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Maruf Kuchimov

"Bulletin of TUIT: Management and Communication Technologies", zafar210980@mail.ru

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METHODOLOGY OF DEVELOPMENT OF MATHEMATICAL MODELS OF INFORMATION PROCESSING SYSTEMS

Kuchimov Maruf Kuchimovich

Art. Rev. Department of Information Technologies, Tashkent Institute of Architecture and Civil Engineering,

Annotation: Modeling as a research method is a powerful cognitive tool throughout the history of human development. The article describes the methodology for the development of mathematical models of information systems, based on materials from various literary sources, author's developments on the system approach, mathematical modeling and programming. The mathematical model of the information system is described and all the characteristics of the IS are given.

Key words: *information system, model, program, probabilistic-temporal characteristics, process,*

INTRODUCTION

Information systems (IS) are one of the effective means of information processing in shared systems and are widely used in office automated control systems, computer-aided design systems, etc. The widespread use of IP is primarily due to their high economic efficiency. In business, having the necessary information on time is the key to success.

In the conditions of a rapidly changing world, engulfed by socio-economic transformations, the issues of the priority of the value of human life and personality are of particular importance. A person and his interests are promoted to the center of the modern scientific picture of the world - a process of humanization of all areas of knowledge takes place.

Teaching mathematical modeling is impossible without attracting

information from various fields of knowledge, therefore, the issues of implementing interdisciplinary connections of mathematics with other academic subjects play a leading role in this process.

The aim of the research is to develop a methodology for teaching mathematical modeling of processes in information systems as a means of enhancing additional mathematical education of students.

The object of the research is the mathematical education of university students. The subject of the research is teaching students mathematical modeling.

Purposeful use of mathematical modeling in information systems research will enhance the humanization of additional mathematical education, which in turn will increase students'

interest in mathematics and programming.

In accordance with the goal, hypothesis, object and subject of research, the following particular tasks were identified:

1. To study the state of the problem of teaching mathematical modeling and programming.
2. To analyze the possibilities of mathematical modeling of information systems as a means of enhancing the optimization of information processing in such systems.
3. To develop a methodology for teaching mathematical modeling of processes in information systems, as a means of increasing the efficiency of information processing using modern computer technology.
4. To carry out an experimental check of the effectiveness of using the developed methodological models for conducting classes in mathematical modeling and programming.

To solve the set tasks, the following research methods were used:

- analysis of psychological - pedagogical and scientific - methodological literature on the research problem,
- studying the pedagogical experience of teachers, analyzing personal experience of working in groups of various profiles as a programming teacher,

- pedagogical experiment;
- conducting open lessons and methodological seminars for university teachers.

The scientific novelty of the research is that it contains:

- revealed the importance of mathematical modeling and programming in the study of processes in information systems;
- developed the structure and content of didactic materials used for teaching mathematical modeling of social processes.
- developed a methodology for organizing mathematical classes for teaching students mathematical modeling of processes in information systems (IS).

The performance and bandwidth of the IS is determined by a complex of systemically interrelated factors:

- characteristics of technical means (choice of computers and workstations, communication equipment, operating systems of workstations, servers and their configurations, etc.);
- the nature of the distribution and storage of information resources;
- modes of access to the system;
- organization of distributed information processing;
- distribution of database files among the system servers;

- organization of a distributed computing process;
- protection, maintenance and restoration of operability in situations of failures and failures.

Investigation of the characteristics of various access modes and the choice of the most optimal for specific modes of operation of the IS and, accordingly, optimization of information processing modes when solving a given class of problems, possibly by developing mathematical models of these processes and organizing simulation using computational experiment tools.

The practical interest of the problems under consideration is determined by the need to develop software for the design, monitoring and optimization of the operation modes of complex distributed ICs.

Let us define the main parameters of the queuing model for a random method of access to the transmission medium:

λ_i , $i=1,k$ - the intensity of requests coming for processing from the i -th subscriber, characterizes the occurrence of communication between the subscriber "i" and the server;

μ_i , $i=1,k$ is the intensity of processing applications in OPI;

μ_{M1} - the intensity of processing requests in the mono-channel, coming from all subscribers - the reciprocal of the average time of information transmission over the mono-channel;

$1/\mu_{M2}$ - the interval of increasing the processing time of the request in the mono-channel due to the occurrence of conflicts (average delay time).

The conflict situation is detected and eliminated during the average time interval $1/\mu_{M2}$. The likelihood of a conflict is determined as follows:

$$P_{HK} = P(h \geq 3) - H \quad (1)$$

where $P(h > 3)$ is the probability of finding three or more requests from all subscribers in the SMO_m ; H is the probability of conflict-free situations when there are three or more applications in the system.

The probability that there are exactly k customers in the system for the $QS M / M / 1$ is $p_k = (1 - \rho)\rho^k$, $k = 0, 1, 2, \dots$

The probability that the system has at least k requirements for the $QS M / M / 1$:

$$P[h \geq k \text{ требований в системе}] = \sum_{i=k}^{\infty} p_i = \sum_{i=k}^{\infty} (1 - \rho)\rho^i = \rho^k,$$

$$\text{where, где } \rho = \sum_{i=1}^k \rho_i, \quad \rho_i = \frac{\lambda_i}{\mu_i}.$$

Thus, the probability $P(h > 3)$ is determined on the basis of the $QS M / M / 1$ model, taking into account the fact that at the input we have a total flow from all subscribers, i.e. $P(h \geq 3) = \rho^3$.

Further in the chapter, the construction of a mathematical model of a random IP access method with an arbitrary number of subscribers is considered.

Analysis of the behavior of the probability of conflict-free situations in expression (1) with an increase in the number of subscribers "h" showed that the value of H decreases and at h = 4 is 3.48%, and at h = 8 - 0.46%. Therefore, for h > 4, the value of H in (1) can be neglected, and then

$$P_{HK} = \rho^3 \tag{2}$$

When using (2), the share of the load factor in the mono channel from the i-th subscriber is equal to:

$$\rho_i = \lambda_i / M_{M1} + (\Lambda / M_{M1})^3 / M_{M2}, \quad i = 1, k, \tag{3}$$

where где $\Lambda = \sum_{i=1}^k \lambda_i$ is the total total intensity of flows entering the input of the mono-channel; k is the total number of incoming streams to the mono-channel; Mm1 is the intensity of processing requests of all subscribers in the mono-channel; Mm2 –

the intensity of processing requests in the mono-channel in the event of a conflict.

Summing over all elements $\rho_i, i=1, k$, we reduce expression (3) to the form

$$\sum_{i=1}^k \rho_i^{MK} = \frac{1}{M_{M1}} \sum_{i=1}^k \lambda_i + \frac{1}{M_{M2}} \left(\frac{\Lambda}{M_{M1}} \right)^3 \sum_{i=1}^k \lambda_i = \frac{\Lambda}{M_{M1}} + \frac{\Lambda}{M_{M2}} \left(\frac{\Lambda}{M_{M1}} \right)^3 \tag{4}$$

When a conflict occurs in expression (4) in the chapter, the value is defined $M_{M2} ::$

$$M_{M2} = 1 / T_{отср}, \tag{5}$$

where $T_{отср}(n) =$

$$\frac{1 - \Omega}{2} \sum_{n=2}^9 [(2^n - 1)\Omega^{n+2}] + 511.5 \sum_{n=10}^{15} (\Omega^{11} - \Omega^{16});$$

$$\Omega = (\sum_{i \in \text{вх. моноканал}} \lambda_i) / M_{M1}$$

At the input of each subscriber node, a stream with an intensity $\lambda_i, i = \overline{1, k}$, arrives, which is processed in a subscriber station with the intensity and $\mu_i, i = \overline{1, k}$, intensity of requests:

$$\lambda_i^{ex} = \lambda_i + \lambda_i^*, \quad \text{где} \tag{6}$$

$$\lambda_i^* = (\sum_{r=1}^k \lambda_r) / (k - 1), \quad i = \overline{1, k}$$

The VVH of the first phase is defined as follows:

Density of the probability distribution of the time spent by the request in the phase:

$$g_i^{1\phi}(t) = (\mu_I - \lambda_i^{ex}) \exp[(\mu_I - \lambda_i^{ex}) t]. \tag{7}$$

Average time spent by an application in the first phase:

$$u_i^{-1\phi} = [\mu_I (1 - \lambda_i^{ex}) / \mu_I]^{-1}. \tag{8}$$

The variance of the time spent by the application in the first phase:

$$D_{g_i}^{1\phi} = [\mu_I - \lambda_i^{ex}]^{-2}. \tag{9}$$

The density of the probability distribution of the order waiting time:

$$f_i^{1\phi}(t) = (\lambda_i^{ex} / \mu_I) (\mu_I - \lambda_i^{ex}) \exp[-(\mu_I - \lambda_i^{ex}) t]. \tag{10}$$

Average waiting time for an application in the first phase:

$$w_i^{-1\phi} = \frac{1}{\mu_i} \cdot \frac{\lambda_i^{ex} / \mu_i}{1 - \lambda_i^{ex} / \mu_i}.$$

The variance of the waiting time for an order in the first phase:

$$D_f^{1\phi} = (\lambda_i^{ex} / \mu_I)^2 (\mu_I - \lambda_i^{ex})^{-2}.$$

Distribution of probabilities of the number of orders located on:

$$P_i^{1\phi}(n) = (1 - \lambda_i^{ex} / \mu_I) (\lambda_i^{ex} / \mu_I)^n. \tag{13}$$

Average number of applications in the first phase:

$$w_i^{-1\phi} = \frac{1}{\mu_i} \cdot \frac{\lambda_i^{ex} / \mu_i}{1 - \lambda_i^{ex} / \mu_i}.$$

$$\tag{14}$$

Dispersion of the number of claims in the first phase of the system:

$$D_f^{1\phi} = (\lambda_i^{ex} / \mu_I)^2 (\mu_I - \lambda_i^{ex})^{-2} \tag{15}$$

Distribution of probabilities of the number of customers waiting for service:

$$P_i^{*1\phi}(0) = 1 - \lambda_i^{ex} / \mu_I, P_i^{*1\phi}(n) = [1 - (\lambda_i^{ex} / \mu_I)] (\lambda_i^{ex} / \mu_I)^{n-1}, \text{ при } n \geq 1. \tag{16}$$

Average queue length in the first phase:

$$\bar{U}_i^{-1\phi} = (\lambda_i^{ex} / \mu_I)^2 / (1 - \lambda_i^{ex} / \mu_I). \tag{17}$$

Variance of the length of the queue of claims:

$$D_{D_i}^{1\phi} = (1 - \lambda_i^{ex} / \mu_I) \sum_{k=1}^{\infty} k^2 / (\lambda_i^{ex} / \mu_I)^{k+1} - (\lambda_i^{ex} / \mu_I)^2 / (1 - \lambda_i^{ex} / \mu_I). \tag{18}$$

The input flow rate of the second phase is determined as follows:

$$\Lambda = \sum_{i=1}^k (\lambda_i + \lambda_i^*) + \Lambda_{\text{вогне}} + \Lambda_{\text{извне}}. \tag{19}$$

Average processing time of a message packet in a mono-channel:

$$\tau_{\text{экс}}^{MK} = \frac{1}{\mu_{\text{экс}}^{MK}} = \frac{\rho_{MK}}{\Lambda}, \quad \Lambda = \sum_{i=1}^k \lambda_i, \tag{20}$$

where ρ_{MK} is defined by relation (4).

Having the obtained values « Λ » и « $\mu_{MK}^{окг}$ » as the initial parameters for the second processing phase, and using the same queuing model (M / M / 1) for determination as for the first phase, we obtain for the second phase a similar set of VVH: $g^{2\delta}(t), \bar{U}^{2\delta}, \bar{D}_{g_i}^{2\delta}, f^{2\delta}(t), \bar{w}^{2\delta}, D_{g_i}^{2\delta}, P^{2\delta}(n), \bar{n}^{2\delta}, D_{n_i}^{2\delta}$ where in all expressions of the BBX of the first phase, μ_I changes to $\mu_{MK}^{окг}$, a λ_I , and $\Lambda.I$ is replaced by .

For the third phase of processing, VVX are determined on the basis of the same M / M / 1 queuing model. Here = and $\Lambda = \sum_{i=1}^k (\lambda_i + \lambda_i^*) + \Lambda_{\text{всего}}$ и μ^* will be used as initial parameters.

By analogy with the VVH obtained in the first and second processing phases, we obtain a set of indicators for the third phase.

Since the IS model is an exponential system, the integral VVH for the three phases of the route "subscriber Ai - subscriber Ai" are determined by the following relations:

$$\bar{\Pi}_m^\Sigma(i, j) = \bar{\Pi}_m^{1\phi}(i) + \bar{\Pi}_m^{2\phi} + \bar{\Pi}_m^{3\phi}(j) \tag{21}$$

- for VVH, which determine the average and variance, where $\bar{\Pi}_m^\Sigma(i, j)$ is the integral indicator, m is the number of the $\bar{\Pi}_m^{n\phi}(i)$ - indicator; - m-th

indicator of the n-th phase of processing;

$$\bar{\Pi}_m^\Sigma(i, j) = \bar{\Pi}_m^{1\phi}(i) * \bar{\Pi}_m^{2\phi} * \bar{\Pi}_m^{3\phi}(j) \tag{22}$$

is the integral indicator for VVH, which determine the probability distribution density and the probability distribution of discrete states, where * is the sign of the composition.

Let's determine the integral indicators for all three phases of processing.

Density of the probability distribution of the time spent by the request in the system:

$$g_i(t) = g_i^{1\phi}(t) * g_i^{2\phi}(t) * g_i^{3\phi}(t) \tag{23}$$

Average time spent by a request in the system:

$$\bar{u}_i = u_i^{1\phi} + u_i^{2\phi} + u_i^{3\phi} \tag{24}$$

Dispersion of the time spent by the request in the system:

$$Dg_i = Dg_i^{1\phi}(t) + Dg_i^{2\phi} + Dg_i^{3\phi} \tag{25}$$

The density of the probability distribution of the order waiting time:

$$f_i(t) = f_i^{1\phi}(t) * f_i^{2\phi}(t) * f_i^{3\phi}(t) \tag{26}$$

Average waiting time for service of a request in the system:

$$\bar{w}_i = \bar{w}_i^{1\phi} + w_i^{2\phi} + w_i^{3\phi} \quad (27)$$

The variance of the waiting time for servicing a request in the system:

$$Df_i = Df_i^{1\phi} + Df_i^{2\phi} + Df_i^{3\phi} \quad (28)$$

Distribution of probabilities of the number of serviced claims:

$$P_i(n) = P_i^{1\phi}(n) * P_i^{2\phi}(n) * P_i^{3\phi}(n) \quad (29)$$

Average number of applications in the system:

$$\bar{n}_i = \bar{n}_i^{1\phi} + \bar{n}_i^{2\phi} + \bar{n}_i^{3\phi} \quad (30)$$

Dispersion of the number of orders in the system:

$$Dn_i = g_i^{1\phi} + g_i^{2\phi} + g_i^{3\phi} \quad (31)$$

Average queue length:

$$\bar{D}_i = \bar{D}_i^{1\phi} + \bar{D}_i^{2\phi} + D_i^{3\phi} \quad (32)$$

Variance of the average queue length:

$$D_{D_i} = D_{D_i}^{1\phi} + D_{D_i}^{2\phi} + D_{D_i}^{3\phi} \quad (33)$$

The rest of the integral VVHs are completely determined by the corresponding expressions (24), (25), (27) - (33).

To obtain the probability that the delivery time of information from the subscriber "i" to the server will exceed

the value $T_{доп}$, is determined as follows:

$$P_i[t > T_{доп}] = 1 - \int_0^{T_{доп}} g_i(t) dt. \quad (34)$$

Thus, we got the opportunity to calculate VVH, both for individual processing phases and for typical routes of information movement.

The described methodology for the development of mathematical models of information systems makes it possible to apply it in pedagogical practice when teaching students of various specialties in those cases when it requires processing large volumes of information in the IS. On the basis of a mathematical model, a software package was developed in the C ++ programming language, which is also successfully used in the educational process.

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