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WORKFLOW SPECIFICATION AND INFERENCE IN SOME EXTENSION OF ALLEN'S INTERVAL LOGIC

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Abstract: *A workflow is an automation of a process, in which agents (people or programs) are involved in activities for solving a set of tasks in order to attain a common goal. The concept of workflow appeared in business informatics. Currently, the workflow techniques are used in many other fields of informatics (medical and bioinformatics, organization of scientific researches, computer-aided design and manufacturing, robotics et al,) Many methods and formalisms were applied for specifying workflows. Specific logical languages were used for this. In particular, temporal logics are popular as workflow specification formalisms. Allen's interval logic is the simplest temporal logic, but only a few kinds of qualitative properties can be specified for workflows, We define a metric extension of Allen's interval logic and show how to use it for specifying workflows. We construct an inference method for this formalism. The method is based on the analytic tableaux techniques. We also show how to use the inference method for query answering over workflows schemas and their states.*

Keywords: *systems specification, workflows, temporal logics, Allen's interval logic, inference methods, query answering*

Introduction. Main definitions

A *workflow* is an automation of a process, in which agents (people or programs) are involved in activities for solving a set of tasks in order to attain a common goal. The concept of workflow has appeared in business informatics. Currently, the workflow techniques are used in many other fields of informatics, such as medical informatics, bioinformatics (in particular, genomics, automation of scientific research, multi-agent systems, computer-aided design and manufacturing, etc. Workflows are applied to the problems of designing Web services [5, 7, 9]. The concept of workflow is the central in Process-Aware Information Systems (PAIS) [6]. Many methods and formalisms were applied for specifying workflows. In particular, specific logics were used [8].

In 1983, J.A. Allen published the seminal paper "Maintaining knowledge about temporal intervals", where he has proposed a simple temporal logic formalism [2]. Allen's interval logic and its extensions were applied to various problems of intelligent information systems designing (knowledge representation, common sense reasoning, natural language understanding, actions planning, ontology modeling et al.; see, for example [3,4]).

Table 1

<table>
<tr><th>Relation</th><th>Illustration</th><th>Inequalities and equalities</th></tr>
<tr><td>b (before)</td><td>|==A==| |==B==|</td><td>$A^+ < B^-$</td></tr>
<tr><td>m (meets)</td><td>|==A==|==B==|</td><td>$A^- = B^+$</td></tr>
<tr><td>s (starts)</td><td>|==A==|
|=====B=====|</td><td>$A^- = B^-$, $A^+ < B^+$</td></tr>
<tr><td>f (finishes)</td><td>|==A==|
|=====B=====|</td><td>$A^- < B^-$, $A^+ = B^+$</td></tr>
<tr><td>d (during)</td><td>|==A==|
|=====B=====|</td><td>$B^- < A^-$, $A^+ < B^+$</td></tr>
<tr><td>o (overlaps)</td><td>|===A===|
|===B===|</td><td>$A^- < B^-$, $B^- < A^+$, $A^+ < B^+$</td></tr>
</table>

e (equals)	|=====*A*=====| |=====*B*=====|	$A^-=B^-$, $A^+=B^+$

In Allen's interval logic (let us denote it by **AL**), there are 7 basic interval relations: ***e*** (equals), ***b*** (before), ***m*** (meets), ***o*** (overlaps), ***f*** (finishes), ***s*** (starts), ***d*** (during). The sense of the basic interval relations is shown in Table 1. Here, A^- and A^+ (B^- and B^+) denote the beginning and the ending of the interval *A* (interval *B*). It is assumed that A^-, A^+, B^- and B^+ are nonnegative integers with $A^-<B^+$ and $B^-<B^+$.

Let Ω = {***b***, ***m***, ***s***, ***f***, ***d***, ***o***, ***e***, ***b****, ***m****, ***s****, ***f****, ***d****, ***o****} where asterisks denote inversion of relations (i.e. $X\ \theta^* Y \Leftrightarrow Y\ \theta\ X$). An *elementary* **AL** *sentence* has the form $A\ \theta\ B$ where $\theta \in \Omega$. Let "•" be any assignment nonnegative numbers to the ends of the intervals *A* and *B* such that "A^-"<"A^+" and "B^-"<"B^+". Then the assignment defines the interpretation "•" of elementary **AL** sentences as follows:

"*A* ***b*** *B*"=1 ⇔ "B^+"<"A^-" ⇔ "B^-"–"A^-" ≥ 1, "*A* ***m*** *B*" = 1 ⇔ "B^+"="A^-" ⇔ "B^-"–"A^-" ≥ 0,
"*A* ***s*** *B*" = 1 ⇔ ("A^-"="B^-") ∧ ("A^+"<"B^+") ⇔ ("B^-"–"A^-"≥ 0) ∧ ("B^+"–"A^+"≥ 1) et al.

An arbitrary **AL** *sentence* has the form *A*ω*B* where ω⊆Ω. For example, *A*{***f****,***s***,***d***}*B* is **AL** sentence which is written briefly as *A* ***f*sd*** *B*. The sentence *A* ω *B* is interpreted as a disjunction of elementary sentences entering it. For example,

"*A* ***f*s*** *B*"="*A* ***f****B*" ∨ "*A* ***s*** *B*" ∨"*A* ***d*** *B*"=1
⇔ ("B^-"–"A^-"≥1) ∧ ("A^+"–"B^+≥ 0) ∧
("B^+"–"A^+"≥ 0) ∨ ("A^-"–"B^-"≥ 0) ∧
("B^-"–"A^-"≥ 0) ∧ ("B^+"–"A^+"≥ 1).

Let **BAL** denote the Boolean extension of **AL**. Its sentences are obtained from **AL** statements by applying Boolean operations ~, ∧, ∨ and →. We also introduce the metrical extension $\mathbf{AL_m}$ and Boolean metric extensions $\mathbf{BAL_m}$ of Allen's interval logic. An *elementary* $\mathbf{AL_m}$ *sentence* has the form *A* θ{σ} *B* and σ where σ is a constraint on ends of the intervals *A* and *B*. Such a constraint is a conjunction of inequalities and equalities of the forms $X–Y \le r$, $X–Y \ge r$ or $r \le X–Y \le s$ where *r*, *s* ∈ ℕ and *X*, *Y* ∈ {A^-,A^+,B^-,B^+}. The sentence *A* θ{σ} *B* is interpreted as "*A* θ *B*" ∧ "σ". Here is an example of a elementary $\mathbf{AL_m}$ sentence:

A ***d***{2≤A^-–B^-≤5; A^+–A^-≥ 3; B^+–A^-= 2} *B*.

It is interpreted as
("A^-"–"B^-"≥ 1) ∧ ("B^+"–"A^-"≥ 2) ∧
(2≤ "A^-"–"B^-"≤ 5) ∧ ("A^+"–"A^-"≥ 3) ∧
("B^+"–"A^-"= 2)
which is equivalent to
("A^-"–"B^-"≥ 2) ∧ ("B^-"–"A^-"≥–5) ∧
("A^+"–"A^-"≥ 3) ∧ ("B^+"–"A^-"≥ 2) ∧
("A^-"–"B^+"≥ –2).

An arbitrary $\mathbf{AL_m}$ sentence is obtained by joining elementary $\mathbf{AL_m}$ sentences and is interpreted as the disjunction of the interpreted elementary sentences. For example, we have

A ***d***{2≤A^-–B^-≤5; A^+–A^-≥3; B^+–A^-= 2}
b{B^-– A^+≥ 2; A^+ – A^-≤ 4} *B*" = 1 ⇔
"*A* ***d***{2≤A^-–B^-≤ ; A^+–A^-≥ ; B^+– A^-= 2} *B*"
∨ "*A* ***b***{B^-– A^+≥ 2; A^+ – A^-≤ 4} *B*" = 1 ⇔
("A^-"–"B^-"≥2) ∧ ("B^-"–"A^-"≥–5) ∧
("A^+"–"A^-"≥ 3) ∧ ("B^+"–"A^-"≥2) ∧
("A^-"–"B^+"≥–2) ∨
("B^-"– "A^+"≥2 ∧ ("A^-"–"A^+"≥–4).

An arbitrary $\mathbf{BAL_m}$ sentence is obtained by Boolean operations ~, ∧, ∨ and → applied to $\mathbf{AL_m}$ sentences.

From the above mentions examples, it easy to understand that the following proposition is true.

Proposition 1. Let "•" be a given assignment non-negative integers to the ends of intervals *A* and *B*. Then every $\mathbf{AL_m}$ sentence about the intervals *A* and *B* is interpreted as a disjunctive normal form with atoms of the form "*X–Y*" ≥ *r* where we set "*X–Y*" = "*X*"–"*Y*".

We call **AL**-*ontology* ($\mathbf{AL_m}$-*ontology* and $\mathbf{BAL_m}$-*ontology*) **a**ny finite set ***O*** of **AL** sentences ($\mathbf{AL_m}$ and sentences).

In this paper we consider how the logics **BAL** and $\mathbf{BAL_m}$ can be applied to workflow modeling problems. In particular, we can specify: (*i*) the order of tasks performance in a given workflows by sentences with the relations ***b*** and *m*; (*ii*) qualitative relations between tasks by arbitrary **BAL** sentences; and (*iii*) quantitative constraints by $\mathbf{BAL_m}$ sentences.

Thus, in a result of such a specification we have some $\mathbf{BAL_m}$-ontology ***O***. Then we can to query answering over the ontology ***O***. For this the logical inference, based on the analytical tableaux, is used. We can used the inference also for proving inconsistence of the ontology ***O***.

1. Tableaux inference for the logic BAL

The analytical tableau inference methodology has been invented by Beth and Hintikka in 1950s. Later the methodology was perfected by Smullyan and Fitting. Today the tableaux inference methods is widely applied in AI systems [2].

In Table 2 and 3, the tableaux inference rules for the logic **BAL** is presented.

Let us consider, by example, how to apply these rules for proving logical consequences.

Example 1. Let us take the ontology ***O*** = {~ (*A* ***m**** *C* → ~ *A* ***bo*** *B*), *C mf A* → *C* ***b**** *B*} and the sentence *A* ***b*** *C*. For proving that *A* ***b*** *C* is the logical consequence of ***O*** we suppose that in some interpretation both sentences of ***O*** are true but the sentence *A* ***b*** *C* is false. This means that the set of formulas E = {+~ (*A* ***m**** *C* → ~ *A* ***bo*** *B*), +*C mf A* → *C* ***b**** *B*, –*A* ***b*** *C*} is inconsistent. Here the sign "plus" ("minus") denotes that a sentence is true (false).

Table 2

Number	***Antecedent***	***Consequents***
1	+ ~ φ	– φ
2	– ~ φ	+ φ
3	+ φ ∧ ψ	+ φ and + ψ
4	– φ ∧ ψ	– φ or – ψ
5	+ φ ∨ ψ	+ φ or + ψ
6	– φ ∨ ψ	– φ and – ψ
7	+ φ → ψ	– φ or + ψ
8	– φ → ψ	+ φ and – ψ

Table 3

Number	***Antecedent***	***Consequents***
1	+*A* ***b*** *B*	$B^{-} - A^{+} \geq 1$
2	–*A* ***b*** *B*	$A^{+} - B^{-} \geq 0$
3	+*A* ***m*** *B*	$A^{+} - B^{-} \geq 0$ and $B^{-} - A^{+} \geq 0$
4	–*A* ***m*** *B*	$B^{-} - A^{+} \geq 1$ or $A^{+} - B^{-} \geq 1$
5	+*A* ***s*** *B*	$A^{-} - B^{-} \geq 0$ and $B^{-} - A^{-} \geq 0$ and $B^{+} - A^{+} \geq 1$
6	–*A* ***s*** *B*	$B^{-} - A^{-} \geq 1$ or $A^{-} - B^{-} \geq 1$ or $B^{+} - A^{+} \geq 1$
7	+*A* ***f*** *B*	$B^{-} - A^{-} \geq 1$and $A^{+} - B^{+} \geq 0$ and $B^{+} - A^{+} \geq 0$
8	– *A* ***f*** *B*	$A^{-} - B^{-} \geq 0$ or $B^{+} - A^{+} \geq 1$ or $A^{+} - B^{+} \geq 0$
9	+ *A* ***d*** *B*	$A^{-} - B^{-} \geq 1$ and $B^{+} - A^{+} \geq 1$
10	– *A* ***d*** *B*	$B^{-} - A^{-} \geq 0$ or $A^{+} - B^{+} \geq 0$
11	+*A* ***o*** *B*	$B^{-} - A^{-} \geq 1$ and $A^{+} - B^{-} \geq 1$ and $B^{+} - A^{+} \geq 1$
12	–*A* ***o*** *B*	$A^{-} - B^{-} \geq 0$ or $B^{-} - A^{+-} \geq 0$ or $A^{+} - B^{+} \geq 0$
13	+*A* ***e*** *B*	$B^{-} - A^{-} \geq 0$ and $A^{-} - B^{-} \geq 0$ and $B^{+} - A^{+} \geq 0$ and $A^{+} - B^{+} \geq 0$
14	– *A* ***e*** *B*	$B^{-} - A^{-} \geq 1$ or $A^{-} - B^{-} \geq 1$ or $B^{+} - A^{+} \geq 1$ or $A^{+} - B^{+} \geq 1$
15	+*A* θω *B*	+*A* θ *B* or +*A* ω *B*
16	– *A* θω *B*	– *A* θ *B* and – *A* ω *B*
17	+ *A* θ**B*	+*B* θ *A*
18	– *A* θ**B*	–*B* θ *A*

Figure 1 shows the inference tree that has been constructed by applying the rules from Tables 2 and 3. The set E is the initial branch of the tree. There are labels for nodes of the tree. For example, the label '[1]T2(1)' is associated with the first node of the tree. This means that at step 1 the rule, placed in Table 2 at the first row, was applied. As the result of the application, the sentence –*A* ***m**** *C* → ~ *A* ***bo*** *B* with left label '1:' was added to the initial branch of the tree (the label indicates that this sentence was obtained at step 1). The label '[5] T2(7)' is associated with the second node of the tree. This means that the rule T2(7) was applied to + *C mf A* → *C* ***b**** *B* at step 7. As the result, the "fork" of two formulas – *C mf A p* and +*C* ***b**** *B* was added to the current branch of the tree. At step 10 the sentences + *C* ***m*** *A* and – *C* ***m*** *A* was considered. Since they are contradictory sentences, the sign 'X' was added to the current branch that means inconsistency of the branch. For proving inconsistency of the second and the third branches of the tree, it is sufficient to show that the following sets of equalities and inequalities are inconsistent:

$$S_1 = \{C^{+} - A^{-} \geq 0,\ A^{-} - C^{+} \geq 0,\ C^{-} - B^{+} \geq 1,\ B^{-} - A^{+} \geq 1,\ A^{+} - A^{-} \geq 1,\ B^{+} - B^{-} \geq 1,\ C^{+} - C^{-} \geq 1\},$$

$$S_2 = \{C^{+} - A^{-} \geq 0,\ A^{-} - C^{+} \geq 0,\ C^{-} - B^{+} \geq 1,\ B^{-} - A^{+} \geq 1,\ B^{+} - B^{+} \geq 1,\ A^{+} - A^{-} \geq 1,\ B^{+} - B^{-} \geq 1,\ C^{+} - C^{-} \geq 1\}.$$

Here S_1 (or S_2) is the set of all inequalities from the second (or third) branch plus the standard inequalities for *A*, *B* and *C*. Indeed, S_1 is inconsistent since it contains the inequalities $B^{-} - A^{+} \geq 1$, $B^{+} - B^{-} \geq 1$, $C^{-} - B^{+} \geq 1$, $A^{+} - C^{-} \geq 0$ from which we obtain, by adding the inequalities, the contradiction 0>3. These inequalities correspond to cycle $(A^+,1,B^-)$, $(B^-,1,B^+)$, $(B^+,1,C^-)$, $(C^-,0,A^+)$ in the graph $\Gamma(S_1)$ shown in Fig.3. Similarly, S_2 contains the inequalities giving the contradiction. ■

In general, let S be a system of equalities of the form $X–Y \geq \varepsilon$ where $\varepsilon \in \{0,1\}$. Then $\Gamma(S)$ is the labeled graph whose nodes are variables X_i and arcs are triples of the form (X,ε,Y). By definition (X,ε,Y) enters the graph $\Gamma(S)$ if and only if the inequality $X–Y \geq \varepsilon$ belongs to S. We say that a cycle is *positive* if it contains at least one arc with $\varepsilon = 1$, in other words, if the length of the cycle (i.e., the sum of its integer lables) is positive. It is easy to understand (from the above example) that the following assertion is true.

Proposition 2. A system S of inequalities is inconsistent if and only if the graph $\Gamma(S)$ contains a positive cycle.

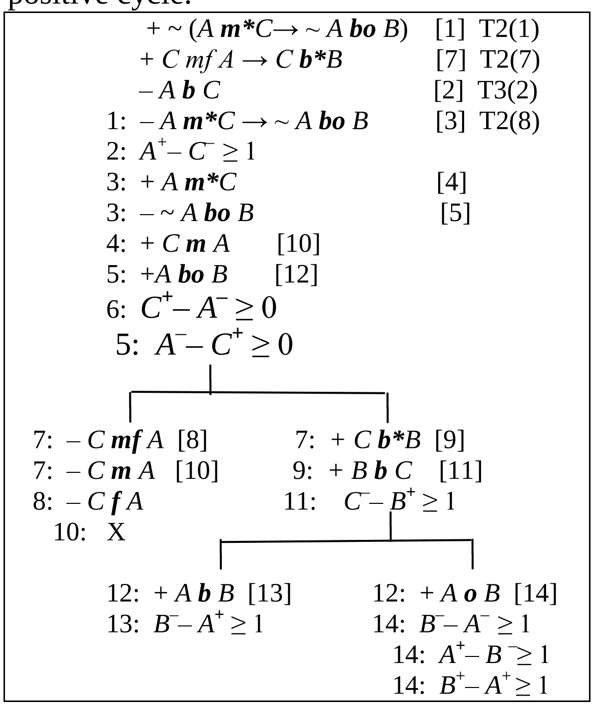

Figure 1. Inference tree for Example 4

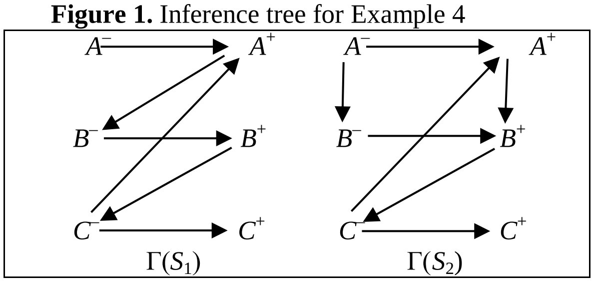

Figure 2. Graphs $\Gamma(S_1)$ and $\Gamma(S_2)$ from Example 5

2. Specification of workflows in the logics BAL and $\mathbf{BAL_m}$ and query answering

Consider an example of a specification of a workflow by ontologies in the logics **AL** and $\mathbf{BAL_m}$..

Example 2. Figure 3 shows the graph defining the order of tasks performance in a simple workflow with four tasks A, B, C and D. Each task takes a certain amount of time for its performance. Therefore, temporal intervals, denoted also by A, B, C and D, are associated with the tasks. The graph nodes represent workflow tasks, and the labeled by '***b***'arrows represent the "before" relation in performance of the tasks. Semicircle between arrows $(A,\boldsymbol{b},B)$ and (A,b,D) denotes that after A should be performed B or D. Suppose also that there is a dependency between the task D and B that D should start together with B or D should carry out during B.

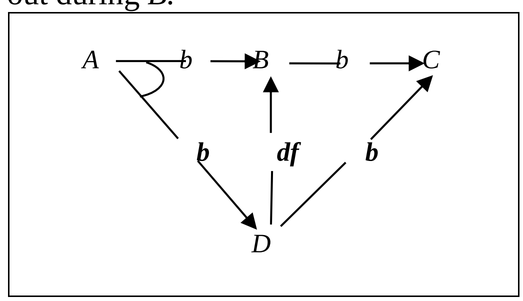

Figure 3. Graph of **BAL-ontology**

The following **BAL-**ontology represents this graph:

$\boldsymbol{O}_1=\{A\ \boldsymbol{b}\ B \vee A\ \boldsymbol{b}\ D,\ D\ \boldsymbol{df}\ B,\ B\ \boldsymbol{b}\ C,$
$D\ \boldsymbol{b}\ C\}.$

Adding some metric information to the ontology $\boldsymbol{O}_1$ we obtain the $\mathbf{BAL_m}$-ontology that can be considered as a refinement of $\boldsymbol{O}_1$:

$\boldsymbol{O}_2 = \{A\ \boldsymbol{b}\{B^-–A^- \leq 5\}B \vee A\ \boldsymbol{b}\{D^+–A^+ \leq 3\}D,$
$B\ \boldsymbol{b}\{C^-–B^- \leq 4\}C,\ D\ \boldsymbol{df}\ B,$
$D\ b\{C^+–D^+ \leq 6\}C\}$. ■

Consider, by example, how to recognize consistency of the $\mathbf{BAL_m}$-ontology for a given workflow.

Example 3. Figure 4 shows the inference tree built for the ontology $\boldsymbol{O}_2$ from Example 2. The initial branch of the tree is the set $E = \{+\alpha \mid \alpha \in \boldsymbol{O}_2\}$. Then the ontology $\boldsymbol{O}_2$ will be inconsistent (or consistent) if and only if the set E will be inconsistent (or consistent). Therefore, for proving that the ontology $\boldsymbol{O}_2$ is consistent it is sufficient to make sure that there is an open branch in the tree.

When building the inference trees for $\mathbf{BAL_m}$-ontologies also the following rules are needed:

(a) $+A\ \theta\{\sigma\}B \mid\!- \mathrm{con}(+A\ \theta\ B)$ and σ;

(b) $-A\ \theta\{\sigma\}B \mid\!- \mathrm{con}(-A\ \theta\ B)$ or σ;

(c) $\sigma\ ;\ \tau \mid\!- \sigma$ and τ.

Here $\mathrm{con}(+A\ \theta\ B)$ (or $\mathrm{con}(-A\ \theta\ B)$) is the consequent of the rule with the antecedent $+A\ \theta\ B$ (or $\mathrm{con}(-A\ \theta\ B)$). In fact, (a) – (d) are schemas of rules in the sense that specifics rules are obtained by replacing θ with specific relations. For example, the rule

$$+A\ \boldsymbol{f}\{\sigma\}B \mid\!- B^-{-}A^- \geq 1 \text{ and } A^+{-}B^+ \geq 0 \text{ and } B^+{-}A^+ \geq 0 \text{ and } \sigma$$

is obtained from (a).

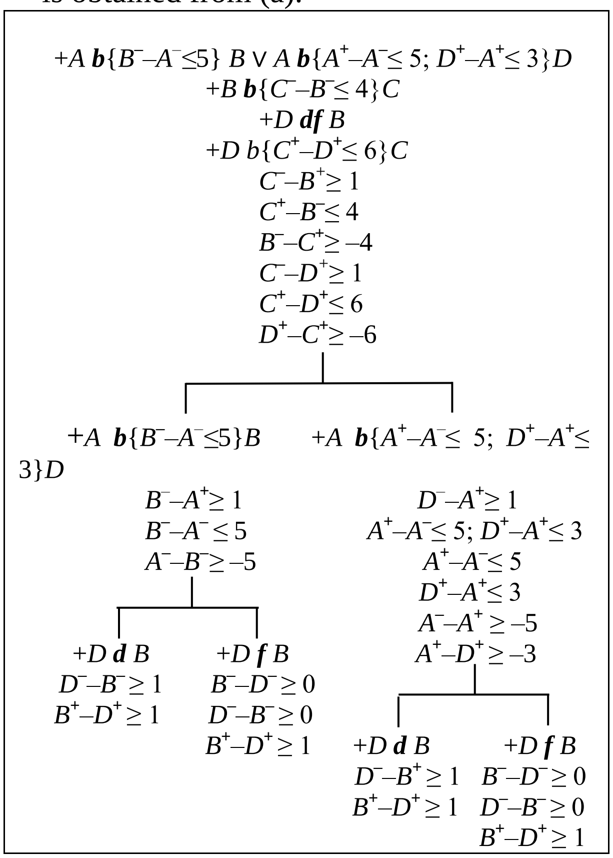

Figure 4. Inference tree for $\mathbf{BAL_m}$-ontology

Let us show that the first branch of the inference tree is open. Indeed, the branch contains the following set of inequalities (see Figure 3):

$$S_1 = \{C^-{-}B^+ \geq 1,\ C^-{-}D^+ \geq 1,\ B^-{-}C^+ \geq -4,\ D^+{-}C^+ \geq -6,\ B^-{-}A^+ \geq 1,\ A^-{-}B^- \geq -5,\ D^-{-}B^- \geq 1,\ B^+{-}D^+ \geq 1,\ A^+{-}A^- \geq 1,\ B^+{-}B^- \geq 1,\ C^+{-}C^- \geq 1,\ C^+{-}D^- \geq 1\}.$$

It easy to see that the graph $\Gamma(S_2)$ has no positive cycles (see Figure 5). Therefore, by Proposition 2, the set S_1 is consistent. ■

Now let us consider how to find the answers to the queries addressed to $\mathbf{BAL_m}$-ontologies.

Example 4. Suppose we want to evaluate the possible time of execution of a given workflow, in other words, to find the low and upper values of that time. If the worlflow is represented by the $\mathbf{BAL_m}$-ontology $\boldsymbol{O}_2$ (from Example 2). Then the above question can be expressed by two queries:

? max x: $C^+{-}A^- \geq x$, ? min x: $C^+{-}A^- \leq x$.

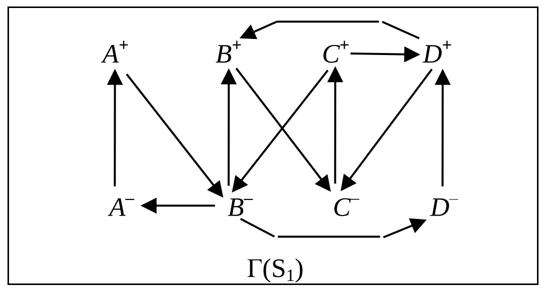

Figure 4. Graph of the system S_2

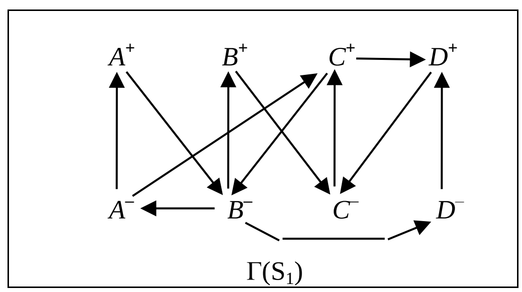

Figure 5. Graph of the system S_1*

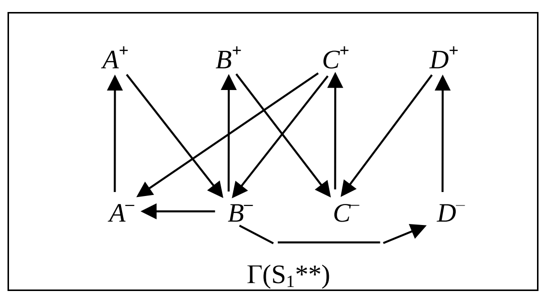

Figure 4. Graph of the system S_1

We have

$$\sim(C^+{-}A^- \geq x) \Leftrightarrow C^+{-}A^- < x \Leftrightarrow A^-{-}C^+ > -x$$

$\Leftrightarrow A^{-}–C^{+} \geq 1– x$

(since $a > b$ is equivalent to $a \geq b+1$ for integers a and b). Similarly,

$\sim(C^{+}–A^{-} \leq x) \Leftrightarrow C^{+}–A^{-}>x \Leftrightarrow C^{+}–A^{-} \geq 1+x$.

Let us add the inequality $A^{-}–C^{+} \geq 1– x$ to all branches of the inference tree. Then it is clear that for a given x this inequality is followed from the ontology $\boldsymbol{O}_2$ if and only if all the sets S_i*= $S_i \cup \{A^{-}–C^{+} \geq 1–x\}$ (i=1, 2,3,4) are inconsistent. Similarly, $C^{+}– A^{-} \leq x$ is followed from $\boldsymbol{O}_2$ if and only if all the sets S_i**= $S_i \cup\{C^{+}–A^{-} \geq 1+x\}$ (i=1, 2,3,4) are inconsistent.

The graph $\Gamma(S_1{*})$ is shown in Figure 4. It contains the cycle

$(C^{+},1–x, A^{-}), (A^{-},1,A^{+}), (A^{+},1,B^{-}),$
$(B^{-},1, D^{-}), (D^{-},1,D^{+}), (D^{+},1,C^{-}),(C^{-}, 1,C^{+}).$

The cycle has the length 7–x. Therefore, the cycle is positive if and only if $x \leq 6$. Hence, 6 is the maximal value of x that the set S_1* is inconsistent. It turns out that 6 is also the maximal value of x that others S_1* are inconsistent.

The graph $\Gamma(S_1{**})$ contains the cycle

$(A^{-},1+x, C^{+}), (C^{+},–4, B^{-}), (B^{-},–5, A^{-})$

The cycle has the length 7–x. Therefore, the cycle is positive if and only if $x \geq 9$. Hence, 9 is the minimal value of x that the set S_1** is inconsistent. It turns out that 6 is also the minimal value of x that others S_1** are inconsistent.

Thus, the answers to the above queries are 6 and 9 (correspondingly). ■

Conclusion

We have defined some temporal logic which is an extension of well known Allen's interval logic by inserting metric properties in qualitative temporal relations. We show how to use the proposed logic for specifying workflows. The result of a specification is an ontology to which we can set queries. For finding the answers to queries we have developed the method related to the type of analytical tableaux. We give the complete system of sound rules for the method.

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