Acta of Turin Polytechnic University in Tashkent

Volume 9
Issue 2 *The Annual Colloquia, Seminars and Exhibition of the Turin Polytechnic University in Tashkent, 2019*

Article 1

6-1-2019

Boundary Value Problem for Nonhomogeneous Mixed-Type Equation with Two Degenerate Lines

K.S. Fayazov
Turin Polytechnic University, Kichik Halka yuli 17, Tashkent 100095, Uzbekistan., kudratillo52@mail.ru

Y. K. Khudayberganov
National University of Uzbekistan, Universitet Str. 4, Tashkent 100174, Uzbekistan, komilyashin@mail.ru

ACTA TTPU

I. FUNDAMENTAL SCIENCE

BOUNDARY VALUE PROBLEM FOR NONHOMOGENEOUS MIXED-TYPE EQUATION WITH TWO DEGENERATE LINES

Fayazov K. S., Khudayberganov Y. K.

[1]Turin Polytechnic University, Kichik Halka yuli 17, Tashkent 100095, Uzbekistan
[2]National University of Uzbekistan, Universitet Str. 4, Tashkent 100174, Uzbekistan
E-mail:komilyashin@mail.ru, E-mail: kudratillo52@mail.ru

Abstract

In this work, we study boundary value problem for nonhomogeneous mixed-type equation with two degenerate lines. A priori estimate for the solution of the problem is obtained, theorems of uniqueness and conditional stability in the set of correctness are proved. The approximate solution by the regularization method has been constructed.

Keywords: Mixed type equations, ill-posed problem, a priori estimate, theorem of the uniqueness, conditional stability, set of correctness, approximate solution, regularization method.

Introduction

In this paper we consider ill-posed boundary value problem for nonhomogeneous partial differential equation of mixed type with two degenerate lines.

Consider the equation

$$u_t = sign(x)u_{xx} + sign(y)u_{yy} + f(x,y,t) \qquad (1)$$

in the region $\Omega = \{(x,y,t) \,|\, (-1;1)^2 \times (0;T),$ $T < \infty,\ x \neq 0,\ y \neq 0\}$.

Problem 1. Find the solution of the equation (1) on the region Ω that satisfies the initial condition

$$u(x,y,0) = \varphi(x,y), \Pi = \left\{(x,y) \,|\, (-1;1)^2\right\}, \qquad (2)$$

corresponding homogeneous boundary conditions, as well as the conditions for gluing the function itself and the corresponding first derivatives with respect to the variable x and y at $x = 0$ and $y = 0$, where $\varphi(x,y)$, $f(x,y,t)$ - are given sufficiently smooth functions.

The theory of boundary value problems for mixed type differential equations with partial derivatives is one of the most important sections of the modern theory of partial differential equations mathematical physics. This is due both to the direct connections of the mixed type equations with the problems of the theory of singular integral equations, the theory of integral transformations and special functions, and applied problems of transonic gas dynamics, the theory of infinitesimal bending's of surfaces, magnetic hydrodynamics, mathematical biology and other fields. Here should be noted the work of A. S. Chaplygin, N. Y. Zhukovsky, F.I. Frankl, L. Bers. The fundamentals of the theory of boundary value problems for equations of mixed type were laid in the fundamental works of F. Tricomi, S. Gellerstedt, A.V. Bitsadze, K. I. Babenko.

Boundary value problems for classical equations of parabolic type have been the subject of research by many authors, including E. M. Landis, S. P. Shishatsky, and for an equation of elliptic type. M. M. Lavrent'ev,[5] E. M. Landis, F. John, L. Hermander and others. Boundary (ill-posed) problems for operator-differential equations were investigated in the works of S.G. Krein, H. A. Levine and others. In these works, proceeding from the idea of A. N. Tikhonov proved uniqueness theorems and obtained estimates characterizing the conditional stability of these problems. The correct boundary value problems for various non-classical equations were considered in the works of A. V. Bitsadze, S. A. Tersenov, V. N. Vragov, A. M. Nakhushev and other authors. The tasks for such types of equations were the subject of research by N. Kislov, A. I. Kozhanov [4], S. G. Pyatkov [6], K. B. Sobitov, A. A. Gimaltdinova [7] and others.

Ill-posed boundary value problems were investigated by A. L. Bukhgeim, V. Isakov, M. Klibanov, K. S. Fayazov [1] and others. The construction of approximate solutions for non-classical equations is devoted to the work of K. S. Fayazov, K. S. Fayazov and I. O. Khajiev[2], K. S. Fayazov and Y. K. Khudayberganov [3]. Inverse problems for this type of equations were considered in the works of M. Klibanov, V. Isakov, A. L. Bukhgeim, I.A. Kaliyev, M. F. Mugafarov, O. V. Fattakhov and others.

The problem 1 under study is related to the class of ill-posed problems of mathematical physics. In the above problem there is no continuous dependence of the solution from the data. We investigate conditional correctness of the initial problem and construct an approximate solution by the regularization method of A. N. Tikhonov.

We need in the results of the following spectral problem:

Find the values of λ for which the next problem has a non-trivial solutions

$$sign(x)\vartheta_{xx}(x,y)+sign(y)\vartheta_{yy}(x,y)=-\lambda\vartheta(x,y),$$

$$(x,y)\in\left((-1;1)^2, x\neq 0, y\neq 0\right), \tag{3}$$

$$\vartheta(x,y)\Big|_{\substack{x=-1\\x=+1}}=0, \vartheta(x,y)\Big|_{\substack{y=-1\\y=+1}}=0, (x,y)\in[-1;1]^2,$$

$$\left.\frac{\partial^i\vartheta(x,y)}{\partial x^i}\right|_{x=-0}=\left.\frac{\partial^i\vartheta(x,y)}{\partial x^i}\right|_{x=+0}, y\in[-1;1], \tag{4}$$

$$\left.\frac{\partial^i\vartheta(x,y)}{\partial y^i}\right|_{y=-0}=\left.\frac{\partial^i\vartheta(x,y)}{\partial y^i}\right|_{y=+0}, x\in[-1;1], i=0,1.$$

On the base of the results [6], it can be proved that problem (3) - (4) has a non-decreasing sequence of eigenvalues $\left\{\lambda_{k,l}^{\pm\pm}\right\}_{k,l=1}^{\infty},\left\{\lambda_{k,l}^{\pm\mp}\right\}_{k,l=1}^{\infty}$ and the corresponding eigenfunctions of $\left\{\vartheta_{k,l}^{\pm\pm}\right\}_{k,l=1}^{\infty},\left\{\vartheta_{k,l}^{\pm\mp}\right\}_{k,l=1}^{\infty}$.

One can present Eigen functions in the form:

$$\vartheta_{k,l}^{\pm\pm}(x,y,\lambda_{k,l}^{\pm\pm})=X_k^{\pm}(x)\times Y_l^{\pm}(y), \quad k,l=1,2,...,$$

$$\vartheta_{k,l}^{\pm\mp}(x,y,\lambda_{k,l}^{\pm\mp})=X_k^{\pm}(x)\times Y_l^{\mp}(y), \quad k,l=1,2,...,$$

where $\mu_k^2+\sigma_l^2=\lambda_{k,l}^{++}$, $\mu_k^2-\sigma_l^2=\lambda_{k,l}^{+-}$,

$-\mu_k^2+\sigma_l^2=\lambda_{k,l}^{-+}$, $\mu_k^2+\sigma_l^2=-\lambda_{k,l}^{--}$,

if $\lambda_{k,l}>0$

$$X_k^+(x)=\begin{cases}\dfrac{\sin\mu_k(x-1)}{\cos\mu_k}, 0\le x\le 1,\\ \dfrac{sh\mu_k(x+1)}{ch\mu_k}, -1\le x\le 0.\end{cases} \quad k=1,2,...,$$

$$Y_l^+(y)=\begin{cases}\dfrac{\sin\sigma_l(y-1)}{\cos\sigma_l}, 0\le y\le 1,\\ \dfrac{sh\sigma_l(y+1)}{ch\sigma_l}, -1\le y\le 0.\end{cases} \quad l=1,2,...,$$

and if $\lambda_{k,l}<0$

$$X_k^-(x)=\begin{cases}\dfrac{sh\mu_k(x-1)}{ch\mu_k}, 0\le x\le 1,\\ \dfrac{\sin\mu_k(x+1)}{\cos\mu_k}, -1\le x\le 0.\end{cases} \quad k=1,2,...,$$

$$Y_l^-(y)=\begin{cases}\dfrac{sh\sigma_l(y-1)}{ch\sigma_l}, 0\le y\le 1,\\ \dfrac{\sin\sigma_l(y+1)}{\cos\sigma_l}, -1\le y\le 0.\end{cases} \quad l=1,2,... .$$

In both cases, $\mu_k,\sigma_l,k,l=1,2,...$ are the positive roots of the transcendental equation $tg\alpha=-th\alpha.$ Let $\|u\|^2=(u,u)$ be the scalar product of

$$(u,\upsilon)=\int_{-1}^{1}\int_{-1}^{1}u\upsilon dxdy$$

Besides

$$(sign(x)sign(y)\vartheta_{k,l}^{\pm\mp},\vartheta_{i,j}^{\mp\pm})=0, \ \forall k,l,i,j;$$

$$(sign(x)sign(y)\vartheta_{k,l}^{\pm\pm},\vartheta_{i,j}^{\pm\pm})=\delta_{k,i}\cdot\delta_{l,j},$$

$$(sign(x)sign(y)\vartheta_{k,l}^{\pm\mp},\vartheta_{i,j}^{\pm\mp})=-\delta_{k,i}\cdot\delta_{l,j},$$

$$\delta_{k,i}=\begin{cases}1, k=i,\\0, k\neq i,\end{cases}$$

where $k,l,i,j=1,2,3,...$.

We represent the spectral projections in the following form

$$P^{+}\phi=\sum_{k,l=1}^{\infty}\Big((sign(x)sign(y)\phi,\vartheta_{k,l}^{++})\vartheta_{k,l}^{++}+$$

$$+(sign(x)sign(y)\phi,\vartheta_{k,l}^{--})\vartheta_{k,l}^{--}\Big),$$

$$P^{-}\phi=-\sum_{k,l=1}^{\infty}\Big((sign(x)sign(y)\phi,\vartheta_{k,l}^{+-})\vartheta_{k,l}^{+-}+$$

$$+(sign(x)sign(y)\phi,\vartheta_{k,l}^{-+})\vartheta_{k,l}^{-+}\Big).$$

Then according to [6]

$$(P^{+}-P^{-})\phi=\phi,\ (sign(x)sign(y)(P^{+}-P^{-})\phi,\phi)=\|\phi\|^{2},$$

$$(sign(x)sign(y)(P^{+},P^{-})\phi,\psi)=$$

$$=(sign(x)sign(y)\phi,(P^{+},P^{-})\psi),\phi,\psi\in\mathrm{H}_0=L_2(-1,1)^2,$$

$$\|\phi\|_0^2=\sum_{k,l=1}^{\infty}\Big\{\left|(sign(x)sign(y)\phi,\vartheta_{k,l}^{++})\right|^2+$$

$$+\left|(sign(x)sign(y)\phi,\vartheta_{k,l}^{+-})\right|^2+\qquad(5)$$

$$+\left|(sign(x)sign(y)\phi,\vartheta_{k,l}^{-+})\right|^2+$$

$$+\left|(sign(x)sign(y)\phi,\vartheta_{k,l}^{--})\right|^2\Big\}.$$

According to the results of [6], the eigenfunctions of the problem (3) - (4) form the Riesz basis in H_0 and the norm in space $L_2(-1,1)^2$, defined by equality (5), is equivalent to the original one.

A priori estimate

Definition1. By the solution of the problem 1, we understand continues in $\overline{\Omega}$ function, that has continuous partial derivatives participating in the equation (1) and satisfies equation in the region Ω and given initial, boundary and gluing conditions.

Theorem1. For any solution of the problem 1 $u(x,y,t)$ by $t\in(0,T)$, the following inequality

$$\|u(x,y,t)\|_0\le 2\Big(\|u(x,y,0)\|_0+\alpha\Big)^{\frac{T-t}{T}}\times$$
$$\times\Big(\|u(x,y,T)\|_0+\alpha\Big)^{\frac{t}{T}}+\alpha,\ t\in(0;T)\qquad(6)$$

is true,

where $\alpha=\left(\int_0^T\|f(x,y,t)\|_0^2dt\right)^{\frac{1}{2}}$.

Proof. One can present the solution of the problem 1 in form

$$u(x,y,t)=\omega(x,y,t)+\nu(x,y,t),$$

where $\omega(x,y,t)$ is the solution of the equation

$$\omega_t=sign(x)\omega_{xx}+sign(y)\omega_{yy}\,,\qquad(7)$$

that satisfies condition (2), boundary and gluing conditions too;

Function $\nu(x,y,t)$ is solution of the equation

$$\nu_t=sign(x)\nu_{xx}+sign(y)\nu_{yy}+f(x,y,t).\quad(8)$$

One can present $\omega(x,y,t)$ and $\nu(x,y,t)$ in the form

$$\omega(x,y,t)=\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\Big(\omega_{k,l}^{++}(t)\vartheta_{k,l}^{++}(x,y)+$$
$$+\omega_{k,l}^{--}(t)\vartheta_{k,l}^{--}(x,y)+\omega_{k,l}^{+-}(t)\vartheta_{k,l}^{+-}(x,y)+$$
$$+\omega_{k,l}^{-+}(t)\vartheta_{k,l}^{-+}(x,y)\Big),$$

$$\nu(x,y,t)=\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\Big(\nu_{k,l}^{++}(t)\vartheta_{k,l}^{++}(x,y)+$$
$$+\nu_{k,l}^{--}(t)\vartheta_{k,l}^{--}(x,y)+\nu_{k,l}^{+-}(t)\vartheta_{k,l}^{+-}(x,y)+$$
$$+\nu_{k,l}^{-+}(t)\vartheta_{k,l}^{-+}(x,y)\Big)$$

where

$$\nu_{k,l}^{++}(t)=\int_0^t e^{\lambda_{k,l}^{++}(\tau-t)}f_{k,l}^{++}(\tau)d\tau,\lambda_{k,l}>0,$$

$$\nu_{k,l}^{+-}(t)=\begin{cases}\int_0^t e^{\lambda_{k,l}^{+-}(\tau-t)}f_{k,l}^{+-}(\tau)d\tau,\lambda_{k,l}>0,\\ -\int_t^T e^{\lambda_{k,l}^{+-}(\tau-t)}f_{k,l}^{+-}(\tau)d\tau,\ \lambda_{k,l}<0,\end{cases}$$

$$\nu_{k,l}^{-+}(t)=\begin{cases}-\int_t^T e^{\lambda_{k,l}^{-+}(\tau-t)}f_{k,l}^{-+}(\tau)d\tau,\lambda_{k,l}<0,\\ \int_0^t e^{\lambda_{k,l}^{-+}(\tau-t)}f_{k,l}^{-+}(\tau)d\tau,\lambda_{k,l}>0,\end{cases}$$

$$\nu_{k,l}^{--}(t)=-\int_t^T e^{\lambda_{k,l}^{--}(\tau-t)}f_{k,l}^{--}(\tau)d\tau,\lambda_{k,l}<0,\ k,l=1,2,\ldots.$$

and

$$\omega_{k,l}^{++}(t) = \varphi_{k,l}^{++} e^{-\lambda_{k,l}^{++} t},\ \lambda_{k,l} > 0,$$

$$\omega_{k,l}^{+-}(t) = \begin{cases} \varphi_{k,l}^{+-} e^{-\lambda_{k,l}^{+-} t}, \lambda_{k,l} > 0, \\ \varphi_{k,l}^{+-} e^{-\lambda_{k,l}^{+-} t} + \int\limits_0^T e^{\lambda_{k,l}^{+-}(\tau - t)} f_{k,l}^{+-}(\tau) d\tau,\ \lambda_{k,l} < 0, \end{cases}$$

$$\omega_{k,l}^{-+}(t) = \begin{cases} \varphi_{k,l}^{-+} e^{-\lambda_{k,l}^{-+} t} + \int\limits_0^T e^{\lambda_{k,l}^{-+}(\tau - t)} f_{k,l}^{-+}(\tau) d\tau,\ \lambda_{k,l} < 0, \\ \varphi_{k,l}^{-+} e^{-\lambda_{k,l}^{-+} t}, \lambda_{k,l} > 0, \end{cases}$$

$$\omega_{k,l}^{--}(t) = \varphi_{k,l}^{--} e^{-\lambda_{k,l}^{--} t} + \int\limits_0^T e^{\lambda_{k,l}^{--}(\tau - t)} f_{k,l}^{--}(\tau) d\tau,\ \lambda_{k,l} < 0.$$

where

$$\varphi_{k,l}^{\pm\pm} = (sign(x)sign(y)\varphi, \vartheta_{k,l}^{\pm\pm}),$$

$$\varphi_{k,l}^{\pm\mp} = -(sign(x)sign(y)\varphi, \vartheta_{k,l}^{\pm\mp}),\ k,l = 0,1,2,\dots\ .$$

$$f_{k,l}^{\pm\pm}(t) = (sign(x)sign(y) f, \vartheta_{k,l}^{\pm\pm}),$$

$$f_{k,l}^{\pm\mp}(t) = -(sign(x)sign(y) f, \vartheta_{k,l}^{\pm\mp}),\ k,l = 0,1,2,\dots\ .$$

According to the definition of the norm for function $v(x,y,t)$, we have

$$\|v(x,y,t)\|_0^2 = \sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\Big(\left|v_{k,l}^{++}(t)\right|^2 + \left|v_{k,l}^{--}(t)\right|^2 + \\ + \left|v_{k,l}^{+-}(t)\right|^2 + \left|v_{k,l}^{-+}(t)\right|^2\Big) \le \int\limits_0^T \|f(x,y,t)\|_0^2\, dt. \qquad (9)$$

It is not difficult to see [3] for $\omega(x,y,t)$

$$\|\omega(x,y,t)\|_0 \le 2\left(\|\omega(x,y,0)\|_0\right)^{\frac{T-t}{T}} \times \\ \times\left(\|\omega(x,y,T)\|_0\right)^{\frac{t}{T}},\ 0 < t < T \qquad (10)$$

is true.

Tking into account equality $\omega(x,y,t) = u(x,y,t) - v(x,y,t)$ and using inequalities (9), (10) we get

$$\|u(x,y,t)\|_0 \le 2\left(\|u(x,y,0)\|_0 + \alpha\right)^{\frac{T-t}{T}} \times \\ \times\left(\|u(x,y,T)\|_0 + \alpha\right)^{\frac{t}{T}} + \alpha,\ 0 < t < T.$$

The theorem is proved.

Conditionally correctness of the problems 1

Theorem 2. The solution of the problem 1 is unique.

Proof. Let $u_1(x,y,t)$ and $u_2(x,y,t)$ be solutions of the problem 1. Then $u(x,y,t) = u_1(x,y,t) - u_2(x,y,t)$ will be a solution of the problem 1 with homogeneous equation and data. For solution of this homogeneous problem we us the results of the Theorem 1. From the inequality (6) followed $\|u(x,y,t)\|_0 = 0$. Hence, for arbitrary $(x,y,t) \in \Omega$, $u(x,y,t) \equiv 0$ and $u_1(x,y,t) = u_2(x,y,t)$. The theorem is proved.

Let M is the set of correctness defined by

$$M = \{u : \|u(x,y,T)\| \le m, m < \infty\}.$$

Theorem3. Let $(u(x,y,t), u_\varepsilon(x,y,t))$ are exact solutions of the problem 1 with exact and approximate data, respectively, and

$$\|\varphi - \varphi_\varepsilon\|_0 \le \varepsilon,\ \|f - f_\varepsilon\|_0 \le \varepsilon. \qquad (11)$$

Then for the solution $U(x,y,t) = u(x,y,t) - u_\varepsilon(x,y,t)$ the following inequality

$$\|U(x,y,t)\|_0 \le 2\left(\varepsilon + \varepsilon\sqrt{T}\right)^{1-\frac{t}{T}} \cdot \left(2m + \sqrt{T}\varepsilon\right)^{\frac{t}{T}} + \varepsilon\sqrt{T}$$

for all $t \in (0;T)$ is valid.

Proof. Function $U(x,y,t) = u(x,y,t) - u_\varepsilon(x,y,t)$ satisfies the equation

$$U_t = sign(x)U_{xx} + sign(y)U_{yy} + (f(x,y,t) - f_\varepsilon(x,y,t))$$

on the Ω,

initial condition

$$U\big|_{t=0} = \varphi(x,y) - \varphi_\varepsilon(x,y), (x,y) \in [-1;0) \cup (0;1],$$

boundary conditions

$$U(x,y,t)\Big|_{\substack{x=-1\\x=+1}}=0,\ (y,t)\in[-1;1]\times[0;T],$$

$$U(x,y,t)\Big|_{\substack{y=-1\\y=+1}}=0,\quad (x,t)\in[-1;1]\times[0;T],$$

and gluing conditions

$$\left.\frac{\partial^i U}{\partial x^i}\right|_{x=-0}=\left.\frac{\partial^i U}{\partial x^i}\right|_{x=+0},(y,t)\in[-1;1]\times[0;T],$$

$$\left.\frac{\partial^i U}{\partial y^i}\right|_{y=-0}=\left.\frac{\partial^i U}{\partial y^i}\right|_{y=+0},i=0,1,(x,t)\in[-1;1]\times[0;T].$$

According to the conditions of Theorem 3

$$\|U(x,y,0)\|_0=\|\varphi-\varphi_\varepsilon\|\le\varepsilon,\ \|U(x,y,T)\|_0\le 2m,$$

we have

$$\|U(x,y,t)\|_0\le 2\left(\varepsilon+\varepsilon\sqrt{T}\right)^{1-\frac{t}{T}}\times$$
$$\times\left(2m+\sqrt{T}\varepsilon\right)^{\frac{t}{T}}+\varepsilon\sqrt{T}.$$

The theorem is proved.

Approximate solution

Let $\varphi(x,y)=0$. Then the solution of the problem1 $u(x,y,t)$ can be represented in form

$$u(x,y,t)=\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\left(\left(\int_0^t e^{\lambda_{k,l}^{++}(\tau-t)}f_{k,l}^{++}(\tau)d\tau\right)\vartheta_{k,l}^{++}(x,y)+\right.$$
$$+\left(\int_0^t e^{\lambda_{k,l}^{--}(\tau-t)}f_{k,l}^{--}(\tau)d\tau\right)\vartheta_{k,l}^{--}(x,y)+$$
$$+\left(\int_0^t e^{\lambda_{k,l}^{+-}(\tau-t)}f_{k,l}^{+-}(\tau)d\tau\right)\vartheta_{k,l}^{+-}(x,y)+$$
$$\left.+\left(\int_0^t e^{\lambda_{k,l}^{-+}(\tau-t)}f_{k,l}^{-+}(\tau)d\tau\right)\vartheta_{k,l}^{-+}(x,y)\right).$$

By $u^N(x,y,t)$ we denote sequence of approximate solutions

$$u^N(x,y,t)=\sum_{k=1}^{N}\sum_{l=1}^{N}\left(\left(\int_0^t e^{\lambda_{k,l}^{++}(\tau-t)}f_{k,l}^{++}(\tau)d\tau\right)\vartheta_{k,l}^{++}(x,y)+\right.$$
$$+\left(\int_0^t e^{\lambda_{k,l}^{--}(\tau-t)}f_{k,l}^{--}(\tau)d\tau\right)\vartheta_{k,l}^{--}(x,y)+$$
$$+\left(\int_0^t e^{\lambda_{k,l}^{+-}(\tau-t)}f_{k,l}^{+-}(\tau)d\tau\right)\vartheta_{k,l}^{+-}(x,y)+$$
$$\left.+\left(\int_0^t e^{\lambda_{k,l}^{-+}(\tau-t)}f_{k,l}^{-+}(\tau)d\tau\right)\vartheta_{k,l}^{-+}(x,y)\right),$$

where $N-$ is regularization parameter.

By $u_\varepsilon^N(x,y,t)$ we denote sequence approximate solutions with approximate data

$$u_\varepsilon^N(x,y,t)=\sum_{k=1}^{N}\sum_{l=1}^{N}\left(\left(\int_0^t e^{\lambda_{k,l}^{++}(\tau-t)}f_{\varepsilon k,l}^{++}(\tau)d\tau\right)\vartheta_{k,l}^{++}(x,y)+\right.$$
$$+\left(\int_0^t e^{\lambda_{k,l}^{--}(\tau-t)}f_{\varepsilon k,l}^{--}(\tau)d\tau\right)\vartheta_{k,l}^{--}(x,y)+$$
$$+\left(\int_0^t e^{\lambda_{k,l}^{+-}(\tau-t)}f_{\varepsilon k,l}^{+-}(\tau)d\tau\right)\vartheta_{k,l}^{+-}(x,y)+$$
$$\left.+\left(\int_0^t e^{\lambda_{k,l}^{-+}(\tau-t)}f_{\varepsilon k,l}^{-+}(\tau)d\tau\right)\vartheta_{k,l}^{-+}(x,y)\right),$$

where

$$f_{k,l\varepsilon}^{\pm\pm}(t)=(sign(x)sign(y)f_\varepsilon,\vartheta_{k,l}^{\pm\pm}),$$
$$f_{k,l\varepsilon}^{\pm\mp}(t)=-(sign(x)sign(y)f_\varepsilon,\vartheta_{k,l}^{\pm\mp}).$$

Let $\|f-f_\varepsilon\|_0\le\varepsilon$ and $u(x,y,t)\in M$. Let us estimate the norm of difference between the exact and approximate solutions for fixed N

$$\|u(x,y,t)-u_\varepsilon^N(x,y,t)\|_0\le$$
$$\le\|u(x,y,t)-u^N(x,y,t)\|_0+$$
$$+\|u^N(x,y,t)-u_\varepsilon^N(x,y,t)\|_0. \tag{12}$$

We estimate second term in the right side of the last inequality using Cauchy-Bunyakovsky, some elementary transformation and inequalities

$$\|u^N(x,y,t)-u_\varepsilon^N(x,y,t)\|_0^2=$$
$$\sum_{k=1}^{N}\sum_{l=1}^{N}\left(\left(\int_0^t e^{\lambda_{k,l}^{++}(\tau-t)}\left(f_{k,l}^{++}-f_{\varepsilon k,l}^{++}\right)d\tau\right)^2+\right.$$

$$+\left(\int_0^t e^{\lambda_{k,l}^{--}(\tau-t)}\left(f_{k,l}^{--}-f_{\varepsilon k,l}^{--}\right)d\tau\right)^2+$$
$$+\left(\int_0^t e^{\lambda_{k,l}^{+-}(\tau-t)}\left(f_{k,l}^{+-}-f_{\varepsilon k,l}^{+-}\right)d\tau\right)^2+ \qquad (13)$$
$$\left.+\left(\int_0^t e^{\lambda_{k,l}^{-+}(\tau-t)}\left(f_{k,l}^{-+}-f_{\varepsilon k,l}^{-+}\right)d\tau\right)^2\right)\le$$
$$\le\left(t\varepsilon e^{-\lambda_{N,N}^{--}t}\right)^2$$

We estimate the first term in the right-hand side of the inequality (12) under the condition $u(x,y,t)\in M$

$$\left\|u(x,y,t)-u^N(x,y,t)\right\|_0^2\le$$
$$\le m^2\left(e^{2\lambda_{N+1,N+1}^{+-}(T-t)}+e^{2\lambda_{N+1,N+1}^{-+}(T-t)}+\right.$$
$$\left.e^{2\lambda_{N+1,N+1}^{--}(T-t)}+\gamma(N)\right),$$

where

$$\gamma(N)=\frac{2T}{N^\alpha}\int_0^T\left\|f(x,y,t)\right\|_0^2dt,\ \alpha>1.$$

As final result we have

$$0.5\left\|u(x,y,t)-u_\varepsilon^N(x,y,t)\right\|_0^2\le\left(t\varepsilon e^{-\lambda_{N,N}^{--}t}\right)^2+$$
$$+m^2\left(e^{2\lambda_{N+1,N+1}^{+-}(T-t)}+e^{2\lambda_{N+1,N+1}^{-+}(T-t)}+\right.$$
$$\left.+e^{2\lambda_{N+1,N+1}^{--}(T-t)}+\gamma(N)\right).$$

Minimizing the right-hand side of the last inequality in N with $\varepsilon>0$, we determine the regularization parameter N. m is selected depending from the application problem.

Results of numerical calculations.

For the numerical solution of problem 1, we select data as follows

$$\varphi(x,y)=0,\ f(x,y,t)=(1-x^2)(1-y^2)e^{-t},$$

and approximate data in the form

$$\varphi(x,y)=0, f_\varepsilon(x,y)=(1-x^2)(1-y^2)e^{-t}(1+\varepsilon).$$

Conditions for N are selected from

$$\inf_{N>0}\left(\left(t\varepsilon e^{-\lambda_{N,N}^{--}t}\right)^2+m^2\left(e^{2\lambda_{N+1,N+1}^{+-}(T-t)}+e^{2\lambda_{N+1,N+1}^{-+}(T-t)}+\right.\right.$$
$$\left.\left.+e^{2\lambda_{N+1,N+1}^{--}(T-t)}+\gamma(N)\right)\right).$$

As $\gamma(N)$ one can choose in the form of $\frac{1}{N^2}$.

As an example, consider

$m=5$, $T=1$, $\varepsilon=10^{-8}$, $t=0.05$, $N=5$,
$m=200$, $T=1$, $\varepsilon=10^{-7}$, $t=0.09$, $N=3$,
$m=6000$, $T=1$, $\varepsilon=10^{-8}$, $t=0.3$, $N=2$.

Here m is chosen arbitrarily, and usually it is determined depending on the specific model.

When $m=60$, $T=1$, $\varepsilon=10^{-3}$, $t=0.3$, $N=1$ the values of the solution of the problem are given in Tables 1 and 2, as well as are reflected in Figs. 1.and Fig.2. From the tables and graphs below, it can be seen that the numerical values of the approximate solution and the approximate solution are fairly close to each other according to the approximate data. Calculations performed for the remaining values of the solutions for other values of the parameters, generally speaking, remain within similar limits of accuracy.

Table 1: The Graph of approximate solution $u^N(x,y,t)$ for accurate data

	y= -0.4	y= -0.2	y= 0.2	y= 0.4
x= -0.4	2212,472	2123,096	960,840	575,337
x= -0.2	2123,096	2037,330	922,025	552,096
x= 0.2	960,840	922,025	417,277	249,859
x= 0.4	575,337	552,096	249,859	149,612

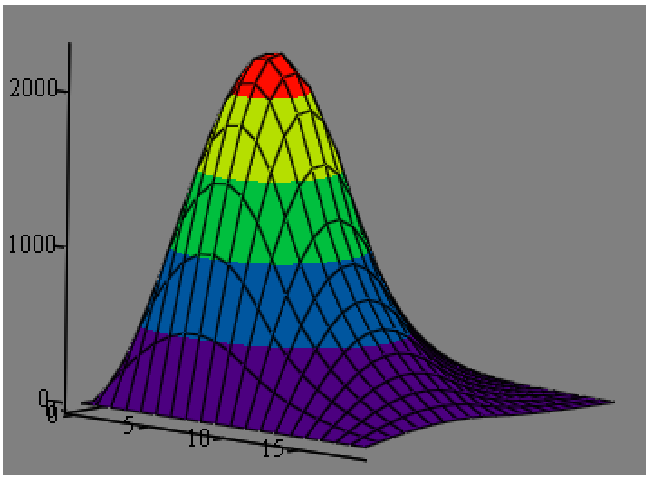

Fig. 1. The graph approximate solution for accurate data.

Table 2: The Graph of approximate solution $u_\varepsilon^N(x,y,t)$ for approximate data

	y= -0.4	y= -0.2	y= 0.2	y= 0.4
x= -0.4	2216,899	2127,344	962,762	576,489
x= -0.2	2127,344	2041,407	923,870	553,201
x= 0.2	962,762	923,870	418,112	250,359
x= 0.4	576,489	553,201	250,359	149,911

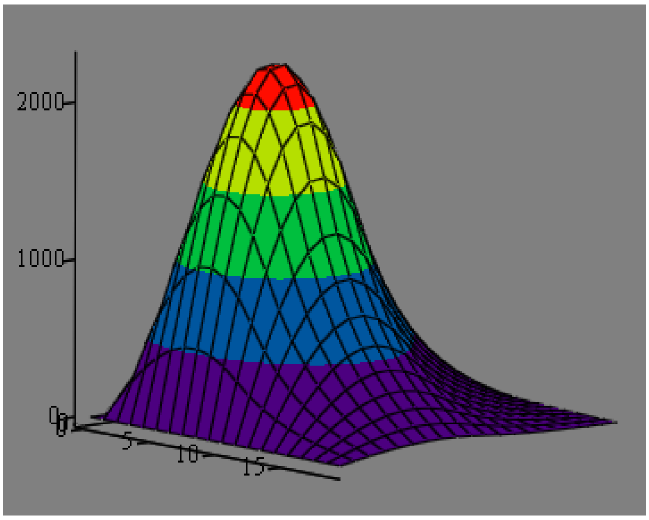

Fig. 2. The graph approximate solution for approximate data.

References

1. Fayazov K. S., An ill-posed boundary-value problem for a second order mixed type equation, UzMat Journal. 1995. №2. P. 89-93.

2. Fayazov K.S., Khazhiev I.O.,Conditional stability of a boundary value problem for a system abstract differential equations of the second order with operator coefficients,Uzbek Mathematical Journal, (2017),№2. P. 145-155.

3. Fayazov K.S., Khudayberganov Y.K., Ill-posed boundary value problem for a mixed type equation with two degenerate lines, Uzbek Mathematical Journal,(2018),№2.P 32-42.

4. Kozhanov A.I., Composite Type Equations and Inverse Problems,Utrecht:VSP 1999

5. Lavrent'ev M. M. and Savelyev L. Ya., Theory of operators and ill-posed problems, -2 th ed., Pererab. and add. Novosibirsk:Published by Institute of Mathematics, 2010. P.941.

6. Pyatkov S. G., On the solvability of a boundary value problem for a parabolic equation with a changing time direction, Dokl. Acad. Sciences of the USSR.1985. T. 285, №6.P.1322-1327.

7. Gimaltdinova A. A., The Dirichlet problem for a mixed-type equation with two perpendicular lines of transition in a rectangular region, 2014. 19 (190). Edition. 36.