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# TASKS AND MECHANISMS OF DISTRIBUTION COSTS AND INCOMES IN MARKET ECONOMY

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***Abstract:*** *The paper considers models and mechanisms of resource allocation that are interpreted in a meaningful way either as problems of allocating costs for the implementation of a common project (program) between participants (investors) interested in this project, or as problems of distributing income or profits received from joint activities of several participants. Agents can be legal entities and individuals, as well as federal and local governments. Various mechanisms of distribution of costs (incomes) are given-priority, competitive, fair play mechanisms, etc.*

***Keywords:*** *project, program, resource allocation, distribution of costs, distribution of income or profits, agents, cost allocation mechanisms, priority function, R - absolute, direct and reverse priorities mechanisms, competitive mechanisms, multi-stage cost allocation mechanisms, Nash equilibrium situation.*

## Introduction

The tasks of allocating costs and incomes are, perhaps, are the most common tasks of resource allocation in a market economy. Indeed, a characteristic feature of modern market relations is the integration of the efforts of market enterprises, firms, other legal and natural persons, as well as federal (republican) and local authorities for the implementation of projects and programs of common interest. How to divide the costs for the implementation of a project or program, how to allocate the revenue obtained as a result of their implementation are central tasks, on the effectiveness of solving which depends the success in achieving the set goals. The tasks of distribution of incomes and expenses are very close to the known problem of distribution of a limited resource, the methods of solving which have been developed in great detail [1,2,3,4]. However, unlike the latter, in this case, the costs (income) are not limited, but depend on the total income (costs) that the participants (hereinafter referred to as agents) wish to receive (can spend). Nevertheless, there is a fairly close relationship between the mechanisms for allocating scarce resources and the mechanisms for allocating income and costs.

The paper provides various mechanisms for allocating costs and revenues.The cost allocation model is taken as the base. The following scheme of agent interaction is considered.

Each agent reports an estimate of $y_i$, the required resources (material or financial), the use of which gives him a certain income $\varphi_i(y_i)$ (in a particular case, this estimate can be interpreted directly as an estimate of the income that the agent expects to receive from the implementation of the overall program) . The costs of $C(Y)$ on a program common to all agents depend on the total resource $Y = \sum_{j=1}^{n} y_j$, which agents demanded. The task is to determine the mechanism for allocating these costs among agents $x_i = \pi_i(y_i)$, where $y = (y_1, y_2, . . . , y_n)$, $\overline{1,n}$, where $n$ is the number of agents, and obviously $\sum_i x_i = C(Y)$.

The functioning of the system with a given mechanism of distribution of costs (incomes) can be considered as a game of $n$ persons (agents), which strategies are the message of estimating the required resource (or estimating the parameters of the revenue function), and the payoff function is equal to the difference in income and costs.

As a solution to the game in this paper, we consider the Nash equilibrium situation or a set of dominant strategies (if they exist).

**1. The tasks of allocating costs and incomes in a market economy**

Task 1. Financing of the joint project. Several firms (agents) decided to jointly implement the construction of an object of common interest.

From the operation of this facility, the firm i expects to receive an income $q_i$.

The cost of the construction of an object depends on the total income that firms expect to receive. We will denote the $y_i$ estimate of the income reported by firm i (the representative of the company reports to the Board of Directors of the joint-stock company established to implement the construction of the facility).

Then the total estimate of the expected income $y_i$ is $Y = \sum_i y_i$

and the costs are $C(Y)$. Obviously, $C(Y)$. is an increasing function of $Y$, $C(0) = 0$. How to allocate these costs between the founding companies of the joint-stock company? Let's designate the mechanism of distribution of expenses

$$x = \pi(y)\left(x_i = \pi_i(y), \quad i = \overline{1,n}, \sum_i \pi_i(y) = C(Y)\right).$$

What is the most fair and preferred cost allocation mechanism? As a rule, for this problem it is assumed that a fair mechanism must satisfy two conditions (axioms): anonymity and monotony.

The axiom of anonymity: the mechanism for allocating costs is called anonymous if the result of the distribution does not depend on the renumbering of agents. In other words, the distribution of costs depends only on estimates of the expected income, and the non-identical agent has no special advantage over other agents.

Axiom of monotony: with an increase in the estimate of the expected income of the i -th agent, its costs $\left(\frac{\partial \pi_i(y)}{\partial y_i} \geq 0\right)$ do not decrease. In a stronger form, the axiom of monotonicity requires that the agent's share of costs increase (as it does not decrease) with an increase in his estimate of expected income $\left(\frac{\partial}{\partial y_i}\left[\frac{\pi_i(y)}{C(Y)}\right] \geq 0\right)$.

The axiom of anonymity reflects the natural requirement of equality of partners, and the axiom of monotony is just as natural a requirement, the essence of which: the more you get, the more you pay.

Task 2. Financing development programs. A large company, uniting several enterprises, is developing a development program. This program is a combination of development programs for individual enterprises that are members of the association. Each enterprise forms and submits its program to the Board of Directors (or boards) of the firm with the justification of the required funding $y_i$. Denote $\varphi_i(y_i)$ the expected income of the i - th enterprise as a result of the program implementation. If the total amount of funds $\sum_i^n y_i = Y$, required for financing all programs exceeds the value of the centralized development fund of firm $R$, that is, $Y = \sum_j y_i > R$ (as a rule, this excess is significant), then it becomes necessary to obtain additional funds by taking a loan, issuing additional shares, etc., which leads to additional costs $Y - R$). ). The difference $Y - R$). determines the amount of additional costs for the implementation of all programs. The task is to allocate these additional costs between enterprises.

Task 3. Distribution of income. In a certain sense this problem is dual to the previous one. Several enterprises are merged to implement a common project. Each enterprise reports the amount of money $y_i$, that it can invest in this project (that is, the amount of costs). The expected income from the project $C(Y)$, of course, depends on the amount of total financing $Y$, $Y = \sum_i y_i$. How to distribute this income $C(Y)$ between enterprises? Here, the axioms of anonymity and monotony are natural, although exceptions are possible (if, for example, state or local authorities act as one of the enterprises).

Task 4. Financing priority development programs.

At present, stabilization and sustainable development of the economy is possible only on the basis of selective state support for priority areas. The forms of such support are different. This includes direct budget financing (partial or full), and concessional lending, and preferential taxation, etc. When forming programs for the development of priority areas, a competition for participation in these programs is organized. Public, private enterprises (state-private partnerships) and organizations submit applications, indicating the amount of required financial resources and justifying the effectiveness of their participation in the program. It is necessary to form a program, defining the composition of participants, the form of state support and the amount of funding.

As noted above, the problem of cost allocation is closely related to the known problem of the distribution of limited resources. Indeed, consider the following cost dependence on the implementation of the program $C(Y)$ on the required funding Y

$$C(Y) = \begin{cases} \lambda Y, & \text{if } Y \le R \\ M, & \text{if } Y > R \end{cases},$$

where M is a large number, certainly exceeding the expected total effect from the program. It is quite obvious that the resulting estimates of the required amount of funding will be such that

$$\sum_{i}^{n} y_i \le R.$$

Therefore, the distribution of costs will correspond to a certain distribution of the limited resource R with the price of the resource $\lambda$.

## 2. Mechanisms for allocating costs and revenues

The cost allocation mechanism assigns a set of estimates of the agents $\{y_i\}_{i=1}^{n}$ to the cost distribution $\{x_i = \pi_i(y)\}_{i=1}^{n}$ such that

$$\sum_i \pi_i(y) = C(Y) \qquad (2.1)$$

We describe the mechanisms of cost allocation. First of all, due to their simplicity, priority mechanisms are singled out. In these mechanisms, for each agent, its priority (weight) $\eta_i(y_i)$, is determined, and costs are allocated directly in proportion to the priorities of the agents

$$x_i = \pi_i(y) = \frac{\eta_i}{\sum_i \eta_i(y_i)} \cdot C(Y). \qquad (2.2)$$

The condition (2.1) is used automatically when priority mechanisms are used.

Depending on the type of functions $\eta_i(y_i)$ the mechanisms of direct, inverse and absolute priorities are distinguished. The direct mechanisms priorities $\eta_i(y_i)$ an increasing function $y_i$, $i = \overline{1,n}$, mechanisms in reverse priorities $\eta_i(y_i)$ - decreasing function $y_i$, $i = \overline{1,n}$, and in the mechanisms of absolute priorities $\eta_i(\cdot)$ does not depend on $y_i$, that is, $\eta_i(y_i) = \alpha_i \ge 0$. Obviously, priority mechanisms satisfy the axiom of monotony (in strong form). If anonymity is required, then the priority functions $\eta_i(y_i)$ must be the same (should not depend on).

A wide class of cost allocation mechanisms can be obtained by analyzing known mechanisms for the distribution of limited resources. Recall that the mechanism for the distribution of limited resources is the mapping of the vector of claims $\{y_i\}$ to the resource allocation vector $x_i = \theta_i(y, R)$, such that $\sum_i \theta_i(y, R) = R$.

We show that any mechanism allocation of limited resources satisfying the axiom of monotony of $R\,(\theta_i(y, R))$ is an increasing function R, $i = \overline{1,n}$), we can associate a certain cost allocation mechanism $\pi(y, R)$. We will take first, that $C(Y)$ is a piecewise-linear continuous function Y with break points $R_k$, $k = \overline{1,q}$, that is:

$$C(Y) = C\,(R_{k-1}) + \lambda_k(Y - R_{k-1});\ R_{k-1} < Y \le R_k,$$
$$\lambda_k \ge 0,$$

where $R_0 = 0$, $C(R_0) = 0$.

Define the segment $[R_{q-1}, R_q]$, such that $R_{q-1} < Y \le R_q$. We allocate in sequence the resource in the number $R_1, R_2, \cdots, R_{q-1}$, Y based on the mechanism $\pi\,(y, R)$. Denote by $\{x_{ik}, i = \overline{1,n}, k = \overline{1,q}\}$ the corresponding resource allocation. The resulting cost allocation is defined as follows:

$$z_i = \sum_{k=1}^{q} \lambda_k\,(x_{ik} - x_{ik-1}); x_{i0} = 0 \qquad (2.3)$$

Now let $C(Y)$ be an arbitrary nondecreasing differentiable function, $\pi_i(y, R)$ – differential functions R. We remark that

$$\sum_i \frac{d\pi_i(y, R)}{dR} = 1$$

Determine the costs of the i - th agent as follows

$$z_i = \int_0^Y \frac{dC(R)}{dR} \cdot \frac{\partial \pi_i(y,R)}{\partial R} \cdot dR \qquad (2.4)$$

It is easy to see that $\sum_i z_i = C(Y)$. Thus, any allocation mechanism of a limited resource R generates a well-defined cost allocation mechanism. The corresponding mechanism for allocating costs will be called the R - mechanism. Let's describe the basic mechanisms of distribution of limited resources and the mechanisms of cost distribution generated by them (R - mechanism).

Priority mechanisms for resource allocation. In these mechanisms, as well as in the priority mechanisms for allocating costs (2.2), the distribution is carried out on the basis of agent priority functions

$x_i = \pi_i(y, R)\min(y_i;\ \gamma \cdot \eta_i(y_i))$ (2.5)

where $\gamma$ is determined from equation

$$\sum_i \min\left(y_i;\ \gamma \cdot \eta_i(y_i)\right) = R.$$

Depending on the type of priority functions, the mechanisms of absolute, direct and reverse priority are singled out.

Let's consider corresponding R-mechanisms (mechanisms of distribution of expenses), satisfying anonymity condition:

R - the mechanism of absolute priorities.

Let $\eta_i(y_i) = 1$ (similar conclusions can be obtained if all priority functions are equal to the same positive quantity), then

$$x_i = \min(y_i;\ \gamma),\ i = \overline{1, n}.,$$

where $\gamma$ is determined from equation

$$\sum_i \min(y_i;\ \gamma) = R.$$

Let $y_1 < y_2 < \cdots < y_n$. We denote by

$$\gamma_i = y_i,\quad R_i = \sum_{j=1}^{i-1} y_j + \gamma_i[n - (i-1)],\ i = \overline{2, n}.$$

Note that $\{R_i\}$ - is an increasing sequence, hence if $R_{i-1} < R < R_i$, , then

$$x_j(y, R) = \begin{cases} y_j, & 1 \le j \le i-1 \\ \dfrac{R - \sum_{k=1}^{i-1} y_k}{n - (i-1)}, & j \ge i \end{cases}$$

those the resource is distributed according to the following procedure:

$$x_j(y, R) = \min(y_j\,;\gamma),$$

where

$$\gamma = \frac{R - \sum_{k=1}^{i-1} y_k}{n-(i-1)};$$

where so

$$z_i = \sum_{k=1}^{i} \frac{C(R_k) - C(R_{k-1})}{n-k+1},\ C(R_0) = 0,\quad C(R_n) = C(Y).$$

R - the mechanism of direct priorities. Consider three types of priority function $\eta(\cdot)$ - convex, linear and concave.

a) Convex priority functions. Let $\eta_i(y_i) = y_i^2$ and $y_i$ order in descending order and all are different, that is $y_1 > y_2 > \cdots > y_n$.

Denote by:

$$\gamma_i = \frac{1}{y_i},\ R_i = \sum_{j=1}^{i-1} y_j + \gamma_i \sum_{j=1}^{n} y_j^2.$$

It is easy to show that

$$z_i = y_i^2 \sum_{k=1}^{i} \frac{C(R_k) - C(R_{k-1})}{A_k^2},$$

where

$$A_k = \sqrt{\sum_{j=1}^{k} y_j^2},\qquad C(R_0) = 0,\qquad C(R_n) = C(Y).$$

For comparison, we note that the usual priority mechanism for allocating costs (2.2) with the same priority functions gives the following distribution of costs:

$$\tilde{z}_i = \frac{y_i^2}{A_1^2} \cdot C(Y)$$

It can be shown that an R-mechanism with convex priority functions gives a certain advantage to agents with high bids. More precisely, we have:

$$\sum_{k=1}^{i} z_k < \sum_{k=1}^{i} \tilde{z}_k,\quad i = \overline{1,(n)}$$

b) Linear priority functions. Let $\eta_i(y_i) = \sqrt{y_i}$, In this case

$$x_i = \pi_i(y, R) = y_i \cdot \min(1; \gamma) = \frac{y_i R}{Y},\qquad R \le Y.$$

and R - the mechanism is completely analogous to the usual priority mechanism with linear priority functions.

c) Concave priority functions. Let $\eta_i(y_i) = \sqrt{y_i}$ and $y_i$ order in ascending order and all are different, that is $y_1 < y_2 < \cdots < y_n$.

Denote

$$\gamma_i = \sqrt{y_i},\ R_i = \sum_{j=1}^{i-1} y_j + \gamma_i \sqrt{B_i}$$

by:

$$B_i = \left(\sum_{j=1}^{n} \sqrt{y_j}\right)^2,\ i = \overline{1, n}.$$

We have

$$z_i = \sqrt{y_i} \cdot \sum_{k=1}^{i} \frac{C(R_k) - C(R_{k-1})}{\sqrt{B_k}}$$

A conventional priority mechanism with the same priority functions is given by the cost distribution

$$\tilde{z}_i = \frac{\sqrt{y_i}}{\sqrt{B_i}} \cdot C(Y), \quad i = \overline{1, n}\,.$$

In this case, the R-mechanism gives an advantage to agents with smaller orders, that is, there is

$$\sum_{k=1}^{i} z_k < \sum_{k=1}^{i} \tilde{z}_k\,.$$

R - the mechanism of reverse priorities. Consider the priority functions $\eta_i(y_i) = {}^1/_{y_i}$. Let $y_i$ be ordered in ascending order and all are different, that is, $y_1 < y_2 < \cdots < y_n$.

Denote by:

$$\gamma_i = y_i\,, \qquad R_i = \sum_{j=1}^{k} y_j + \gamma_i \frac{1}{Q_i}\,, \text{где } Q_i = \left(\sum_{k=1}^{n} \frac{1}{y_i}\right)^{-1}\,, \quad i = \overline{2, n}\,.$$

We have

$$z_i = \frac{1}{y_i} \sum_{k}^{i} [C(R_k) - C(R_{k-1})] \cdot Q_k\,, \quad C(R_0) = 0\,, \; C(R_n) = C(Y)\,.$$

It is not possible to compare R - the mechanism of reverse priorities with usual priority mechanisms in this case, since the priority mechanism with decreasing priority functions does not satisfy the monotonicity condition. However, R - the mechanism of reverse priorities gives very serious advantages to agents with smaller applications. Namely, such agents pay for the same amount of resource less than agents with higher bids. This follows from relation

$$\pi_i(y, R) < \pi_j(y, R)\,, \quad \text{for all } i > j\,, \quad R < R_j\,.$$

Competitive mechanisms of resource allocation.

These mechanisms constitute a special class of priority mechanisms [1,2,3,4]. Agents are ranked by priority. The agent with the highest priority is in a sense a dictator. He gets the resource first. The remaining agents get the resource in descending order of priorities. The distribution of costs can be done in various ways. However, the following condition must be met: agent costs may depend only on his application and on applications of agents with a higher priority.

We confine with a description of the R - mechanism on the basis of competition, on the condition of anonymity. In this case, the agents are ordered in ascending order. Let $y_1 < y_2 < \cdots < y_n$. We denote by $Y_i = \sum_{j=1}^{i} y_i$.

In the literature, two mechanisms of cost allocation are considered based on the competition [1, 4]. In the first, the costs of the agent are determined by the expression:

$$z_i = C(Y_i) - C(Y_{i-1}), \quad Y_0 = 0\,, \quad i = \overline{1, n}\,.$$

(in the case of identical applications, costs are also taken equal).

In the second mechanism:

$$z_i = z_{i-1} + \frac{1}{n - i + 1}[C(ny_i) - C(ny_{i-1})]\,.$$

Obviously, in both cases:

$$\sum_{i=1}^{n} z_i = C(Y).$$

Multi-stage cost allocation mechanisms.

Let $C(Y)$ be a piecewise linear convex function of Y with breakpoints

$$R_k, k = \overline{1, \ell}, \quad \text{that is } C(Y) = C(R_{k-1}) + \lambda_k(Y - R_{k-1}), \; Y \in [R_{k-1}, R_k], \; \lambda_k \geq \lambda_{k-1}.$$

In this case it is natural to treat $\lambda_k$ as the price of the resource on the segment $[R_{k-1}, R_k]$. Let's consider mechanisms of distribution of expenses in which basis the stage-by-stage procedure of resource allocation lays. At the first stage, the resource is distributed in the quantity $\Delta_1 = R_1$ at the price $\lambda_1$, on the second - the resource in the quantity $\Delta_2 = R_2 - R_1$ at the price $\lambda_2$, etc., until at the next stage there are people wishing to receive the resource. At each stage, agents submit an application $s_{ik}$ on the resource they want to receive at this stage. Possible the different organization of multi-stage procedures. You can do several iterations at each stage, approaching the equilibrium situation at this stage. It is possible, on the contrary, at each stage to allow only one iteration (one message of applications), repeating the procedure after the orders at the next stage are zero.

Multistage mechanisms are attractive because they allow to apply the distribution procedures for a limited resource for cost allocation. Note that due to the increase in the resource price with the growth of the stage number, it is preferable for agents to receive a resource at an early stage. This fact further brings together a multi-stage procedure for allocating costs with procedures for allocating a limited resource. Indeed, we denote $\upsilon_{ik}$ ikoptimal number of resources for the $i-$ th agent at the price $\lambda_k$. Obviously, the goal of the $i-$ th agent at some stage is to get the resource in the amount of $\upsilon_{ik}$ (then at the subsequent stages the resource is no longer needed). Thus, gaming analysis of multi-stage procedures, in fact, breaks down into a phased analysis of the procedures for allocating a limited resource.

Two-prong mechanisms with the message of an estimation of effect or efficiency of distribution of expenses.

In cases where the authority allocating the resource (hereinafter referred to as its center) is able to obtain information on the actual effect of agents $\varphi_i(y_i)$ on the use of the resource $y_i$, the cost allocation can be carried out on the basis of two estimates - the required resource $y_i$ and the expected efficiency of its use $\xi_i$, where efficiency is understood as the ratio of the effect $\varphi_i(y_i)$ to the resource $y_i$. The fact that the center has information on the actual effect allows it to apply a system of sanctions (penalties and bonuses) in the case when the expected (or promised by the agent) effect $\xi_i \cdot y_i$ does not coincide with the actual one [2,3,4]. So, in the case of linear sanctions, the agent's objective function takes the form

$$f_i(y_i\,,\,\xi_i) = \varphi_i(y_i) - \alpha(\xi_i y_i - \varphi_i(y_i))$$

where $\alpha$ is the coefficient of penalty (premiums).

Often, sanctions are applied only in the form of fines in the case when the actual effect is lower than expected. In this case

$$f_i = \begin{cases} \varphi_i(y_i) - z_i\,, & \text{if } \xi_i y_i \le \varphi_i(y_i) \\ \varphi_i(y_i) - \alpha\big(\xi_i y_i - \varphi_i(y_i)\big) - z_i, & \text{if } \xi_i y_i > \varphi_i(y_i). \end{cases}$$

If $\alpha$ is so large that the excess of the expected effect over the actual is clearly unprofitable to the agent, we get a case of "heavy fines". Typically, cost allocation mechanisms that use performance ratings are arranged in such a way that the agent is interested in overestimating the estimate. With heavy penalties, such data manipulation is disadvantageous for agents and therefore the reported estimate is equal to $\xi_i = \frac{\varphi_i(y_i)}{y_i}$ [1,2].

All the above mechanisms of cost allocation can be applied in the case of two estimates. To do this, it is sufficient to make the priority function $\eta_i(\xi_i)$ dependent on the efficiency estimate $\xi_i$ (naturally, $\eta_i(\xi_i)$ increasing functions $\xi_i$). For two-pronged mechanisms, the condition of anonymity seems natural and fair, as the effectiveness estimates fully reflect the differences between agents.

## Conclusion

The models and mechanisms for allocating costs and incomes cover a broad class of applied problems related to the choice of financing schemes for investment projects, the distribution of income in corporate structures, the implementation of large social programs affecting federal and regional interests, etc. Many tasks in the work are only delivered and require additional research.

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