the same time, at the first stage, where there is no task of determining the saturation field, the count can be carried out with a large step (up to ten days), and at the second and third stages the initial time step $\tau_0^i (i = 2,3)$ you have to take about hundreds of fractions of a minute Figure 4 shows the pressure distribution curves for the following points in time:

curve 1-at $t = t_{m_1} = 35,78 day$;

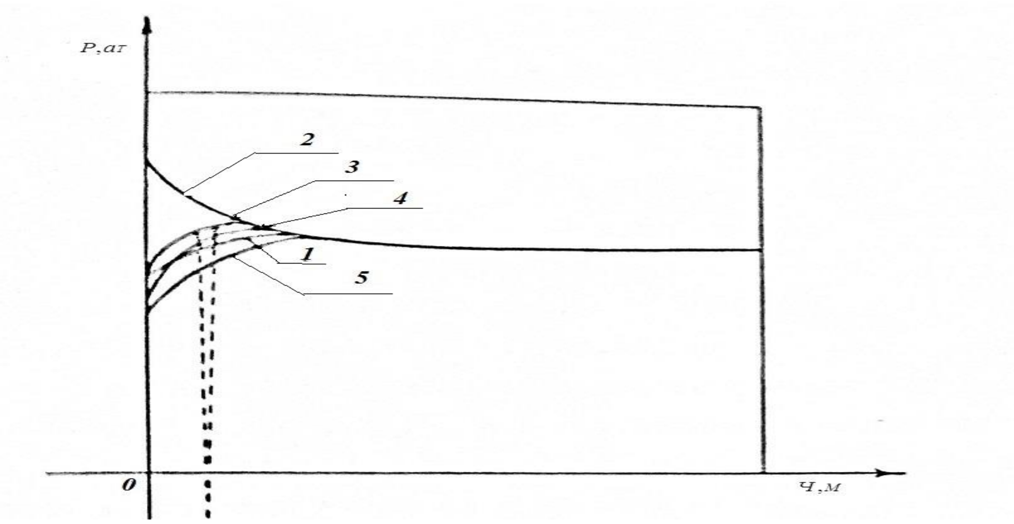
curve 2-at $t = t_{m_2} = 35,78 day + 168 \min$;

curve 3-at $t = t_{m_2} + 0,6 \min$;

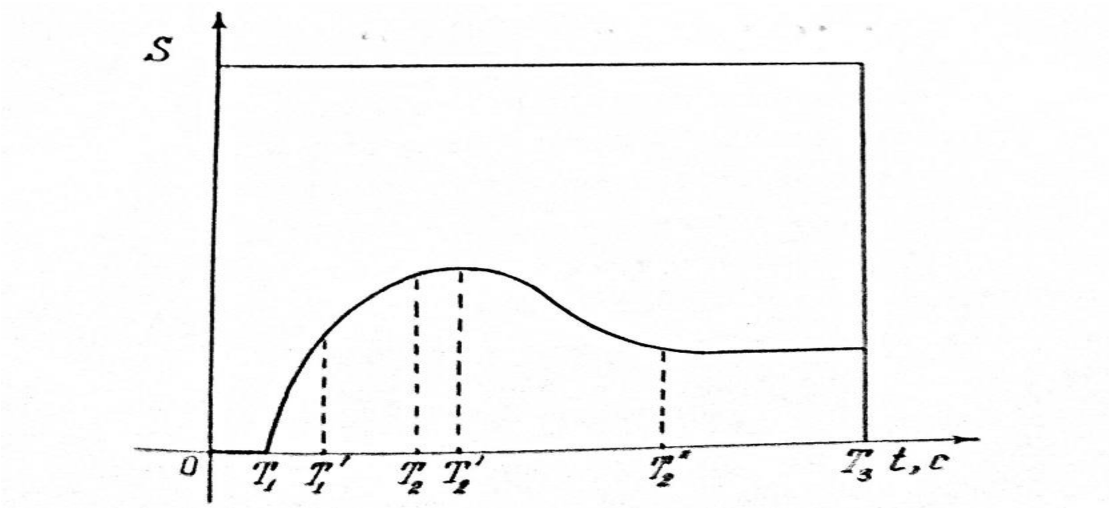
curve 4-at $t = t_{m_2} + 6 \min$;

curve 5-at $t = t_{m_3} = t_{m_2} + 40 day$.

The dashed lines show the calculation results for τ^3=0,5 days and α_3=1,2. In this case, after 5 time layers, the pressure at the point cm is put negative, apparently,

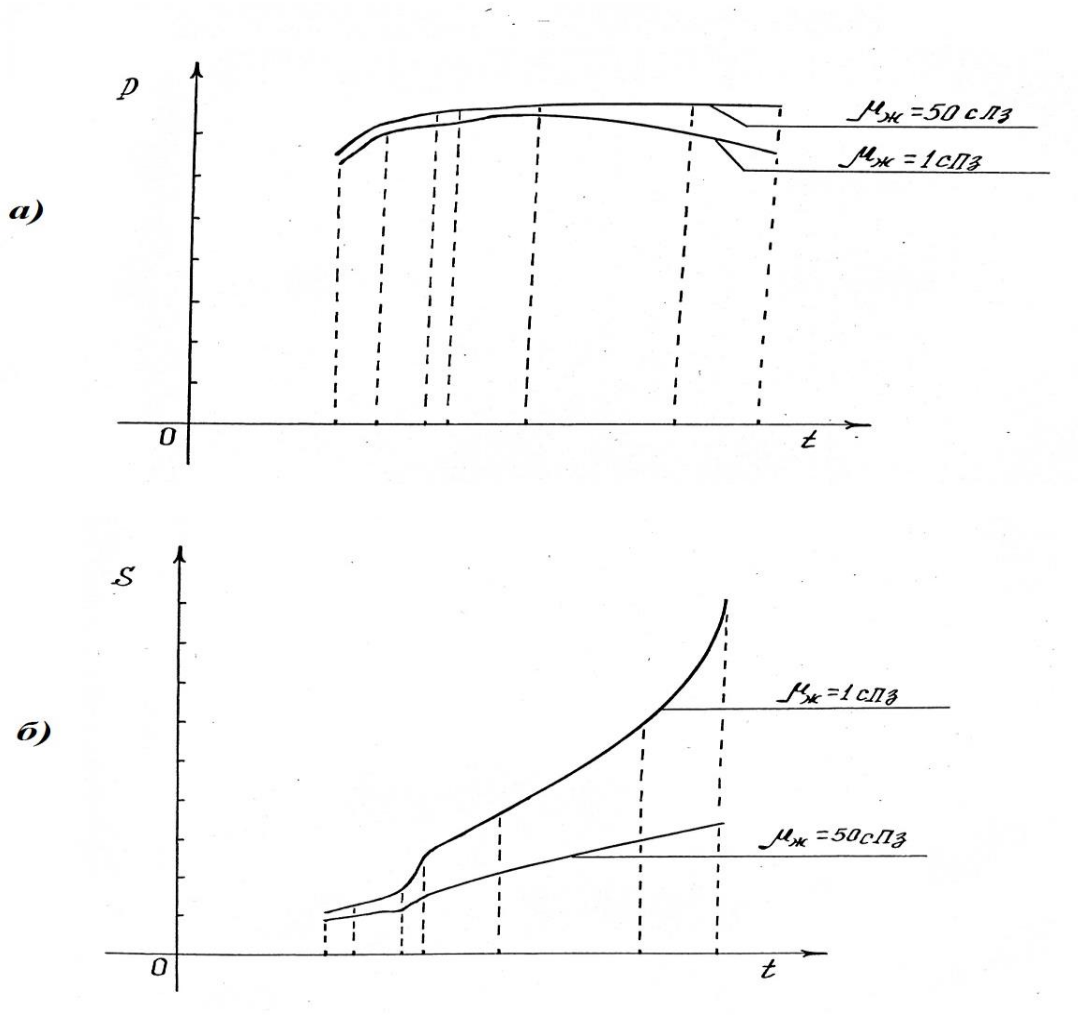


Pic. 3 Pressure distribution graphs along the length of the reservoir at different points in time



Pic.4 Distribution of the saturation of

the flushing fluid over time by increasing the "artificial viscosity". Figure 6 shows a graph of the time variation of the rock saturation with flushing fluid at the point r=50 cm, the beginning of the second stage is highlighted on it t_{m_1}, start of filtration of flushing fluid T_1, time of maximum saturation with flushing liquid T_2 and residual saturation T_3. Increase saturation at a point r=50 after the start of pumping out the drilling fluid is explained by the fact that the gas filtration rate is greater than the fluid filtration rate, therefore it accumulates near the well.



Pic.6 Distribution of pressure (a) and saturation (b) of the drilling fluid in time near the well, where $t = 0{,}0021$, $h_0 = 0{,}001$, $\alpha_2 = 1{,}01$.

The graphs shown in Fig. 7 indicate the dependence of the filtration process on the viscosity of the flushing fluid. With fluid viscosity $\mu_{жс} = 1$ cPz a lower pressure is observed near the well than at $\mu_{жс} = 50$ cPz, at the same time, more fluid with a lower viscosity is observed near the well.

It is seen that scheme (19) gives an overestimated result in comparison with scheme (20). The obtained results of the computational experiment show that using the above algorithms it is possible to calculate the pressure and saturation fields under the conditions under consideration and the scheme (19)

It is seen that scheme (19) gives an overestimated result in comparison with scheme (20). The obtained results of the computational experiment show that using the above algorithms it is possible to calculate the pressure and saturation field under the conditions under consideration, and scheme (19) is stable in time.

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Сувонов Олим Омонович
Заведующей кафедры “Методика точных и естесственных наук” Навоийского регионального центра переподготовки и повышения квалификации кадров народного образования, кандидат технических наук, доцент.
Тел.: +998(99) 731-35-12
Эл. Почта: olimsuvonov54@umail.uz