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Numerical Study of the Process of Unsteady Filtration of a Fluid in Interacting Porous Pressure Layers

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ABSTRACT
A review of the fundamental studies conducted in 2010 - 2020 is given in the article to develop a mathematical model related to the fluid and gas filtration processes in porous media. To conduct a comprehensive study of the process of unsteady filtration of fluid in multi-layer porous pressure media and to make a management decision, a mathematical model described by a system of partial differential equations with corresponding initial and boundary conditions and a conservative numerical algorithm were developed. On the basis of the developed software of the problem posed, computational experiments were conducted on a computer; the calculation results were presented in the form of tables and graphical objects. The schemes of location and capacity of vertical drainage wells to protect irrigated and non-irrigated areas from flooding were proposed on the basis of the developed software. Using the proposed mathematical tool, it is possible to obtain the prognostic groundwater levels for any area for the required period of time, considering a number of factors, for example, the formation heterogeneity in plan, the gradient of the permeability barrier, and other hydrogeological, hydro-technical, and natural conditions; to calculate the capacity and optimal drilling pattern of vertical drainage wells to protect the territory and to develop oil and gas fields.

Key words : Mathematical model, differential equation, numerical algorithm, filtration, porous medium, computational experiment.

1. INTRODUCTION

In the process of designing and developing multilayer oil reservoirs (as the oil reservoir is depleted), there arises the need to increase the efficiency of oil production, to substantiate and select optimal control actions, and to assess their technological efficiency. This requires comprehensive processing of all accumulated geological, geophysical, and field information on the structure and properties of the productive formation.

Technologists, geologists, and geophysicists accumulated significant volumes of information in the process of exploration, drilling of fields, research and operation of wells that characterize the structure, properties, and processes in oil formations. Integration of these data, their complex processing with adequate models and high-performance computers, reveals substantial reserves to increase the efficiency of field exploitation, prolonging their life cycle, including the optimal use of formation energy.

The issues of analyzing the complex technological systems functioning (including oil and gas fields) and their control come into the view of specialists in continuum mechanics and theory of automatic control of mathematical modeling and computer software.

These tasks are of great scientific and practical importance, and the mathematical and software support developed improves the efficiency of technological process control and optimizes calculations of oil and gas production indices.

Leading scientific centers and higher educational institutions worldwide conduct complex scientific research aimed at the development of mathematical models, effective algorithms, and software for solving problems of fluid filtration in a porous medium; sufficient results of a theoretical and applied nature were obtained. Some of such scientific centers that should be mentioned are Mineral Engineering at the Pennsylvania State University, the University of Texas at Austin, Missouri University of Science and Technology (USA), Robert Gordon University (UK), Institut Français du Pétrole (France), Polytechnic University of Turin (Italy), Technical University of Denmark (Denmark), East China University of Science and Technology (China), University of Petroleum & Energy Studies Dehradun (India), Petroleum Institute Abu Dhabi (UAE), Azerbaijan State University (Azerbaijan), Gubkin Russian State University of Oil and Gas (Russian Federation).

Many authors considered the problem of mathematical modeling of the process of filtration of liquids and gases in multilayer porous media and substantial theoretical and applied results were obtained.

In particular, [1] considers a mathematical model of the multiscale process of fluid filtration in periodic porous fabrics. The model is based on the three-dimensional Navier-Stokes equation with Carro-Yasuda viscosity using the asymptotic averaging method. A numerical algorithm was developed for solving local problems of non-Newtonian fluids in periodic cells, and the distribution of velocity, pressure, and viscosity in an individual pore was obtained. An algorithm for calculating the permeability tensor was developed, and the effects of fluid rheology were highlighted.

The study in [2] is devoted to the mathematical modeling of the displacement of a single-phase liquid in synthetic porous samples. The basis of the mathematical model used is the system of equations of poroelasticity in terms of the Biot model, which accounts for the processes of liquid filtration and the dynamics of changes in the stress-strain state of a continuous medium. A numerical algorithm and software were developed in that study to conduct a computational experiment on a computer.

A mathematical model was developed in [3] to study the process of gas filtration in a porous medium; the model was described by a nonlinear partial differential equation and the corresponding boundary and internal conditions. The article presents the main stages of building a mathematical model of the gas filtration process in porous media, taking into account changes in the hydrodynamic parameters of the object. To solve this problem, the following numerical methods were used: locally one-dimensional schemes and schemes of longitudinal-transverse direction. To solve the problem of nonlinear gas filtration in a porous medium, several methods for constructing an iterative process were tested.

In [4], a mathematical model for non-isothermal gas filtration was developed taking into account the phase transition. An algorithm to solve the problem was constructed, which allows calculating the key parameters of the heat effect in a hydrate-saturated reservoir, taking into account the dissociation process of gas hydrate.

In [5], a fractional-differential mathematical model of the process of anomalous geo-migration based on the MIM approach (movable-immovable medium) was used in engineering practice, where the process of modeling the object of research was simplified. When formulating the problem, two-dimensional initial-boundary conditions were posed for convective diffusion of soluble substances under vertical stationary filtration of groundwater with a free surface from the reservoir to the coastal drain. To simplify the modeling domain, the technique of transition to the region of complex flow potential through conformal mapping was used. In the case of averaging the filtration rate over the region of complex flow potential, an analytical solution to the problem under consideration was obtained. A numerical algorithm for solving the problem was developed in the article for the case of a variable filtration rate.

The study in [6] solves the problem related to the process of changing the groundwater level and mineral salts transfer in soils. The problem was described by a system of partial differential equations and corresponding initial, internal, and boundary conditions of various kinds. To build a mathematical model of the process under consideration, a detailed review of scientific research devoted to various aspects of the research object and its software was given. To conduct a comprehensive study of the filtration process and of the changes in the salt regime of groundwater, mathematical models and an effective numerical algorithm were proposed taking into account external sources and evaporation.

A mathematical model of filtration of two-component suspensions in a dual-zone porous medium is considered in [7]. The model consists of mass balance equations, kinetic equations for active and passive zones of the porous medium for each component of the suspension, and Darcy's law. To solve this problem, a numerical algorithm was developed to conduct computer experiments. On the basis of the numerical calculations performed, the main characteristics of suspension filtration in a porous medium were established. The influence of the model parameters on the transfer and deposition of suspended particles of a two-component suspension in porous media was analyzed. It was shown that the polydispersity of the suspension and the multistep nature of the deposition kinetics could lead to various effects that are not typical for the transport of one-component suspensions with one-stage particle deposition kinetics. In particular, in the distribution of the concentration of suspended particles in a moving fluid, non-monotonic dynamics was obtained at individual points of the medium.

The study in [8] is devoted to the study of suspension filtration in a porous medium with modified deposition kinetics. It is assumed that there are two types of deposition: reversible and irreversible deposition. A suspension filtration model in a porous medium consists of a mass balance equation and kinetic equations for each type of deposition. The model includes dynamic factors and kinetics of multistage deposition. In that article, the cases of higher dimension are reduced to the one-dimensional case using the symmetry of the porous medium. To solve this problem, a stable, efficient, and simple numerical algorithm based on the finite difference method was proposed. Sufficient conditions for the scheme stability were found. On the basis of numerical results, the influence of dynamic factors on the characteristics of the solid particles transfer and deposition was analyzed. It is shown that dynamic factors mainly affect the profiles of changes in the concentration of active zone deposits.

A solution related to the technological process of filtration and dehydration of liquid solutions from small particles and unwanted ionic compounds is discussed in [9]. A mathematical model and a numerical algorithm were developed taking into account key factors and parameters that substantially affect the process under consideration. The article resolves the problems associated with determining the key parameters and ranges of their change. Analyzing conducted numerical experiments, conclusions are formulated that serve as the basis for making appropriate management decisions.

In [10] a mathematical model of the process of filtration-consolidation of moisture-saturated microporous media is developed. Computational algorithms for its discretization are proposed. Explicit expressions of functional residual gradients are obtained for the identification of model parameters by the gradient methods.

To describe the dynamics of the relaxation process of convective diffusion under planned filtration, a mathematical model based on the fractional time derivative equation was proposed in [11]; a parallel computing algorithm based on a locally one-dimensional scheme was developed, and the results of the numerical implementation of the problem solution on a computer were presented.

In [11], a probabilistic model of the lattice Boltzmann and cellular automata (LB-CA) method was developed for modeling the filtration processes for four types of particles: non-circular fibers with rectangular, three-edged, four-leafed, and triangular cross-sections at a low Reynolds number. The article investigates the pressure drop and the rate of particle deposition in the filter pores in a diffusion-dominated mode when the Peclet number is from 235 to 1000. The authors of that article stated that the pressure drop of non-circular fibers depends on the orientation angle and aspect ratio. Diffusion collection efficiency is almost independent of orientation angle but is proportional to aspect ratio for all non-circular fibers.

The study in [13] considers moisture and heat transfer in porous media when a moist medium consists of solid and gaseous components, with distributed or localized heat sources. The modes of temperature change during heating of the materials of different initial moisture were investigated. With the developed mathematical model, moisture and heat transfer in a porous multicomponent medium with internal heat sources was investigated, taking into account heat transfer (when the coefficient of thermal conductivity changes), chemical reactions, drying, and humidification of solids.

A mathematical model of gas production from a reservoir, initially saturated with methanol and its hydrate, under conditions of negative (below 0°C) initial reservoir temperature is presented in [14]. An algorithm is proposed and a numerical scheme is constructed that allows the determination of the key parameters of non-isothermal filtration flow in a hydrate-saturated reservoir, taking into account the decomposition of the hydrate into gas and ice. The analysis of the influence of the mass flow rate of the extracted gas, the permeability of the porous medium, the initial reservoir temperature and the initial hydrate saturation on the mode, and rate of gas hydrate decomposition is performed.

In [15] a mathematical model, finite-difference schemes, and algorithms for calculating transient thermo- and hydrodynamic processes are developed during the commissioning of a unified system, that includes an oil production well, an electric pump submerged into a fractured-porous reservoir with near-bottom water. The mathematical model is implemented using the Oil-RWP software package, and the research results are visualized as graphical objects. An important feature of the package is its interaction with a special external program GCS, which simulates the operation of a ground electric control station and the data exchange between these two programs. Oil-RWP package transfers telemetry data and current parameters of the operating submersible unit to the GCS software module (direct connection). The station controller analyzes the incoming data and generates the necessary control parameters for the submersible pump. These parameters are sent to Oil-RWP (feedback). This approach allows considering the developed software as an “intelligent system of wells”.

The problem of modeling the dynamics of a locally in-equilibrium in time process of convective diffusion of soluble substances is considered in [16] under conditions of plane-vertical stationary filtration of groundwater with a free surface, taking into account the presence of interphase mass transfer. The urgency of solving this problem is related to the need to develop measures for soil washing, desalination, and purification of groundwater from pollutants. A fractional-order integro-differentiation apparatus was used for mathematical modeling of the corresponding transport process in media with the property of temporal nonlocality. The corresponding nonlinear fractional-differential model of the migration process was developed using the generalized Caputo-Katugampola fractional derivative of a function with respect to another function, which allows, in a sense, to control the modeling process.

In [17] inverse problems of recovering the coefficients of a linear pseudo-parabolic equation arising in the filtration theory were investigated. Boundary conditions of the Neumann type are supplemented by overdetermination conditions, which are the values of the solution at some interior points of the domain. In that paper, the theorems of existence and uniqueness of solution in Sobolev spaces are proved.

The problem related to the actual problem of the process of water and salt transfer in soil was solved in [18]. The article proposes a mathematical model for a comprehensive study of the process taking into account colmatation of soil pores with fine particles in time; change in soil permeability coefficient, fluid loss and filtration coefficient; changes in the initial porosity and the settled mass porosity, and an effective numerical algorithm based on the Samarsky-Fryazinov vector scheme with a second-order approximation. In the article, in deriving the mathematical model of salt transport it is assumed that the pressure gradient in the channel is constant and equal to atmospheric pressure. It was found that with insufficient sprinkling, the maximum water absorption and accumulated transfer of salts occur in the upper soil layers.

Numerical calculations have established that the change in the water transfer rate in soil depends on the porosity, soil permeability, filtration coefficient, soil composition and structure, and on the porosity of the settled mass. The salinization process has reached equilibrium after using saline water for irrigation for several years.

In [19], a mathematical model of thermo-hydrodynamic processes occurring in an oil reservoir and a horizontal wellbore was developed. On the basis of the proposed model and regularization methods, a computational algorithm for solving the inverse problem was proposed to determine the main parameters of the process under consideration. In the article, the data on temperature changes are taken as initial information, recorded simultaneously by several depth gauges installed in different places in the horizontal part of the wellbore.

In [20], a mathematical model of isothermal internal soil erosion was studied without consideration of the porous medium deformation. When a certain filtration rate is reached, soil particles are removed from the flow area. As a mathematical model of the problem, the equations of conservation of mass for water, of moving solid particles and stationary porous skeleton, and an analog of Darcy's law for water and moving solid particles and the ratio for the intensity of the suffusion aquifer are used. Moving soil particles are considered as a separate phase, the filtration rate of which is determined when solving the problem. This assumption allows the construction of a closed model. An algorithm is proposed for the numerical solution of the initial-boundary value problem of groundwater filtration, which takes into account internal soil erosion. Numerical test calculations were performed. The calculation results correlate well with the experimental data.

The process of water and oil filtration in a porous medium is studied in [21]. To research and predict the main indices of the process and to make management decisions, a mathematical model and a numerical algorithm were developed with the concepts of differential-difference schemes and the differential sweep method, on the basis of which computer experiments can be conducted. When modeling the object of research, the process of displacement of one fluid by another in a porous medium was considered.

The study in [22] deals with the problem related to the processes of filtration of liquid (water, oil) in multilayer interacting porous pressure media, predicting the groundwater level in any region for the required period of time, taking into account the heterogeneity of the reservoir in plan, the gradient of the permeability barrier, infiltration supply or evaporation, and other hydrogeological, hydro-technical and natural conditions, and the development of the optimal drilling pattern of vertical drainage wells to protect the territory and oil and gas fields development.

In [23] the problems related to the environmental protection, protection of groundwater from pollution sources, that is, the technology of ion-exchange filtration of liquid solutions from heavy ionic compounds emitted from production facilities, are considered; a mathematical model and its approximate analytical solution describing a nonlinear system are presented in partial differential equations.

A mathematical model of groundwater uptake, water accumulation and salt filtration is developed in [24], for a two-layer aquifer. A mathematical model of the problem of groundwater recovery in a two-layer aquifer is built. The research is of great practical importance since in some regions it is necessary to restore saline soils and create fresh water resources to change an environmentally unfavorable situation.

In [25], computer simulation of filtration processes based on the laws of hydrodynamics is considered. The adequacy of the developed models was verified by conducting a series of computational experiments. The developed mathematical and software support serves the purpose of research, forecasting and decision-making in the development and design of oil and gas fields.

A problem related to solving the problems of salt transfer in soils is considered in [26]; numerical modeling of the process of salt transfer and diffusion in soils is presented, in which the total soil porosity is divided into fractions of "through" and "dead-end" pores with the corresponding concentration of the solution. A brief overview of scientific publications devoted to this problem is given. To study and predict the spread of harmful substances, a mathematical model and a numerical algorithm were developed to conduct a computational experiment on a computer.

Determination of dynamic indices of production in oil and gas deposits development is associated with the solution of boundary value problems of non-stationary filtration of a three-phase mixture during the joint movement of oil, gas and water in the reservoir. Since small oil and gas fields, in most cases, are developed with a few production and injection wells randomly located in the reservoir, the filtration process cannot be systematized as a one-dimensional flow. Consequently, the determination of pressure fields and saturation of deposits with water, oil, and gas is one of the important and difficult tasks to be solved when drawing up technological schemes for the development of oil and gas deposits.Depending on the various combinations of fluid phases in a porous medium, different formulations of joint filtration of multiphase fluids can be considered.

Analyzing the results obtained above, the current article considers the process of fluid filtration in multi-layer interacting porous pressure layers when unsteady fluid inflows to the circular and linear gallery of wells.

2. PROBLEM STATEMENT

Let us consider the process of unsteady fluid inflow to circular well galleries in a three-layer bounded reservoir.

We will assume that there are galleries in each well-permeable aquifer.

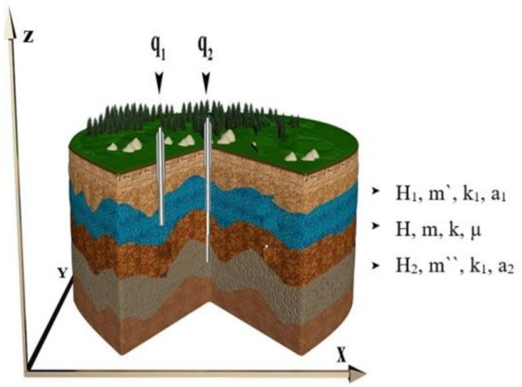

Figure 1: Schematic representation of a three-layer area of filtration

Let one battery of perfect vertical drains be drilled into the well-permeable layers, each with radii R_{c_1}, R_{c_2}. We assume that the three-layer formation is bounded in plan and has the shape of a circle with a radius R_s and it is impermeable. We assume that the wells on batteries are located and operated in such a way that their work can be replaced by the work of the gallery.

Then, due to radial symmetry, this problem is reduced to the integration of the system of partial differential equations with corresponding initial, boundary, and internal conditions, that is, the fluid flow in such a three-layer reservoir has the following form (accounting for the elastic regime in a low-permeable layer and the process of unsteady fluid flow to the central well):

$$\frac{1}{a_1}\frac{\partial H_1}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial H_1}{\partial r}\right)-\frac{k}{T_1}\frac{\partial H\left(r,m+m'',t\right)}{\partial z}\ , \quad (1)$$

$$\frac{1}{a}\frac{\partial H}{\partial t}=\frac{\partial^2 H}{\partial z^2}\ , \quad (2)$$

$$\frac{1}{a_2}\frac{\partial H_2}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial H_1}{\partial r}\right)+\frac{k}{T_2}\frac{\partial H\left(r,m'',t\right)}{\partial z} \quad (3)$$

Under following initial and boundary conditions:

$$H_1(r,0)=H_{10}\ ,\ \ H(r,z,0)=H_0\ ,\ \ H_2(r,0)=H_{20}\ , \quad (4)$$

$$\frac{\partial H_1(R_{c_1},t)}{\partial r}=\frac{Q_1}{2\pi T_1 R_{c_1}}\ , \quad (5)$$

$$\frac{\partial H_1(R_s,t)}{\partial r}=0\ , \quad (6)$$

$$H(r,m'',t)=H_2(r,t)\ ,\ \ H(r,m+m'',t)=H_1(r,t) \quad (7)$$

$$\frac{\partial H_2(R_{c_2},t)}{\partial r}=\frac{Q_2}{2\pi T R_{c_2}}\ , \quad (8)$$

$$\frac{\partial H_2(R_s,t)}{\partial r}=0\ , \quad (9)$$

where H_{10}, H_0, H_{20} are the given functions, $Q=Q_1+Q_2$ is the flow rate at the well, T_1, and T_2 are the conductivities of the upper and lower well-permeable layers, respectively, a_1 and a_2 are the piezo conductivity coefficients of well-permeable layers, k and a are the coefficients of filtration and piezo conductivity of a low-permeable layer, respectively, $m^{'}, m''$ are the thicknesses of the upper and lower layers, respectively.

3. SOLUTION METHOD

We introduce dimensionless quantities by the following formulas

$$\left.\begin{array}{l} H_1^*=\dfrac{H_1}{H_{xap}}, H_2^*=\dfrac{H_2}{H_{xap}}, H^*=\dfrac{H}{H_{xap}} \\ \xi=\dfrac{r}{R_K}, \eta=\dfrac{z}{m}, \xi_{\Gamma I}=\dfrac{R_{1\Gamma I}}{R_K}, \xi'_{\Gamma I}=\dfrac{R_{2\Gamma I}}{R_K} \\ \tau=\dfrac{Q_2}{R_K^2}t \end{array}\right\}$$

where $R_{\Gamma 1l}$ and $R_{\Gamma 2l}$ are the positions of the galleries in different layers. Then the system of equations (1) - (9) with initial and boundary conditions is rewritten as

$$\alpha\frac{\partial H_1^*}{\partial \tau}=\frac{1}{\xi}\frac{\partial}{\partial \xi}\left(\xi\frac{\partial H_1^*}{\partial \xi}\right)-\sigma_1\frac{\partial H^*\left(\xi,\dfrac{m+m''}{m},\tau\right)}{\partial \eta}\ , \quad (10)$$

$$\beta\frac{\partial H^*}{\partial \tau}=\frac{\partial^2 H^*}{\partial \eta^2}\ , \quad (11)$$

$$\frac{\partial H^*}{\partial \tau}=\frac{1}{\xi}\frac{\partial}{\partial \xi}\left(\xi\frac{\partial H_2^*}{\partial \xi}\right)+\sigma_2\frac{\partial H^*\left(\xi,\dfrac{m''}{m},\tau\right)}{\partial \eta} \quad (12)$$

$$H_{10}^*=\frac{H_{10}}{H_{char}}\ , \quad (13)$$

$$H_0^*=\frac{H_0}{H_{xap}}\ , \quad (14)$$

$$H_2^*=\frac{H_2}{H_{xap}}\ , \quad (15)$$

$$\frac{\partial H_1^*(0,\tau)}{\partial \xi}=0\ ,\ \frac{\partial H_1^*(1,\tau)}{\partial \xi}=0\ , \quad (16)$$

$$\frac{\partial H_1^*(\xi_{\Gamma I}+0,\tau)}{\partial \xi}-\frac{\partial H_2^*(\xi_{\Gamma I}-0,\tau)}{\partial \xi}=\frac{1}{\xi_{\Gamma I}}Q_{\Gamma 1l}^*\ ,\ \ l=1,2,...,\mathrm{N}_1\ , \quad (17)$$

$$H^*\left(\eta,\frac{m+m''}{m},\tau\right)=H_1^*\left(\eta,\tau\right)\ , \quad (18)$$

$$H^*\left(\eta,\frac{m''}{m},\tau\right)=H_2^*\left(\eta,\tau\right)\ , \quad (19)$$

$$\frac{\partial H_2^*(0,\tau)}{\partial \xi}=0\ ,\ \frac{\partial H_2^*(1,\tau)}{\partial \xi}=0\ , \quad (20)$$

$$\frac{\partial H_2^*(\xi'_{\Gamma I}+0,\tau)}{\partial \xi}-\frac{\partial H_2^*(\xi'_{\Gamma I}-0,\tau)}{\partial \xi}=\frac{Q_{\Gamma 2l}^*}{\xi'_{\Gamma I}}\ , \quad (21)$$

where

$$\left.\begin{aligned}\sigma_i&=\frac{KR_K^2}{T_i m}\\ Q_{il}^*&=\frac{Q_{i\Gamma l}}{2\pi T_i H_{char}}\end{aligned}\right\},\quad i=1,2\ .$$

$Q_{i\Gamma l}$ - is the flow rate at the well , $R_{i\Gamma l}$, $\alpha=\frac{a_2}{a_1}$, $\alpha=\frac{a_2}{a}\frac{m^2}{R_K^2}$,

solution of H_1^* and H_2^* are given in the forms:

$$H_1^*=U-\varphi_1(\xi)\,, \tag{22}$$

$$H_2^*=V-\varphi_2(\xi)\,, \tag{23}$$

where

$$\varphi_1(\xi)=\frac{1}{2}\xi^2\sum_{l=1}^{N_1}Q_{1l}^*-\sum_{l=1}^{N_1}\alpha_{1l}Q_{1l}^*\ln\frac{\xi}{\xi_{\Gamma l}}\,,$$

$$\varphi_2(\xi)=\frac{1}{2}\xi^2\sum_{l=1}^{N_2}Q_{2l}^*-\sum_{l=1}^{N_2}\alpha_{2l}Q_{2l}^*\ln\frac{\xi}{\xi'_{\Gamma l}}\ ,\quad \alpha_{1l}=\begin{cases}0, & 0\le\xi<\xi_{\Gamma l}\\ 1, & \xi_{\Gamma l}\le\xi\le 1\end{cases},$$

$$\alpha_{2l}=\begin{cases}0, & 0\le\xi<\xi'_{\Gamma l}\\ 1, & \xi'_{\Gamma l}\le\xi\le 1\end{cases},$$

Then (10) - (21) under condition (22), is written as

$$\alpha\frac{\partial U}{\partial\tau}=\frac{1}{\xi}\frac{\partial}{\partial\xi}\left(\xi\frac{\partial U}{\partial\xi}\right)-$$
$$-\sigma_1\left[(U-V)+(\varphi_1-\varphi_2)+\delta\frac{\partial W\left(\xi,\frac{m+m''}{m},\tau\right)}{\partial\xi}\right]-2\sum_{e=1}^{n_1}Q_{1l}^* \tag{24}$$

$$\frac{\partial W}{\partial\tau}=\frac{1}{\beta}\frac{\partial^2 W}{\partial\eta^2}-\left(1-\eta+\frac{m''}{m}\right)\frac{\partial V}{\partial\tau}-\left(\eta-\frac{m''}{m}\right)\frac{\partial U}{\partial\tau}\,, \tag{25}$$

$$\alpha\frac{\partial V}{\partial\tau}=\frac{1}{\xi}\frac{\partial}{\partial\xi}\left(\xi\frac{\partial V}{\partial\xi}\right)+$$
$$+\sigma_2\left[(U-V)-(\varphi_1-\varphi_2)+\delta\frac{\partial W\left(\xi,\frac{m''}{m},\tau\right)}{\partial\eta}\right]-2\sum_{e=1}^{n_2}Q_{2l}^*, \tag{26}$$

$$U(\xi,0)=H_{10}^*+\varphi_1(\xi)\,, \tag{27}$$

$$\frac{\partial U(0,\tau)}{\partial\xi}=0\ ,\ \frac{\partial U(1,\tau)}{\partial\xi}=0\,, \tag{28}$$

$$W(\xi,\eta,0)=H_0^*-\left(\eta-\frac{m''}{m}\right)H_{10}^*-\left(1-\eta+\frac{m''}{m}\right)H_{20}^*\,, \tag{29}$$

$$W\left(\xi,\frac{m''}{m},\tau\right)=0\ ,\ W\left(\xi,\frac{m+m''}{m},\tau\right)=0\,, \tag{30}$$

$$V(\xi,0)=H_{20}^*+\varphi_2(\xi)\,, \tag{31}$$

$$\frac{\partial V(0,\tau)}{\partial\xi}=0\ ,\ \frac{\partial V(1,\tau)}{\partial\xi}=0\,, \tag{32}$$

Thus, system (24) - (26) with conditions (27) - (32) coincides with the problem considered in [22]. Therefore, the algorithm described in that article, is suitable here if $Q_i^*=0$, $(i=1,2)$, and the region of continuous variation of the argument $\eta_c\le\eta\le 0$, $0\le\tau\le T<\infty$ is replaced with a finite set of points (the grid) with the following coordinates $\{\eta_i=\eta_c+i\Delta\eta,\tau_j=j\Delta\tau, i=0,1,2,...,I;\ j=1,2,...\}$, where $\Delta\eta$, $\Delta\tau$ are the grid steps.

The derivatives entering the problem posed in (24) - (32), are replaced by difference relations, then we obtain the finite-difference equation [27,28]

$$a_i u_{i-1}-b_i u_i+c_i u_{i+1}+d_i v_i=-f_i\,, \tag{33}$$

$$a_i' v_{i-1}-b_i' v_i+c_i' v_{i+1}+d_i' u_i=-f_i'\,, \tag{34}$$

$$i=1,2,...,I-1\,.$$

Coefficients $a_i,b_i,c_i,d_i,f_i,a_i',b_i',c_i',d_i',f_i'$ are determined by the following formulas:

$$a_i=\frac{\Delta\tau}{\alpha\Delta\xi}\left(\frac{1}{\Delta\xi}-\frac{1}{2\xi_i}\right),\ b_i=\frac{2\Delta\tau}{\alpha(\Delta\xi)^2}+1+\frac{\sigma_1}{m}\frac{\Delta\tau}{\alpha}+\delta\frac{a}{2\alpha\Delta\eta}\,,$$

$$c_i=\frac{\Delta\tau}{\alpha\Delta\xi}\left(\frac{1}{\Delta\xi}+\frac{1}{2\xi_i}\right)\ ,\ d_i=\frac{\sigma_1\Delta\tau}{\alpha m}-\delta\frac{\sigma_1 b}{2\alpha\Delta\eta}\,,$$

$$f_i=\hat{U}_i+\frac{\sigma_1}{2\alpha\Delta\eta}\left(a\hat{U}_i+b\hat{V}_i-c\Delta\tau\right)-2\Delta\tau\sum_{l=1}^{N_1}Q_{1l}^*+\frac{\sigma_2}{\alpha}(\varphi_{1i}-\varphi_{2i})\,,$$

$$a_i'=\frac{\Delta\tau}{(\Delta\xi)}\left(\frac{1}{\Delta\xi}-\frac{1}{2\xi_i}\right),\ b_i'=\frac{2\Delta\tau}{\alpha(\Delta\xi)^2}+1+\frac{\sigma_2}{m}\Delta\tau+\delta\frac{\sigma_2 a}{2\Delta\eta}\ ,$$

$$c_i'=\frac{\Delta\tau}{(\Delta\xi)}\left(\frac{1}{\Delta\xi}+\frac{1}{2\xi_i}\right),\ d_i'=\frac{\sigma_2\Delta\tau}{m}\Delta\tau-\delta\frac{\sigma_2 b}{2\Delta\eta}\ ,$$

$$f_i'=\hat{V}_i+\delta\frac{\sigma_2}{2\Delta\eta}\left(a\hat{V}_i+b\hat{U}_i-\bar{c}\Delta\tau\right)-2\Delta\tau\sum_{l=1}^{N_2}Q_{2l}^*+\sigma_1(\varphi_{2i}-\varphi_{1i})\,.$$

4. RESULTS

On the basis of the above algorithms for solving problems, a software tool has been developed to conduct a comprehensive study of the process of unsteady inflow to linear and circular galleries. The results of the performed numerical calculations are shown in Figs. 1-9 and Tables 1-2.

To illustrate the above algorithm, consider examples with the following reservoir parameters

$$a_1=a_2=1000000\,m^2/day,\ \ a=100\,m^2/day,\ \ m'=m''=100meters\,,$$
$$m=10meters,\ \ K=0,01meter/day\ ,\ \ T_1=T_2=1000\,m^2/day,\ .$$

With unsteady inflow to the central well $S^*=\frac{T(H-H_0)}{10QR_K}$, $R_c=0,1meters$, $R_K=2000meters$ with/without taking into account the elastic regime.

Figures 1 - 2 and Tables 1 - 2 show the decrease in pressure in the formations along the length of the filtration layers without taking into account the elastic regime.

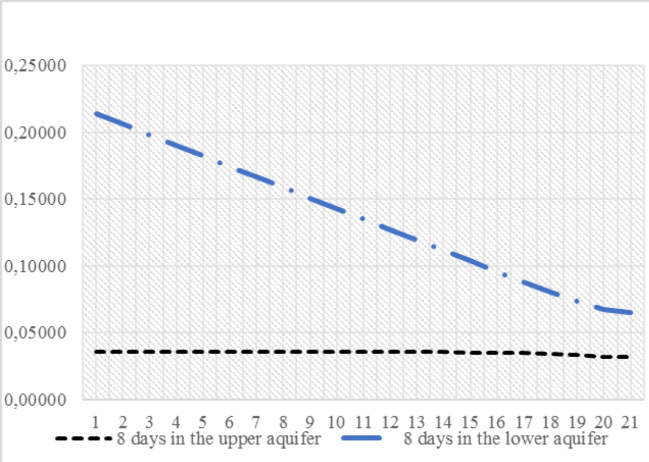

Figure 1: Changes in pressure in the layers along the length of filtration layers without taking into account the elastic regime.

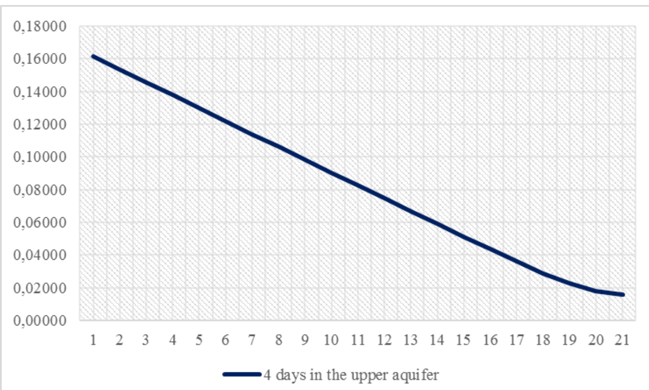

Figure 2: Changes in pressure in the upper filtration layer without taking into account the elastic regime.

Table 1: Table of overflows and balance relations in the upper aquifer

$$\bar{Q} = Q^* \frac{R_c}{R_K} = \frac{1}{20\pi}$$

	Time in days	$\bar{Q}\tau$	$\frac{\partial H_{av}}{\partial z}\tau$	$\frac{\partial W_{av}}{\partial z}\tau$	U_{av}	ΔU_{av}
1	0.50	0.00000	-0.00080	0.00000	0.00082	0.00002
2	1.00	0.00000	-0.00234	0.00000	0.00243	0.00009
3	1.50	0.00000	-0.00417	0.00000	0.00437	0.00019
4	2.00	0.00000	-0.00614	0.00000	0.00644	0.00030
5	2.50	0.00000	-0.00815	0.00000	0.00856	0.00041
6	3.00	0.00000	-0.01018	0.00000	0.01070	0.00052
7	3.50	0.00000	-0.01222	0.00000	0.01285	0.00062
8	4.00	0.00000	-0.01427	0.00000	0.01500	0.00073
9	4.50	0.00000	-0.01631	0.00000	0.01715	0.00084
10	5.00	0.00000	-0.01836	0.00000	0.01930	0.00095
11	5.50	0.00000	-0.02040	0.00000	0.02146	0.00106
12	6.00	0.00000	-0.02245	0.00000	0.02361	0.00116
13	6.50	0.00000	-0.02449	0.00000	0.02577	0.00127
14	7.00	0.00000	-0.02654	0.00000	0.02792	0.00138
15	7.50	0.00000	-0.02859	0.00000	0.03007	0.00149
16	8.00	0.00000	-0.03063	0.00000	0.03223	0.00160
17	8.50	0.00000	-0.03268	0.00000	0.03438	0.00170

Table 2: Table of overflows and balance relations in lower aquifer.

$$\bar{Q} = Q^* \frac{R_c}{R_K} = \frac{1}{20\pi}$$

	Time in days	$\bar{Q}\tau$	$\frac{\partial H_{av}}{\partial z}\tau$	$\frac{\partial W_{av}}{\partial z}\tau$	U_{av}	ΔU_{av}
1	0.50	-0.00398	0.00080	0.00000	0.00321	0.00004
2	1.00	-0.00796	0.00234	0.00000	0.00583	0.00021
3	1.50	-0.01194	0.00417	0.00000	0.00818	0.00042
4	2.00	-0.01592	0.00614	0.00000	0.01042	0.00064
5	2.50	-0.01989	0.00815	0.00000	0.01260	0.00086
6	3.00	-0.02387	0.01018	0.00000	0.01477	0.00108
7	3.50	-0.02785	0.01222	0.00000	0.01693	0.00130
8	4.00	-0.03183	0.01427	0.00000	0.01909	0.00152
9	4.50	-0.03581	0.01631	0.00000	0.02124	0.00174
10	5.00	-0.03979	0.01836	0.00000	0.02340	0.00196
11	5.50	-0.04377	0.02040	0.00000	0.02555	0.00218
12	6.00	-0.04775	0.02245	0.00000	0.02770	0.00218
13	6.50	-0.05173	0.02449	0.00000	0.02986	0.00263
14	7.00	-0.05570	0.02654	0.00000	0.03201	0.00285
15	7.50	-0.05968	0.02859	0.00000	0.03417	0.00307
16	8.00	-0.06366	0.03063	0.00000	0.03632	0.00329
17	8.50	-0.06764	0.03268	0.00000	0.03847	0.00351

Analysis of the numerical calculations performed on a computer showed (Figs. 1, 2 and Tables 1, 2) that in the upper aquifer the pressure does not change substantially over time, it remains almost constant, and the water pressure in the lower aquifer decreases linearly with time. This is seen from the curves given in Fig. 2.

Figures 3 - 10 show the interactions between the upper and lower aquifers at unsteady fluid filtration and fluid inflow to the central well.

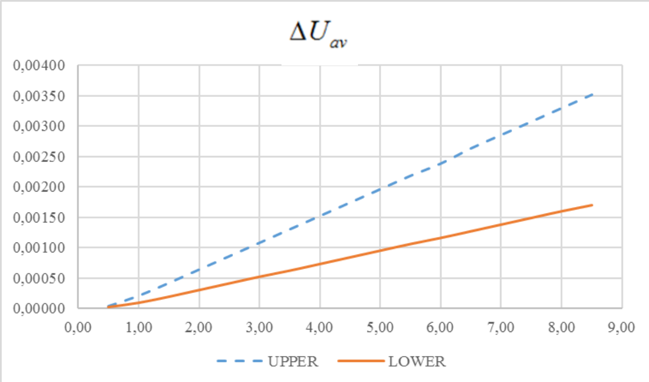

Figure 3: Change in the average pressure in the upper and lower aquifers

As follows from the analysis of the numerical calculations conducted on a computer, the interaction between the layers depends substantially on the hydrodynamic parameters of the layers, that is, on the conductivity of the upper and lower well-permeable layers, the coefficients of piezo conductivity of well-permeable and low-permeable layers, respectively.

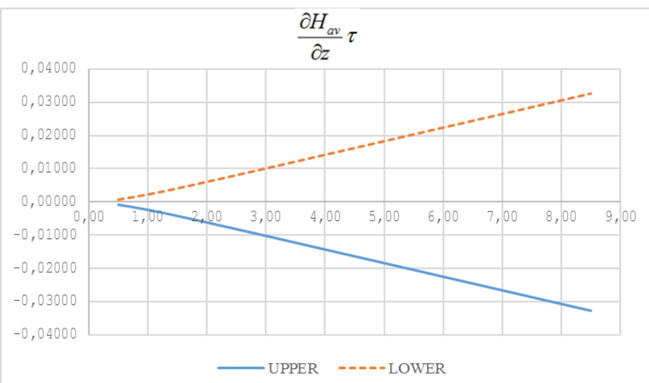

Figure 4: Average change in pressure gradient in the upper and lower aquifers

From the curves in Figs. 6, 8 it is seen that the balance ratio and the overflow in the upper and lower aquifers depend on the hydrodynamic parameters of the middle layer and the value of the pressure in the layers. Numerical experiments conducted have shown that the overflow and outflow between the layers depend on the coefficients of piezo conductivity of the layers.

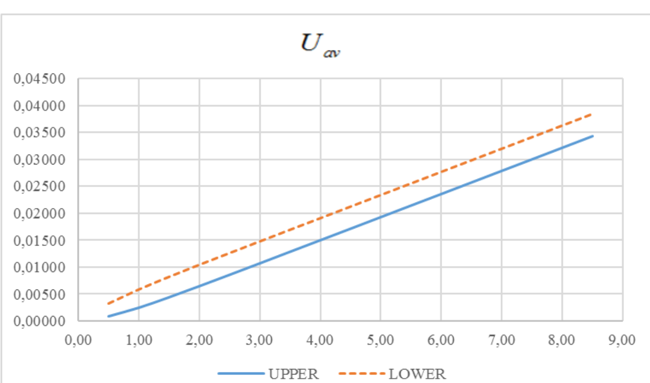

Figure 5: Average overflow rate in the upper and lower aquifers

As seen from Fig. 7, the average change in the gradient of the filtration rate in the upper and lower aquifers grows linearly with time. As follows from the numerical calculations (Fig. 9), the overflow rate in the upper layer is lower than the rate in the lower aquifer.

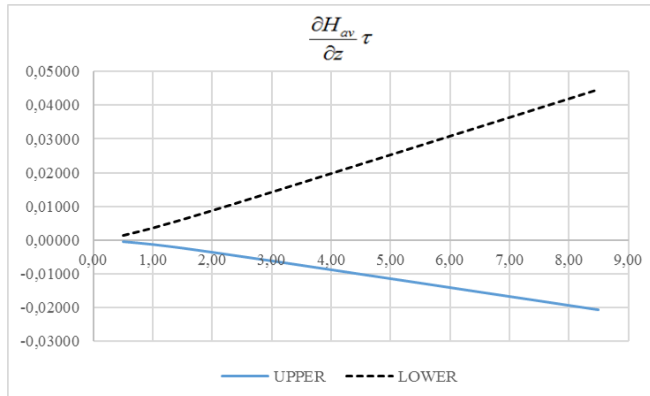

Figure 6: Balance ratio and overflow in the upper and lower aquifers

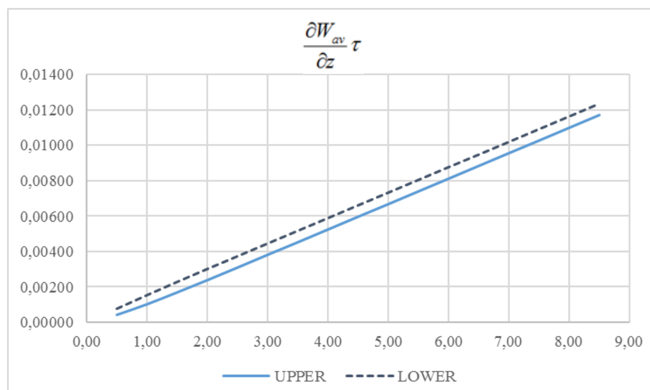

Figure 7: Average change in the gradient of the filtration rate in the upper and lower aquifers

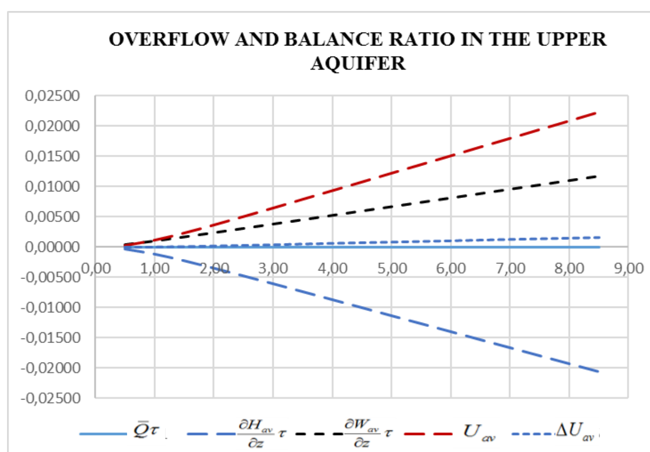

Figure 8: Overflow and balance ratio in the upper aquifer

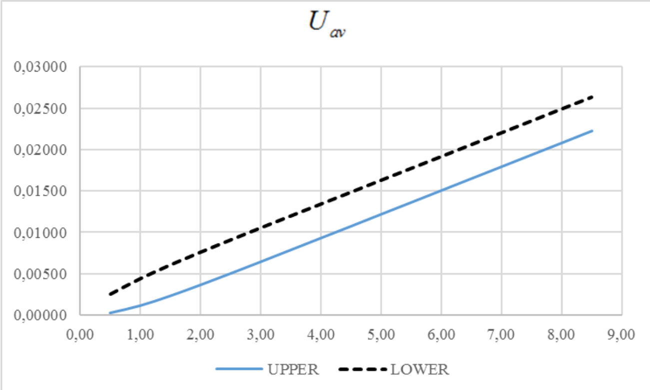

Figure 9: Average overflow rate in the upper and lower aquifers

5. CONCLUSION

The analysis of the numerical calculations performed for various values of the hydrodynamic parameters of the process showed that the overflow through the boundary between the filtration layers of fluids substantially depends on the conductivity of the upper and lower well-permeable layers, the piezoconductivity coefficients of well-permeable layers, and on the filtration coefficient and piezoconductivity of the low-permeable layer, respectively.

To perform a comprehensive study of the process of fluid filtration in multilayer interacting pressure layers, a mathematical model and a numerical algorithm were developed, which can be used to conduct computational experiments on a computer. The results of the numerical calculations of the problems made it possible to establish the degrees of influence of the elastic filtration regime in a low-permeable layer without overflow into adjacent layers.
Based on the developed mathematical tool, it is possible to propose schemes for the location and capacity of vertical drainage wells to protect irrigated and non-irrigated areas from flooding. Using the proposed mathematical tool, it is possible to obtain the predicted groundwater levels for any area for the required period of time, taking into account a number of factors, such as the heterogeneity of the reservoir in plan, the gradient of the permeability barrier, infiltration recharge or evaporation and other hydrogeological, hydro-technical and natural conditions; to calculate the capacity and optimal drilling pattern of vertical drainage wells to protect the territory and the development of oil and gas fields

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