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Kompakt operatorlar algebrasidagi cheksiz Pirs yoyilmasi va uning lokal avtomorfizmlari tavsifi

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Annatotsiya. Ushbu maqolada cheksiz o'lchamli separabel Gilbert fazosidagi kompakt operatorlar algebrasidaning norma bo'yicha cheksiz Pirs yoyilmasi fundamental ketma-ketlik bo'lishlik shartiga o'shash shart asosida qurilgan va uning C^*-algebra bo'lishligi hamda bir qator xossalari isbotlangan. Bundan tashqari, ushbu maqola kompakt operatorlar algebrasidaning norma bo'yicha cheksiz Pirs yoyilmasidagi har bir uzluksiz biyektiv lokal avtomorfizm, yoki avtomorfizm, yoki antiavtomorfizm bo'lishi isbotlangan.

Kalit so'zlar: Kompakt operatorlar algebrasi, C^*-algebra, cheksiz Pirs yoyilmasi, avtomorfizm, lokal avtomorfizm.

Аннотация. В данной статье по норме и по аналогу условия фундаментальности последовательности построено бесконечное пирсовское разложение алгебры компактных операторов в бесконечномерном сепарабельном гильбертовом пространстве. А также, доказано, что каждый локальный автоморфизм бесконечного пирсовского разложения алгебры компактных операторов в бесконечномерном сепарабельном гильбертовом пространстве является, или автоморфизмом, или анти автоморфизмом.

Ключевые слова: алгебра компактных операторов, C^*-алгебра, бесконечное пирсовское разложение, автоморфизм, локальный автоморфизм.

Kirish

Ushbu maqolada H cheksiz o'lchamli separabel Gilbert fazosidagi $K(H)$ – kompakt operatorlar algebrasidagi o'zaro ortogonal minimal proyektorlarning, ya'ni o'z-o'ziga qo'shma, idempotent elementlarning, maksimal to'plami bo'yicha $K(H)$ algebraning norma bo'yicha cheksiz Pirs yoyilmasi qurilgan. C^*-algebraning norma bo'yicha cheksiz Pirs yoyilmasi birinchi avtorning [2] maqolasida ham qurilgan. Ammo, norma bo'yicha cheksiz Pirs yoyilmasi qurish uchun [2] maqolada olingan shart ushbu yoyilmaning algebra bo'lishi uchun yetarli bo'lmay qolgan. Shuning uchun biz berilgan maqolada norma bo'yicha cheksiz Pirs yoyilmasi qurish uchun fundamental ketma-ketlik bo'lishlik shartiga o'shash shart kiritdik va $K(H)$ algebraning norma bo'yicha cheksiz Pirs yoyilmasi C^*-algebra bo'lishligini hamda uning bir qator xossalarini isbotladik. Operator algebralarida cheksiz Pirs yoyilmasi qurish g'oyasi birinchi bor birinchi avtorning [1] maqolasida qo'llanilgan.

Bundan tashqari, ushbu maqola kompakt operator algebralarida lokal avtomorfizmlarni tavsiflashga bag'ishlangan. Lokal differensiallashlar va lokal avtomorfizmlar tarixi Gleason-Kahane-Żelazko teoremasidan [3] va [6] boshlangan. Ushbu teorema Banax algebralari nazariyasida fundamental natija hisoblanadi. Ushbu teoremada takidlanadiki, har bir A kompleks birlik Banach algebrasida aniqlangan va, har bir $a \in A$ uchun, $F(a)$ qiymat $\sigma(a)$ spektrga tegishli bo'lgan F birlik chiziqli funksional multiplikativdir. Zamonaviy terminologiyada bu quyidagi shartga ekvivalent: har bir A kompleks birlik Banach algebrasini **C** kompleks sonlar maydoniga akslantiruvchi lokal gomomorfizm multiplikativdir. Eslatib o'tamiz, agar A Banach algebrasini B Banach algebrasiga akslantiruvchi T chiziqli akslantirish uchun, har bir $a \in A$ element uchun $\Phi_a A \to B$ – a elementga bog'liq bo'lgan gomomorfizm mavjud bo'lib, $T(a) = \Phi_a(a)$ shart bajarilsa, u holda T lokal gomomorfizm deb ataladi.

Keyinchalik [5] maqolada, R.Kadison lokal differensiallash konsepsiyasini kiritadi va har bir fon Neyman algebrasini uning dual Banax bimoduliga akslantiruvchi uzluksiz lokal differensiallash global differensiallash bo'lishini isbotlaydi. B.Jonson o'zining [4] maqolasida yuqoridagi natijani kengaytiradi va har bir C^*-algebrani uning dual Banax bimoduliga akslantiruvchi lokal differensiallash global differensiallash bo'lishini isbotlaydi. Xususan, Johnson A C^*-algebrani X Banax A-bimodulga akslantiruvchi lokal differensiallashlar uzluksiz bo'lishini isbotlab, avtomatik uzluksizlik haqidagi natijani oladi ([Teorema 7.5] [4]). Ushbu natijalarga tayangan holda ko'pchilik mualliflar operator algebralaridagi lokal differensiallashlarni o'rganishgan.

Berilgan maqolada kompakt operatorlar algebralaridagi avtomorfizmlar va lokal avtomorfizmlar o'rganilgan.

Ma'lumki, agar A algebra ustida aniqlangan Φ biyektiv chiziqli akslantirish uchun, A algebraning har bir x, y elementlari juftligida $\Phi(xy) = \Phi(x)\Phi(y)$ shart bajarilsa, u holda Φ avtomorfizm deb ataladi. Agar A algebra ustida aniqlangan ∇ chiziqli akslantirish uchun, A algebraning har bir x elementida $\nabla(x) = \Phi(x)$ shartni qanoatlantiruvchi $\Phi: A \to A$ avtomorfizm mavjud bo'lsa, u holda ∇ lokal avtomorfizm deb ataladi.

Ushbu maqolada biz $\sum_{ij}^{o} p_i\, B(H)p_j$ algebraning har bir uzluksiz biyektiv lokal avtomorfizmi, yoki avtomorfizm, yoki antiavtomorfizm bo'lishini isbotladik.

1. Gillbert fazosida aniqlangan kompakt operatorlar algebrasi

Aytaylik $B(H)$ – bu H separabel Gilbert fazosida aniqlangan barcha chegaralangan chiziqli operatorlar algebrasi va $\{p_i\}_{i=1}^{\infty}$ – bu $B(H)$ algebradagi o'zaro ortogonal minimal proyektorlarning, ya'ni $p^2 = p, p^* = p$ va har qanday $p \geq q$ bo'lgan proyektor uchun $p = q$ shartlarni qanoatlantiruvchi elementlarning, maksimal to'plami bo'lsin.

U holda, $\{p_i\}_{i=1}^{\infty}$ – bu $B(H)$ algebradagi ekvivalent proyektorlarning sanoqli ortoganal to'plami bo'lib, $sup_i p_i = 1$ o'rinli bo'ladi. Aytaylik,

$$\sum_{ij}^{o} p_i\, B(H)p_j =$$

$\{\{a_{ij}\}$: ihtiyoriy i, j idekslar uchun, $a_{ij} \in p_i B(H)p_j$,va har qanday $\varepsilon > 0$ son uchun shunday $n_o \in N$ natural son mavjudki har qanday $n \geq m \geq n_o$ natural sonlar uchun quyidagi shart bajariladi

$$\left\| \sum_{i=m}^{n} \sum_{k=1}^{i-1} (a_{ki} + a_{ki}) + a_{ii} \right\| \leq \varepsilon \;\}.$$

Agar biz ushbu to'plamda mos komponentlar bo'yicha algebraik amallarni kiritadigan bo'lsak, u holda $\sum_{ij}^{o} p_i\, B(H)p_j$ vektor fazoga aylanadi. Shuningdek $\sum_{ij}^{o} p_i\, B(H)p_j$ – $B(H)$ algebraning vektor qism fazosi ekanligini ko'rish mumkin. Aytaylik, $\{a_{ij}\} \in \sum_{ij}^{o} p_i\, B(H)p_j$ element va har bir $n \in N$ natural son uchun

$$a_n = \sum_{ij=1}^{n} a_{ij}$$

bo'lsin. U holda, har bir $n \in N$ uchun $a_n \in B(H)$ va $\sum_{ij}^{o} p_i\, B(H)p_j$ vektor fazoning ta'rifiga ko'ra (a_n) ketma-ketlik $B(H)$ algebrada fundamental ketma-ketlikni tashkil qiladi. Shuning uchun $B(H)$ algebrada shunday element a topiladiki $a = \lim_{n\to\infty} a_n$ tenglik o'rinli bo'ladi. $\{a_{ij}\}$ elementga a elementni mos keltirsak $\sum_{ij}^{o} p_i\, B(H)p_j$ vektor fazoni $B(H)$ algebraga yotqizish mumkin. Shu ma'noda $\sum_{ij}^{o} p_i\, B(H)p_j$ – vektor fazo $B(H)$ algebraning normalangan qism fazosi bo'lishini va ixtiyoriy $\{a_{ij}\} \in \sum_{ij}^{o} p_i\, B(H)p_j$ uchun $n \to \infty$ da

$$\left\| \sum_{ij=1}^{n} a_{ij} - \sum_{ij=1}^{n+1} a_{ij} \right\| \to 0$$

ekanligini ko'rishimiz mumkin. Ushbu yotqizishdagi $\sum_{ij}^{o} p_i\, B(H)p_j$ fazoning $B(H)$ algebradagi obrazini ham $\sum_{ij}^{o} p_i\, B(H)p_j$ orqali belgilaylik. Quyida, umumiylikni buzmagan holda ushbu aynan tenglashtirishdan foydalanamiz.

Teorema 1. Aytaylik $B(H)$ – bu H separabel Gilbert fazosida aniqlangan barcha chegaralangan chiziqli operatorlar algebrasi va $\{p_i\}_{i=1}^{\infty}$ – bu $B(H)$ algebradagi o'zaro ortogonal minimal proyektorlarning maksimal to'plami bo'lsin. U holda $\sum_{ij}^{o} p_i\, B(H)p_j$ fazo $B(H)$ algebradagi algebraik amallar, assotsiativ ko'paytirish amaliga nisbatan $B(H)$ algebraning C^*-qismalgebrasi bo'ladi.

Isbot. Yuqorida asoslanganidek $\sum_{ij}^{o} p_i\, B(H)p_j$ fazoni $B(H)$ algebraning qism fazosi sifatida olamiz.

Aytaylik (a_n) – bu $\sum_{ij}^{o} p_i\, B(H)p_j$ fazoning elementlari ketma-ketligi bo'lib, qandaydir $a \in B(H)$ elementga yaqinlashsin. U holda, barcha i va j indekslar uchun, $n \to \infty$ bo'lganda $p_i a_n p_j \to p_i a p_j$ yaqinlashish mavjud. Shuning uchun, barcha i, j indekslar uchun $p_i a p_j \in p_i B(H) p_j$. Aytaylik, ihtiyoriy n uchun

$$b_n = \sum_{k,l=1}^{n} p_k a p_l, \quad c_m^n = \sum_{k,l=1}^{n} p_k a_m p_l$$

bo'lsin. U holda $m \to \infty$ bo'lganda $c_m^n \to b_n$.

Biz (b_n) ketma-ketlik fundamental bo'lishini isbotlashimiz kerak.

Aytaylik $\varepsilon \in R$ – ihtiyoriy musbat son bo'lsin. U holda shunday n_1, n_2 natural sonlar topiladiki, har qanday $n, m > n_1$ uchun

$$\|a - a_n\| < \varepsilon, \; \|a_m - a\| < \varepsilon, \qquad (1)$$

hamda, har qanday $m_1, m_2 > n_2$ uchun

$$\|a_{m_1} - a_{m_2}\| < \varepsilon \qquad (2)$$

tengsizliklar o'rinli bo'ladi. Shu bilan (1) va (2) tengsizliklar har qanday $n, m, m_1, m_2 > n_3$ uchunham o'rinli bo'ladi, bu yerda $n_3 = \max\{n_1, n_2\}$. Ta'rifga ko'ra shunday n_4 natural son topiladiki, har qanday $k, l > n_4$ uchun

$$\left\|\left(\sum_{k=1}^{k} p_k\right) a_{m_1}\left(\sum_{k=1}^{k} p_k\right) - \left(\sum_{k=1}^{l} p_k\right) a_{m_1}\left(\sum_{k=1}^{l} p_k\right)\right\| < \varepsilon,$$

ya'ni

$$\left\|c_{m_1}^k - c_{m_1}^l\right\| < \varepsilon \qquad (3)$$

tengsizlik o'rinli bo'ladi. Ushbu (3) tengsizlik har qanday $n, m > n_o$ uchun ham o'rinli bo'ladi, bu yerda $n_o = \max\{n_3, n_4\}$. Shu bilan birga, barcha l va $\{p_k\}_{k=1}^{l} \subset \{p_i\}_{i=1}^{\infty}$ qismto'plam uchun

$$\left\|\left(\sum_{k=1}^{l} p_k\right)(a - a_n)\left(\sum_{k=1}^{l} p_k\right)\right\| < \varepsilon,$$

$$\left\|\left(\sum_{k=1}^{l} p_k\right)(a_m - a)\left(\sum_{k=1}^{l} p_k\right)\right\| < \varepsilon,$$

$$\left\|\left(\sum_{k=1}^{l} p_k\right)(a_{m_1} - a_{m_2})\left(\sum_{k=1}^{l} p_k\right)\right\| < \varepsilon$$

tengsizliklar, ya'ni

$$\left\|b_l - c_{m_1}^l\right\| < \varepsilon,$$

$$\left\|c_{m_1}^l - c_{m_2}^l\right\| < \varepsilon,$$

$$\left\|c_{m_2}^l - b_l\right\| < \varepsilon$$

tengsizliklar o'rinli bo'ladi. Shu sababli,

$$\|b_n - b_m\| = \left\|b_n - c_{m_1}^n + c_{m_1}^n - c_{m_1}^m + c_{m_1}^m - c_{m_2}^m + c_{m_2}^m - b_m\right\|$$

$$\leq \left\|b_n - c_{m_1}^n\right\| + \left\|c_{m_1}^n - c_{m_1}^m\right\| + \left\|c_{m_1}^m - c_{m_2}^m\right\| + \left\|c_{m_2}^m - b_m\right\| < 4\varepsilon$$

tengsizliklar o'rinli bo'ladi. Demak, $\varepsilon \in R$ – ihtiyoriy musbat son uchun shunday n_o natural son topiladiki, har qanday $n, m > n_o$ uchun

$$\|b_n - b_m\| < \varepsilon$$

tengsizlik o'rinli bo'ladi. Bundan, ta'rifga ko'ra, $a \in \sum_{ij}^{o} p_i\, B(H) p_j$. U holda, (a_n) ketma-ketlik ixtiyoriy tanlanganligi sababli $\sum_{ij}^{o} p_i\, B(H) p_j$ normalangan fazo – to'la normalangan fazo, ya'ni Banax fazosi bo'ladi.

Endi, $\sum_{ij}^{o} p_i\, B(H) p_j$ Banax fazosining C^*-algebra ekanligini isbotlaymiz. Aytaylik $\{a_{ij}\}$ va $\{b_{ij}\}$ – $\sum_{ij}^{o} p_i\, B(H) p_j$ Banax fazosidan olingan ihtiyoriy elementlar bo'lsin. Barcha m natural sonlar uchun

$$a_m = \sum_{k,l=1}^{m} a_{kl}, b_m = \sum_{k,l=1}^{m} b_{kl}$$

bo'lsin. U holda, barcha n natural sonlar uchun $a_n, b_n \in \sum_{ij}^{o} p_i\, B(H) p_j$ va (a_m) ketma-ketlik $\{a_{ij}\}$ elementga, (b_m) ketma-ketlik esa $\{b_{ij}\}$ elementga $\sum_{ij}^{o} p_i\, B(H) p_j$ Banax fazosida yaqinlashuvchi bo'ladi. Unda ixtiyoriy n uchun $m \to \infty$ yaqinlashishda, $B(H)$ algebrada assotsiativ ko'paytirish amalining uzluksizligiga ko'ra, $a_m b_n \to \{a_{ij}\} b_n$ yaqinlashish orinli bo'ladi. Bundan $\{a_{ij}\} b_n \in \sum_{ij}^{o} p_i\, B(H) p_j$ kelib chiqadi. Yuqorida takidlanganidek, biz $\sum_{ij}^{o} p_i\, B(H) p_j$ fazo $B(H)$ algebrada yotadi deb xisoblashimiz mumkin. Shuning uchun $n \to \infty$ bo'lganda $(\{a_{ij}\} b_n)$ ketma-ketlik $\{a_{ij}\}\{b_{ij}\}$ elementga $B(H)$ algebrada yaqinlashadi. $\sum_{ij}^{o} p_i\, B(H) p_j$ fazo Banax fazosi bo'lgani uchun $\{a_{ij}\}\{b_{ij}\} \in \sum_{ij}^{o} p_i\, B(H) p_j$ natijaga ega bo'lamiz.

$$\sum_{ij}^{o} p_i\, B(H) p_j \subset A$$

tegishlilik orinli bo'lgani uchun $\sum_{ij}^{o} p_i\, B(H) p_j$ fazo $B(H)$ algebraning C^*-qismalgebrasi bo'ladi. Isbot yakunlandi.

Aytaylik H – cheksiz o'lchamli separabel Gilbert fazosi bo'lsin, $B(H)$ – bu barcha chegaralangan chiziqli operatorlar algebrasi bo'lsin, va $\{p_i\}$ – $B(H)$ algebradagi ekvivalent proyektorlarning sanoqli ortogonal to'plami bo'lib, $sup_i p_i = 1$ shart o'rinli bo'lsin. Aytaylik $\left\{\{p_j^i\}_j\right\}_i$ – bu $\{p_i\}$ to'plamning cheksiz qismto'plamlarining to'plami bo'lib, har qanday α va β indekslar uchun

$$\{p_j^\alpha\}_j \cap \{p_j^\beta\}_j = \emptyset, \left|\{p_j^\alpha\}_j\right| = \left|\{p_j^\beta\}_j\right|, \{p_i\} = \cup_i \{p_j^i\}_j$$

shartlar bajarilsin. U holda, agar, $B(H)$ algebrada, barcha i indekslar uchun $q_i = sup_j p_j^i$ belgilash kiritsak, u holda $sup_i\, q_i = 1$ va $\{q_i\}$ – bu o'zaro ekvivalent proyektorlarning sanoqli ortoganal to'plami bo'ladi. U holda biz ekvivalent proyektorlarning sannoqli ortoganal $\{q_i\}$ to'plami $B(H)$ algebrada $\{p_i\}$ to'plam orqali aniqlanadi deymiz. Bundan quyidagi hulosaga ega bo'lamiz.

Hulosa 2. H – cheksiz o'lchamli separabel Gilbert fazosi bo'lsin, $B(H)$ – bu barcha chegaralangan chiziqli operatorlar algebrasi bo'lsin, va $\{p_i\}$ – $B(H)$ algebradagi ekvivalent minimal proyektorlarning sanoqli ortogonal to'plami bo'lib, $sup_i p_i = 1$ shart o'rinli bo'lsin. Aytaylik $\{q_i\}$ to'plam $B(H)$ algebrada $\{p_i\}$ to'plam orqali aniqlangan bo'lsin. U holda

$$\sum_{ij}^{o} q_i B(H) q_j$$

$B(H)$ algebraning C^*-qismalgebrasi bo'ladi.

Isbot. Aytaylik $\left\{\{p_j^i\}_j\right\}_i$ – bu $\{p_i\}$ to'plamning cheksiz qismto'plamlarining to'plami bo'lib, har qanday α va β indekslar uchun

$$\{p_j^\alpha\}_j \cap \{p_j^\beta\}_j = \varnothing, \left|\{p_j^\alpha\}_j\right| = \left|\{p_j^\beta\}_j\right|, \ \{p_i\} = \cup_i \{p_j^i\}_j$$

shartlar bajarilsin. U holda, $B(H)$ algebradagi barcha i indekslar uchun

$$q_i = sup_j p_j$$

tenglik o'rinli bo'ladi. Bundan, barcha i va j indekslar uchun

$$q_i B(H) q_j = \{q_i a q_j : a \in B(H)\}$$

tenglikka ega bo'lamiz. Shuning uchun, barcha i va j indekslar uchun $q_i B(H) q_j \subseteq B(H)$. Isbotning qolgan qismida 1-teoremaning isboti takrorlanadi. Isbot yakunlandi.

Hulosa 3. H – cheksiz o'lchamli separabel Gilbert fazosi bo'lsin, $B(H)$ – bu barcha chegaralangan chiziqli operatorlar algebrasi bo'lsin, va $\{p_i\}$ – $B(H)$ algebradagi ekvivalent minimal proyektorlarning sanoqli ortogonal to'plami bo'lib, $sup_i p_i = 1$ shart o'rinli bo'lsin. U holda, $\sum_{ij}^{o} p_i B(H) p_j$ C^* -qismalgebra $K(H)$ – barcha kompakt operatorlar algebrasining qismalgebrasidir.

Isbot. Aytaylik a – bu $\sum_{ij}^{o} p_i B(H) p_j$ algebraning ihtiyoriy elementi va ihtiyoriy n uchun

$$a_n = \sum_{k,l=1}^{n} p_k a p_l$$

bo'lsin. U holda $n \to \infty$ bo'lganda, norma bo'yicha, $a_n \to a$ yaqinlashish o'rinli bo'ladi. Har bir n natural son uchun a_n – bu chekli o'lchamli operator bo'ladi. Haqiqatan ham, H – Gilbert fazosida $\{p_i\}$ minimal proyektorlar to'plamiga mos $\{\xi_i\}$ to'la ortonormal sistemani olsak, ya'ni

$$p_i : H \to \mathbf{C}\xi_i,$$

bo'lsa, u holda, a_n element

$$a_n : H \to \mathbf{C}\xi_1 \oplus \mathbf{C}\xi_2 \oplus \ldots \oplus \mathbf{C}\xi_n$$

ko'rinishdagi akslantirish bo'ladi. Demak, a_n chekli o'lchamli operatordir. U holda, har bir n natural son uchun, a_n kompakt operator bo'ladi. U holda, kompakt operatorlarning norma bo'yicha limiti yana kompakt operator bo'lgani uchun a ham H Gilbert fazosida kompakt operator bo'ladi. Isbot yakunlandi.

Hulosa 4. H – cheksiz o'lchamli separabel Gilbert fazosi bo'lsin, $B(H)$ – bu barcha chegaralangan chiziqli operatorlar algebrasi bo'lsin, va $\{p_i\}$ – $B(H)$ algebradagi ekvivalent minimal proyektorlarning sanoqli ortogonal to'plami bo'lib, $sup_i p_i = 1$ shart o'rinli bo'lsin. U holda, $\sum_{ij}^{o} p_i B(H) p_j$ algebra **0** elementdan boshqa markaziy elementga, ya'ni, har qanday $b \in \sum_{ij}^{o} p_i B(H) p_j$ uchun $ab = ba$ tenglikni qanoatlantiruvchi a elementga, ega emas.

Isbot. $\{p_i\}$ to'plam yordamida $B(H)$ algebraning $\{p_{ij}\}$ birlik matritsalar sistemasini, ya'ni

$$p_i = p_{ii}, p_i = p_{ij} p_{ji}, i, j = 1,2,3, \ldots, n, \ldots$$

shartlarni qanoatlantiruvchi sistemani, quramiz. Ta'rifga ko'ra

$$\{p_{ij}\} \subset \sum_{ij}^{o} p_i B(H) p_j.$$

Ravshanki, $\sum_{ij}^{o} p_i B(H) p_j$ algebra $\{p_{ij}\}$ to'plamning chiziqli qobig'ining norma bo'yicha yopig'i bilan ustma-ust tushadi. Hatto $B(H)$ algebrada ham har qanday $b \in \{p_{ij}\}$ uchun $ab = ba$ tenglikni qanoatlantiruvchi noldan va $\lambda \cdot 1$, $\lambda \in \mathbf{C}$, ko'rinishdagi elementlardan farqli a element mavjud emas. Isbot yakunlandi.

2. Kompakt operatorlar qismalgebralaridagi lokal avtomorfizmlar tavsifi haqida

Aytaylik H – cheksiz o'lchamli separabel Gilbert fazosi bo'lsin, $B(H)$ – bu barcha chegaralangan chiziqli operatorlar algebrasi bo'lsin, va $\{p_i\}$ – $B(H)$ algebradagi ekvivalent minimal proyektorlarning sanoqli ortogonal to'plami bo'lib, $sup_i p_i = 1$ shart o'rinli bo'lsin. U holda, 3-hulosaga ko'ra, $\sum_{ij}^{o} p_i B(H) p_j$ fazo $K(H)$ barcha kompakt operatorlar algebrasining qismalgebrasi bo'ladi. $\sum_{ij}^{o} p_i B(H) p_j$ algebraning lokal avtomorfizmlari tavsifini beramiz.

Birlamchi tuchunchalar va belgilashlarni kiritaylik.

Ta'rif. Aytaylik A algebra bo'sin. $\Phi: A \to A$ biyektiv chiziqli akslantirish avtomorfizm deyiladi, agar, ihtiyoriy $x, y \in A$ elementlar uchun $\Phi(xy) = \Phi(x)\Phi(y)$ tenglik bajarilsa. Agar $\nabla: A \to A$ chiziqli akslantirish uchun, har bir $x \in A$ element uchun shunday avtomorfizm $\Phi: A \to A$ topilsaki $\nabla(x) = \Phi(x)$ shart bajarilsa, u holda ∇ chiziqli akslantirish lokal avtomorfizm deyiladi.

Lemma 2.1. Aytaylik $\nabla: \sum_{ij}^{o} p_i B(H) p_j \to \sum_{ij}^{o} p_i B(H) p_j$ – lokal avtomorfizm va $p, q \in \sum_{ij}^{o} p_i B(H) p_j$ o'zaro ortogonal idempotentlar bo'lsin (ya'ni $p^2 = p$, $q^2 = q$, $pq = 0$). U holda

$$(\nabla(p))^2 = \nabla(p), \qquad \nabla(p)\nabla(q) + \nabla(q)\nabla(p) = 0.$$

Isbot. Aytaylik Φ_p – bu $\nabla(p) = \Phi_p(p)$ shartni qanoatlantiruvchi avtomorfizm bo'lsin. U holda

$$(\nabla(p))^2 = (\Phi_p(p))^2 = \Phi_p(pp) = \Phi_p(p) = \nabla(p). \qquad (1)$$

Bundan tashqari, quyidagiga ham ega bo'lamiz

$$\nabla(p+q) = \nabla(p) + \nabla(q),$$

$$(\nabla(p+q))^2 = (\nabla(p) + \nabla(q))^2.$$

Birinchi (1) tenglikka ko'ra

$$(\nabla(p+q))^2 = \nabla(p+q) = \nabla(p) + \nabla(q) = (\nabla(p) + \nabla(q))^2$$

$$= (\nabla(p))^2 + (\nabla(q))^2 + \nabla(p)\nabla(q) + \nabla(q)\nabla(p).$$

Bundan

$$\nabla(p) + \nabla(q) = \nabla(p) + \nabla(q) + \nabla(p)\nabla(q) + \nabla(q)\nabla(p)$$

va

$$\nabla(p)\nabla(q) + \nabla(q)\nabla(p) = 0.$$

Isbot yakunlandi.

Asosiy teoremaning isboti $\sum_{ij}^{o} p_i\, B(H) p_j$ algebraning quyidagi xossasiga asoslangan:

(J) $\sum_{ij}^{o} p_i\, B(H) p_j$ algebra ustidagi har qanday Yordan avtomorfizm, yoki avtomorfizm bo’ladi, yoki antiavtomorfizm bo’ladi.

Teorema 2.1. $\sum_{ij}^{o} p_i\, B(H) p_j$ algebraning har bir uzluksiz biyektiv lokal avtomorfizmi, yoki avtomorfizm, yoki antiavtomorfizm bo’ladi.

Isbot. Aytaylik ∇ – bu $\sum_{ij}^{o} p_i\, B(H) p_j$ algebraning ihtiyoriy uzluksiz biyektiv lokal avtomorfizmi va x – bu $\sum_{ij}^{o} p_i\, B(H) p_j$ algebraning

$$x = \lambda_1 p_1 + \lambda_2 p_2 + \cdots + \lambda_n p_n$$

ko’rinishdagi elementi bo’lsin, bu yerda $p_1, p_2, \ldots, p_n$ – bu o’zaro ortogonal proyektorlar, $\lambda_1, \lambda_2, \ldots, \lambda_n$ – bu haqiqiy sonlar. U holda, ∇ akslantirish chiziqli bo’lganligi sababli

$$\nabla(x^2) = {\lambda_1}^2\nabla(p_1) + {\lambda_2}^2\nabla(p_2) + \cdots + {\lambda_n}^2\nabla(p_n).$$

2.1 lemmaga ko’ra

$$(\nabla(x))^2 = (\lambda_1\nabla(p_1) + \lambda_2\nabla(p_2) + \cdots + \lambda_n\nabla(p_n))^2 = {\lambda_1}^2\nabla(p_1) + {\lambda_2}^2\nabla(p_2) + \cdots + {\lambda_n}^2\nabla(p_n) = \nabla(x^2).$$

Bundan,

$$\nabla(x^2) = (\nabla(x))^2. \qquad (3)$$

$\sum_{ij}^{o} p_i\, B(H) p_j$ algebraning ta’rifiga ko’ra ushbu algebraning har bir elementi ushbu algebradagi chekli o’lchamli chiziqli operatorlar ketma-ketligi yordamida approksimatsiyalanadi. O’z navbatida, ushbu algebradagi har bir chekli o’lchamli chiziqli operator ushbu algebradagi ma’lum bir ortogonal proyektorlarning chekli chiziqli kombinatsiyasidir. Shuning uchun, $\sum_{ij}^{o} p_i\, B(H) p_j$ algebraning har bir elementi ushbu algebradagi ortogonal proyektorlarning chekli chiziqli kombinatsiyalari bilan approksimatsiyalanadi.

Aytaylik x – bu $\sum_{ij}^{o} p_i\, B(H) p_j$ algebraning ihtiyoriy elementi bo’lsin. U holda, x elementni approksimatsiyalovchi (x_n) $\sum_{ij}^{o} p_i\, B(H) p_j$ algebraning ortogonal proyektorlarning chekli chiziqli kombinatsiyalari ketma-ketligi mavjud. Yuqorida isbotlanganidek

$$\nabla({x_n}^2) = \nabla(x_n)^2, n = 1,2,3, \ldots$$

Ma’lumki (x_n) ketma-ketlikning x elementga yaqinlashishidan $({x_n}^2)$ ketma-ketlikning x^2 elementga yaqinlashishi kelib chiqadi. Haqiqatan ham, agar $\varepsilon \in R$ – ihtiyoriy musbat son bo’lsa, u holda, shunday n natural son topiladiki, har qanday $m > n$ uchun

$$\|x - x_n\| < \varepsilon$$

tengsizlik bajariladi. U holda,

$$\|x^2 - x_n^2\| = \|(x - x_n + x_n)^2 - x_n^2\| = \|(x - x_n)^2 + (x - x_n)x_n + x_n(x - x_n) + x_n^2 - x_n^2\|$$
$$\leq \|(x - x_n)^2\| + \|(x - x_n)x_n\| + \|x_n(x - x_n)\| \leq \|x - x_n\|^2 + \|x - x_n\|\|x_n\| + \|x_n\|\|x - x_n\| \leq \varepsilon^2 + 2\varepsilon\|x_n\|,$$

va (x_n) ketma-ketlik chegaralangan bo’lganligi sababli $({x_n}^2)$ ketma-ketlik x^2 elementga yaqinlashadi. U holda, ∇ uzluksiz bo’ganligi sababli $(\nabla({x_n}^2))$ ketma-ketlik $\nabla(x^2)$ elementga yaqinlashadi. Hamda, o’z navbatida, $(\nabla(x_n))$ ketma-ketlikning $\nabla(x)$ elementga yaqinlashishidan $(\nabla(x_n)^2)$ ketma-ketlikning $\nabla(x)^2$ elementga yaqinlashishi kelib chiqadi. Har qanday n natural son uchun

$$\nabla({x_n}^2) = \nabla(x_n)^2$$

bo’lgani uchun

$$\nabla(x^2) = \nabla(x)^2$$

tenglikka ega bolamiz. Demak, ∇ Yordan avtomorfizmi bo’ladi. U holda, yuqorida keltirilgan (J) hossaga ko’ra ∇ uzluksiz biyektiv lokal avtomorfizm, yoki avtomorfizm, yoki antiavtomorfizm bo’ladi. Isbot yakunlandi

Foydalanilgan adabiyotlar

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Infinite Peirce distribution in the algebra of compact operators and description of its local automorphisms

Abstract. In the present paper the infinite Peirce decomposition of the algebra $K(H)$ of compact operators on an infinite dimensional separable Gilbert space H is constructed, using the norm of the algebra $K(H)$ and a maximal family of mutually orthogonal minimal projections, i.e., self-adjoint,

idempotent elements. The infinite Peirce decomposition on the norm of a C^*-algebra is also constructed in 2012 by the first author. But, it turns, the condition, applied then, is not sufficient for the infinite Peirce decomposition on the norm constructed in 2012 to be an algebra. Therefore, in the present paper, the infinite Peirce decomposition on the norm we defined by an analog of the criterion of fundamentality of a sequence and we proved that the infinite Peirce decomposition on the norm of the algebra $K(H)$ is a C^*-algebra and a number of its properties. First, the idea of the infinite Peirce decomposition on the norm was realized in 2008 by the firs author.

Also, the present paper is devoted to the description of local derivations on algebras compact operators. The history of local derivations and local automorphisms begins with the Gleason-Kahane-Żelazko theorem proved in 1967-1968. This theorem is a fundamental contribution in the theory of Banach algebras. This theorem asserts that every unital linear functional F on a complex unital Banach algebra A, such that $F(a)$ belongs to the spectrum $\sigma(a)$ of a, for every $a \in A$, is multiplicative. In modern terminology this is equivalent to the following condition: every unital linear local homomorphism from a unital complex Banach algebra A into $\mathbf{C}$ is multiplicative. We recall that a linear map T from a Banach algebra A into a Banach algebra B is said to be a local homomorphism if for every a in A there exists a homomorphism $\Phi_a A \to B$, depending on a, such that $T(a) = \Phi_a(a)$.

Later, in 1990, R. Kadison introduces the concept of local derivation and proves that each continuous local derivation from a von Neumann algebra into its dual Banach bemodule is a derivation. B. Jonson in 2001 extends the above result by proving that every local derivation from a C*-algebra into its Banach bimodule is a derivation. In particular, Johnson gives an automatic continuity result by proving that local derivations of a C*-algebra A into a Banach A-bimodule X are continuous even if not assumed a priori to be so. Based on these results, many authors have studied local derivations on operator algebras.

In the present paper, automorphisms and local automorphisms of compact operator algebras are studied. Recall that a bijective linear mapping Φ on an algebra A, satisfying, for each pair x, y of elements in A, $\Phi(xy) = \Phi(x)\Phi(y)$, is called an automorphism. A linear mapping ∇ on the algebra A is called a local automorphism if, for every element x in the algebra A, there exists an automorphism $\Phi: A \to A$ such that $\nabla(x) = \Phi(x)$.

In the present paper, every continuous bijective local automorphism on the infinite Peirce decomposition $\sum_{ij}^{o} p_i B(H) p_j$ on the norm of the algebra $K(H)$ of compact operators on an infinite dimensional separable Gilbert space H, is, weather an automorphism, or antiautomorphism.

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