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**MATHEMATICAL POWER FLOW MODEL IN AN ELECTRICAL SYSTEM  
CONTAINING A SERIAL COMPENSATOR  
THRISTOR CONTROLLED REACTIVE COMPONENT**

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**Abstract**

*A unified mathematical model of the power flow in a system containing a reactive component compensator consisting of capacitor banks connected in series to a thyristor control reactor is presented. The application of the FACTS (Flexible Alternative Current Transmission System - Controlled flexible DC power transmission) technology is shown to reduce the gap between the controlled and unmanaged modes of operation of the electric power system (EPS), presenting dispatching personnel with additional degrees of freedom in the management of power flows and voltages in excess and deficit areas of the electric network. The main objectives of the FACTS technology application are studied: increasing the transmission line capacity to the thermal limit; optimizing power flows in a complex heterogeneous network; increasing the static and dynamic stability of the electric power system. To assess the action of the new generation of regulators of the power system, two alternative models of power flow in the electric power system are considered. In the first model, the concept of alternating series reactance is used as a state variable. In the second model, the characteristic of the advance angle is used, given in the form of a nonlinear dependence in the problem of calculating the power flow using the Newton-Raphson method. Conclusions are made on the presented models of power flow allowing to estimate possibilities of the serial capacitor with thyristor control TCSC (Thyristor Controlled Series Capacitor), as FACTS device, on improvement of modes of functioning of electric power system.*

**Key words:** *power system, electric mode, reactive power, regulator, controlled flexible power transmission, power flow, static compensator, capacitor, stability.*

The installation of compensating devices on power lines and load nodes is used to improve voltage modes, increase transmission system capacity, and increase the reserves of static and dynamic stability. These devices use static unregulated devices - capacitor banks, adjustable synchronous compensators, astatic thyristor regulated sources of reactive power - FACTS devices (Flexible Alternative Current Transmission System).

To determine the effectiveness of the new generation of EES regulators on the scale of the entire electric network, it is necessary to modernize most of the analysis tools used in planning the modes of operation of the power system. Reliable calculation of power flows in real transmission and distribution power transmission networks is far from a trivial problem, because of its practical nature, many computational methods have been proposed.

The use of FACTS technology allows us to reduce the gap between the controlled and uncontrolled modes of operation of the electric power system (EPS), providing supervisory personnel with additional degrees of freedom when controlling power flows and voltages in excess and scarce areas of the electric network. The main goals of applying the FACTS technology are as follows [1, 6]: increasing the transmission line throughput to the thermal limit; optimization of power flows in a complex heterogeneous network; increase of static and dynamic

stability of EPS.

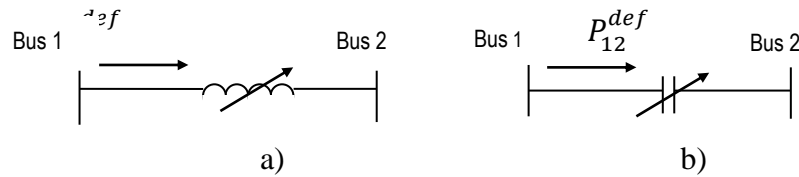
Among FACTS serial devices is the so-called TCSC device, which is a capacitive compensator, consisting of series-compensated battery compensators connected in parallel with a thyristor control reactor to ensure a smooth change in capacitance.

To evaluate the action of TCSC, we consider its application as FACTS devices, and two alternative models of power flow in EPS.

In a simpler model, the concept of variable series reactance is used as a state variable. This reactance connected in series to the line is automatically adjusted within the set limits to ensure the overflow of the set amount of active power. In a more progressive model, a lead angle characteristic defined in the form of a nonlinear dependence is used. In this case, the lead angle TCSC is selected as a state variable in the problem of calculating the power flux using the Newton – Raphson method [2, 3, 8–12].

### 1. Power flow model for sequential impedance control.

The mathematical model of the power flow of EPSs with sequentially installed TCSC thyristor-controlled capacitors is based on the concept of variable series impedance. Its value is automatically set to limit a certain value of the power flux in the line and can be effectively determined using the Newton-Raphson method. Let the  $X_{TCSC}$  variable reactance shown in fig. 1, represents the equivalent reactivity of all series-connected modules that make up the  $X_{TCSC}$  and operate either individually or in capacitive mode.



**Fig. 1.** TCSC equivalent circuit: a) inductive mode; b) capacitive mode

The TCSC matrix of mutual total conductivity, the equivalent circuits of which are shown in Fig. 1 is defined by the equation

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} jB_{11} & jB_{12} \\ jB_{21} & jB_{22} \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} \quad (1)$$

For inductive mode, we obtain

$$B_{11} = B_{22} = -\frac{1}{X_{TCSC}} ; \quad B_{12} = B_{21} = \frac{1}{X_{TCSC}} \quad (2)$$

In capacitive mode

$$B_{11} = B_{22} = \frac{1}{X_{TCSC}} ; \quad B_{12} = B_{21} = -\frac{1}{X_{TCSC}}, \text{ i.e. there is a change of signs in (2).}$$

The equations of active and reactive powers on bus 1 (Fig. 1) have the following form [4, 8]:

$$P_1 = U_1 U_2 B_{12} \sin(\delta_1 - \delta_2); \quad (3)$$

$$Q_1 = -U_1^2 B_{11} - U_1 U_2 B_{12} \cos(\delta_1 - \delta_2). \quad (4)$$

To obtain power equations on bus 2 in equations (3) and (4), you need to perform a dual replacement of the lower indices 1 and 2:

$$P_2 = U_2 U_1 B_{21} \sin(\delta_2 - \delta_1);$$

$$Q_2 = -U_2^2 B_{22} - U_2 U_1 B_{21} \cos(\delta_2 - \delta_1).$$

In solutions obtained using the Newton-Raphson method, the power equations lead to a linear form with respect to the series reactance. For the conditions shown in fig. 1, where the value of the active energy flux from bus 1 to bus 2 is regulated using a series reactive resistance, the system of power flux equations reduced to a linear form will look as follows

$$\begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \Delta Q_1 \\ \Delta Q_2 \\ P_{12}^{TCSC} \end{bmatrix}^{(i)} = \begin{bmatrix} \frac{\partial P_1}{\partial \delta_1} & \frac{\partial P_1}{\partial \delta_2} & \frac{\partial P_1}{\partial \delta_1} U_1 & \frac{\partial P_1}{\partial \delta_2} U_2 & \frac{\partial P_1}{\partial X_{TCSC}} U_{TCSC} \\ \frac{\partial P_2}{\partial \delta_1} & \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial U_1} U_1 & \frac{\partial P_2}{\partial U_2} U_2 & \frac{\partial P_2}{\partial X_{TCSC}} U_{TCSC} \\ \frac{\partial Q_1}{\partial \delta_1} & \frac{\partial Q_1}{\partial \delta_2} & \frac{\partial Q_1}{\partial U_1} U_1 & \frac{\partial Q_1}{\partial U_2} U_2 & \frac{\partial Q_1}{\partial X_{TCSC}} U_{TCSC} \\ \frac{\partial Q_2}{\partial \delta_1} & \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial U_1} U_1 & \frac{\partial Q_2}{\partial U_2} U_2 & \frac{\partial Q_2}{\partial X_{TCSC}} U_{TCSC} \\ \frac{\partial P_{12}^{TCSC}}{\partial \delta_1} & \frac{\partial P_{12}^{TCSC}}{\partial \delta_2} & \frac{\partial P_{12}^{TCSC}}{\partial U_1} U_1 & \frac{\partial P_{12}^{TCSC}}{\partial U_2} U_2 & \frac{\partial P_{12}^{TCSC}}{\partial X_{TCSC}} U_2 \end{bmatrix}^{(i)} \begin{bmatrix} \Delta \delta_1 \\ \Delta \delta_2 \\ \frac{\Delta U_1}{U_1} \\ \frac{\Delta U_2}{U_2} \\ \frac{\Delta U_2}{U_2} \\ \frac{\Delta X^{TCSC}}{X^{TCSC}} \end{bmatrix}^{(i)}, \quad (5)$$

Where

$P_{12}^{TCSC} = P_{12}^{pez} - P_{12}^{6bi4}$  - mismatch of the active power flow for series reactance;

$\Delta X^{TCSC} = X_{TCSC}^{(i)} - X_{TCSC}^{(i-1)}$  - - increment of series reactance;

power flow  $P_{12}^{6bi4} = U_1 U_2 B_{12} \sin(\delta_1 - \delta_2)$ .

The elements of the last column of the Jacobian in the matrix iterated linear equation (5) have the following form

$$\begin{aligned} \frac{\partial P_1}{\partial X} X &= -U_1 U_2 B_{12} \sin(\delta_1 - \delta_2); \\ \frac{\partial Q_1}{\partial X} X &= -U_1^2 B_{11} + U_1 U_2 B_{12} \cos(\delta_1 - \delta_2); \end{aligned}$$

$$\frac{\partial P_{12}^X}{\partial X} X = \frac{\partial P_1}{\partial X} X.$$

At the end of each  $i$ th iteration, the variable component is updated according to the formula [7]:

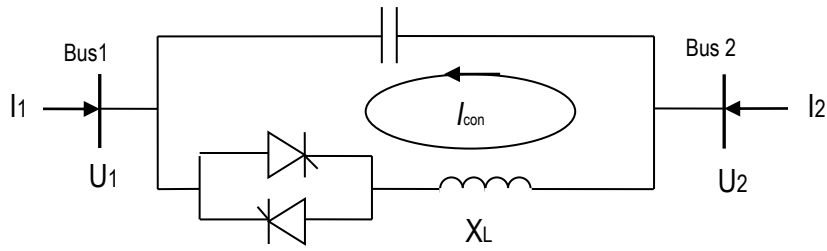
$$X_{TCSC}^{(i)} = X_{TCSC}^{(i-1)} + \left[ \frac{\Delta X_{TCSC}}{X_{TCSC}} \right]^{(i)} X_{TCSC}^{(i-1)}$$

## 2. The model of power flow when controlling the lead angle.

In this model, after determining the reactance value  $X_{TCSC}$ , the lead angle  $\beta_{TCSC}$  can be calculated using the Newton-Raphson method. This makes practical sense only when all the TCSC thyristor series capacitor modules have identical technical characteristics and are designed to operate with the same lead angles. The calculation of the lead angles implies an iterative solution, since the reactance TCSC and the lead angles are connected by a nonlinear dependence. One way to avoid an additional iterative process is to use the alternative power flow model below (Fig. 2).

Consider the circuit of a series capacitor with thyristor control, also called a static thyristor

compensator:



**Fig. 2.** Thyristor-controlled series capacitor equivalent circuit

The reactance corresponding  $X_{TCSC}^{HOM}$  to the rated frequency of the network is [2, 5, 8-10]:

$$X_{TCSC}^{HOM} = -X_c + C_1(2(\pi - \beta) + \sin(2(\pi - \beta))) - C_2 \cos(\pi - \beta) (\omega \operatorname{tg}(\omega(\pi - \beta)) - \operatorname{tg}(\pi - \beta)), \quad (6)$$

where

$$C_1 = \frac{X_c + X_{LC}}{\pi}; \quad C_2 = \frac{4X_{LC}^2}{\pi X_L}; \quad X_{LC} = \frac{X_c X_L}{X_c - X_L}; \quad \omega = \sqrt{\frac{X_c}{X_L}}.$$

In this case, the equivalent reactance  $X_{TCSC}^{HOM}$  in equation (6) replaces the resistance  $X_{TCSC}$  present in equations (1) and (2), and the active and reactive power equations take the following form:

$$\left. \begin{aligned} P_1 &= U_1 U_2 B_{12}^{HOM} \sin(\delta_1 - \delta_2), \\ Q_1 &= -U_1^2 B_{11} - U_1 U_2 B_{12} \cos(\delta_1 - \delta_2) \end{aligned} \right\}, \quad (7)$$

Where  $B_{11}^{HOM} = -B_{12}^{HOM} = B_{TCSC}^{HOM}$ .

To obtain power equations for bus 2, it suffices to perform dual substitution of lower indices 1 and 2 in (7).

In the case where the TCSC controls the flow of active power from bus 1 to bus 2, the system of iterated equations reduced to a linear form has the form:

$$\begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \Delta Q_1 \\ \Delta Q_2 \\ \Delta P_{12}^{TCSC} \end{bmatrix}^{(i)} = \begin{bmatrix} \frac{\partial P_1}{\partial \delta_1} & \frac{\partial P_1}{\partial \delta_2} & \frac{\partial P_1}{\partial \delta_1} U_1 & \frac{\partial P_1}{\partial \delta_2} U_2 & \frac{\partial P_1}{\partial \Delta \beta^{TCSC}} \beta^{TCSC} \\ \frac{\partial P_2}{\partial \delta_1} & \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial U_1} U_1 & \frac{\partial P_2}{\partial U_2} U_2 & \frac{\partial P_2}{\partial \beta^{TCSC}} \beta^{TCSC} \\ \frac{\partial Q_1}{\partial \delta_1} & \frac{\partial Q_1}{\partial \delta_2} & \frac{\partial Q_1}{\partial U_1} U_1 & \frac{\partial Q_1}{\partial U_2} U_2 & \frac{\partial Q_1}{\partial \beta^{TCSC}} \beta^{TCSC} \\ \frac{\partial Q_2}{\partial \delta_1} & \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial U_1} U_1 & \frac{\partial Q_2}{\partial U_2} U_2 & \frac{\partial Q_2}{\partial \beta^{TCSC}} \beta^{TCSC} \\ \frac{\partial P_{12}^{TCSC}}{\partial \delta_1} & \frac{\partial P_{12}^{TCSC}}{\partial \delta_2} & \frac{\partial P_{12}^{TCSC}}{\partial U_1} U_1 & \frac{\partial P_{12}^{TCSC}}{\partial U_2} U_2 & \frac{\partial P_{12}^{TCSC}}{\partial \beta^{TCSC}} \end{bmatrix}^{(i)} \begin{bmatrix} \Delta \delta_1 \\ \Delta \delta_2 \\ \frac{\Delta U_1}{U_1} \\ \frac{\Delta U_2}{U_2} \\ \Delta \beta^{TCSC} \end{bmatrix}^{(i)},$$

where the increment of  $\Delta P_{12}^{\beta^{TCSC}}$  power is set as the difference

$$\Delta P_{12}^{\beta^{TCSC}} = P_{12}^{pe2} - P_{12}^{\beta^{TCSC} \text{ bi}^4},$$

and the increment of the lead angle

$$\Delta \beta^{TCSC} = \beta_{TCSC}^{(i)} - \beta_{TCSC}^{(i-1)}.$$

Power  $P_{12}^{\beta^{TCSC} \text{ bi}^4}$  is calculated by formula (7).

The partial derivatives in the last column of the Jacobian in the iterated equation (8) are found by the formulas:

$$\frac{\partial P_1}{\partial \beta} = P_1 B_{TCSC}^{HOM} \frac{\partial X_{TCSC}^{HOM}}{\partial \beta}; \quad \frac{\partial Q_1}{\partial \beta} = Q_1 B_{TCSC}^{HOM} \frac{\partial X_{TCSC}^{HOM}}{\partial \beta};$$

$$\frac{\partial B_{TCSC}^{HOM}}{\partial \beta} = (B_{TCSC}^{HOM})^2 \frac{\partial X_{\beta}^{HOM}}{\partial \beta};$$

$$\frac{\partial X_{TCSC}^{HOM}}{\partial \beta} = -2C_1(1 + \cos(2\beta)) + C_2 \sin(2\beta)(\varpi \operatorname{tg}(\varpi(\pi - \beta)) - \operatorname{tg}(\pi - \beta) + C_2 \left( \varpi^2 \frac{\cos^2(\pi - \beta)}{\cos^2(\varpi(\pi - \beta))} - 1 \right)).$$

The behavior of the TCSC mathematical model is influenced by several internal resonances. These resonant points are determined by the following expressions [7, 11-20]:

$$\beta_{TCSC}^1 = \pi \left[ 1 - \frac{\omega \sqrt{LC}}{2} \right]; \quad \beta_{TCSC}^2 = \pi \left[ 1 - \frac{3\omega \sqrt{LC}}{2} \right];$$

$$\beta_{TCSC}^3 = \pi \left[ 1 - \frac{5\omega \sqrt{LC}}{2} \right]; \quad \dots \quad \beta_{TCSC}^k = \pi \left[ 1 - \frac{(2k-1)\omega \sqrt{LC}}{2} \right].$$

Theoretically, a TCSC may have n resonance points; in practice, in a well-designed controller in the operating range, there can be only one resonance peak.

Thus, the presented models of the power flow make it possible to evaluate the possibilities of a TCSC thyristor-controlled series capacitor as a FACTS device to improve the operating modes of the electric power system.

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