Chemical Technology, Control and Management

Volume 2018
Issue 3 *SPECIAL ISSUE 4-5*

Article 21

10-19-2018

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Recommended Citation

Zeinalova, Lala M. (2018) "A Z-number valued analytical hierarchy process.," *Chemical Technology, Control and Management*: Vol. 2018 : Iss. 3 , Article 21.
DOI: https://doi.org/10.34920/2018.4-5.88-94
Available at: https://uzjournals.edu.uz/ijctcm/vol2018/iss3/21

Tashkent State Technical University, SSC «UZSTROYMATERIALY», SSC «UZKIMYOSANOAT», JV «SOVPLASTITAL», Agency on Intellectual Property of the Republic of Uzbekistan

ISSN 1815-4840
CHEMICAL TECHNOLOGY.
CONTROL AND MANAGEMENT
2018, Special issue №4-5 (82-83) pp.88-94. https://doi.org/10.34920/2018.4-5.88-94
International scientific and technical journal
journal homepage: https://uzjournals.edu.uz/ijctcm
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A Z-NUMBER VALUED ANALYTICAL HIERARCHY PROCESS

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Abstract. *The Z-number concept is the attempt to model real-world uncertainty and relates to the issue of reliability of information, especially in the realms of economics and decision analysis. In this paper, we present an approach that can handle Z-numbers in the context of multi-criteria decision-making applying direct computations. In this paper we consider an Analytical Hierarchy Process based on Z-number valuations, taking into account the uncertainty of the experts` opinion in estimation of the options. We considered a case of estimation of technical institutions with 7 criteria: campus infrastructure, faculty, students, academic ambience, teaching learning process, use of advance teaching aid, supplementary process and 3 alternatives: technical institutions.*

Keywords: *Z-number, Analytical Hierarchy Process (AHP), multi-criteria decision making, utility value.*

Introduction

Analytical Hierarchy Process (AHP) refers to multi-criteria decision making methods. Decision-making problems in real-world are usually too complicated and require a joint examination of multiple criteria. The non-inclusion of an important criterion or the consideration of all the criteria individually leads to inaccurate and often unrealistic results. AHP "organizes the decision making criteria as a hierarchy and aims quantifying relative priorities for a given set of alternatives based on the decision makers' pairwise judgments" [1]. The Analytic Hierarchy Process can use a linear additive model. The weights for alternatives are obtained by special procedures or are reached by using of questionnaires for pairwise comparison of the corresponding alternatives. Despite the convenience of its use, there are some questions about the theoretical foundations of the AHP and about some properties. One of them is a rank reversal question, according to which addition of the new option can cause a reverse in ranking of other options not related to the new one [2]. This is considered by many experts as not corresponding to rational evaluation of options. In many decision-making problems, data may be presented in a crisp format, interval, fuzzy, fuzzy intuitionist [3]. Taking into account the uncertainty of the experts` opinion in estimation of the options, fuzzy AHP method combining the advantages of usual AHP method and fuzzy logic have spread widely. AHP is a widely used multi-criteria decision making method and can be applied in different areas [4]. In a standard AHP model, the goal can be found in the first level. The criteria and sub criteria are in the second and third levels respectively, the corresponding alternatives are located in the fourth level [5]. An important aspect in decision making is the issue of confidence of the information [3]. In [6] Zadeh introduced the theory of Z-numbers for description of the imperfectness in a general form. A Z-number consists of a pair of fuzzy numbers $(\tilde{A},\tilde{B})$. Here $\tilde{A}$ is a value of some variable and $\tilde{B}$ has a meaning of certainty, confidence, reliability, strength of truth, or probability. It should be noted that in everyday real life most decisions are made by expressions with Z-numbers. The extension principle was a base for suggested by Zadeh computation operations with Z-numbers. This issue was considered in [7].

Author demonstrates how to use these Z-numbers to provide information about imperfect information expressed in the form of Z-valuations, taking into account that an uncertain variable is random. In [7] author considers an issue of Z-valuation, representing decision making and answer multiple questions. An author uses an alternative formulation for expression of the information from Z-valuations applying a Dempster-Shafer belief structure and arising an issue of compatibility of probability distributions. Z-valuation of decision making information is demonstrated in [8]. In [9] an application of AHP model with Z-numbers was considered. For purpose of decision making, authors applied method of conversion of Z-numbers into fuzzy numbers and thus, authors constructed usual fuzzy AHP model. In [10], [11], [12] an authors presented an Analytical Hierarchy Process (AHP) method based on Z-valuations for solving of linguistic decision-making problems, using a method of converting of the describing reliability $\tilde{B}$ of $\tilde{A}$ into a crisp number suggested in [8]. In this paper we consider an Analytical Hierarchy Process based on Z-number valuations. We use the direct method considered in [6], [13], [14] for computations over Z-numbers.

1.Preliminaries

Definition 1. A discrete Z-number [13,14]. A discrete Z-number is an ordered pair $Z=(A,B)$, where A is a discrete fuzzy number that has a meaning of a fuzzy constraint on values of a random variable X : X *is* A . B is a discrete fuzzy number with a membership function $\mu_B:\{b_1,...,b_n\}\to[0,1],\ \{b_1,...,b_n\}\subset[0,1]$. Thus, B has a meaning of a fuzzy constraint on the probability measure of A : $P(A)$ *is* B. Z^+-number Z^+ is a pair consisting of a fuzzy number, A, and a random number R: $Z^+=(A,R)$, where A is a discrete fuzzy number that has a meaning of a fuzzy constraint on values of a random variable X and R has a role of the probability distribution p, such that

$$P(A)=\sum_{i=1}^{n}\mu_A(x_i)p(x_i),\ P(A)\in\operatorname{supp}(B)$$

Definition 2. An addition of discrete Z-numbers [13,14]. Let $Z_1=(A_1,B_1)$ and $Z_2=(A_2,B_2)$ be discrete Z-numbers where A_1 and A_2 are the discrete fuzzy number that have a meaning of a fuzzy constraint on values of the random variables X_1 and X_2. Consider a computation of addition $Z_{12}=Z_1+Z_2$. At first we compute the corresponding discrete Z^+-numbers. The discrete Z^+-number $Z_{12}^+=Z_1^++Z_2^+$ is determined as follows:

$$Z_1^++Z_2^+=(A_1+A_2,R_1+R_2)$$

where R_1 and R_2 are represented by discrete probability distributions:

$$p_1=p_1(x_{11})\backslash x_{11}+p_1(x_{12})\backslash x_{12}+...+p_1(x_{1n})\backslash x_{1n},$$
$$p_2=p_2(x_{21})\backslash x_{21}+p_2(x_{22})\backslash x_{22}+...+p_2(x_{2n})\backslash x_{2n}.$$

Definition 3. A subtraction of discrete Z-numbers [13,14]. Let us consider standard subtraction $Z_{12}=Z_1-Z_2$ of discrete Z-numbers $Z_1=(A_1,B_1)$ and $Z_2=(A_2,B_2)$. First, a discrete Z^+-number $Z_{12}^+=Z_1^+-Z_2^+$ is determined:

$$Z_1^+-Z_2^+=(A_1-A_2,R_1-R_2)$$

where R_1 and R_2 are represented by discrete probability distributions:

$$p_1=p_1(x_{11})\backslash x_{11}+p_1(x_{12})\backslash x_{12}+...+p_1(x_{1n})\backslash x_{1n},$$
$$p_2=p_2(x_{21})\backslash x_{21}+p_2(x_{22})\backslash x_{22}+...+p_2(x_{2n})\backslash x_{2n}.$$

Definition 4. A multiplication of discrete Z-numbers [13,14]. Let us consider multiplication $Z_{12}=Z_1\cdot Z_2$ of $Z_1=(A_1,B_1)$ and $Z_2=(A_2,B_2)$. First, $Z_{12}^+=Z_1^+\cdot Z_2^+$ is determined:

$$Z_1^+\cdot Z_2^+=(A_1\cdot A_2,R_1\cdot R_2),$$

where R_1 and R_2 are represented by discrete probability distributions:

$$p_1=p_1(x_{11})\backslash x_{11}+p_1(x_{12})\backslash x_{12}+...+p_1(x_{1n})\backslash x_{1n},$$
$$p_2=p_2(x_{21})\backslash x_{21}+p_2(x_{22})\backslash x_{22}+...+p_2(x_{2n})\backslash x_{2n}.$$

Definition 5. A division of discrete Z-numbers [13,14]. Let us consider standard division $Z_{12}=Z_1/Z_2$ of $Z_1=(A_1,B_1)$ and $Z_2=(A_2,B_2)$, where $0\notin\operatorname{supp}(A_2)$. First, $Z_{12}^+=(A_{12},p_{12})$ is determined:

$Z^+_{12}=(A_{12},p_{12})$, where $A_{12}=A_1/A_2$ is the standard division of discrete fuzzy numbers and a convolution $p_{12}=p_1\circ p_2$ of discrete probability distributions is obtained as follows:

$$p_{12}(x)=\sum_{\substack{x=x_1/x_2,\\ x_2\neq 0}} p_1(x_1)p_2(x_2).$$

*Definition 6. A degree of discrete Z-numbers [13,14].*Let $Z_Y=Z_X^n, Z_X^+=(\tilde{A}_X,R_X)$ where R_X is represented as

$$p_X=p_X(x_1)\backslash x_1+p_X(x_2)\backslash x_2+...+p_X(x_n)\backslash x_n$$

Then the discrete Z^+-number Z_Y^+ is determined as follows: $Z_Y^+=(\tilde{A}_Y,R_Y)$, where $\tilde{A}_Y=\tilde{A}_X^n$ are determined as fuzzy numbers with degree n and R_Y is represented by a discrete probability distribution

Table 1. Modified scale of relative importance with the corresponding Z-numbers

	Definition	Z-number valued Intensity of relative importance
1	Equal importance of criteria i and j	$Z_1=((1,1,1),(B_1))$
3	The criterion i is not more important than the criterion j The criterion i is more important than the criterion j	$Z_3=((2,3,4),(B_3))$
5	The criterion i is much more important than the criterion j	$Z_5=((4,5,6),(B_5))$
7	The criterion i is predominantly more important than the criterion j	$Z_7=((6,7,8),(B_7))$
9	The intermediate values between two neighbouring scales	$Z_9=((9,9,9),(B_9))$
2		$Z_2=((1,2,3),(B_2))$
4		$Z_4=((3,4,5),(B_4))$
6		$Z_6=((5,6,7),(B_6))$
8		$Z_8=((7,8,9),(B_8))$

$$p_Y=p_Y(y_1)\backslash y_1+p_Y(y_2)\backslash y_2+...+p_Y(y_n)\backslash y_n$$

such that $y_k=x_k^n$ and $p_Y(y_k)=p_X(x_k)$.

Definition 7. A comparison of discrete Z-numbers [13,14]. The Z-number is considered as a pair of values of two attributes – "one attribute measures value of a variable, the other one measures the associated reliability". Thus, Z-numbers are compared as multiattribute alternatives. The Fuzzy Pareto optimality principle is used for comparison of Z-numbers.

2.Statement of a Problem

A structure of an Analytical Hierarchy of Z-number valued AHP model consists of the goal at the top node. The corresponding criteria are values:located at the second level. Sub-criteria, subsub criteria and etc. (if they exist) are at the next level and finally the alternatives are at the last level [15,16]. We have to compare the criteria on the lower level compared to each of the criterion of the corresponding upper level. Let`s suppose that $C=(C_1,C_2,...,C_m)$ is a set of criteria and $A=(A_1,A_2,...,A_m)$ is a set of alternatives. The matrix of pairwise comparison is shown in Eq.1. Thus the decision maker`s preferences of i-th criterion (alternative) over j-th criterion (alternative) are represented as Z-numbers:

$$D=\begin{bmatrix} d_{11}=Z(A_{11},B_{11}) & d_{12}=Z(A_{12},B_{12}) & ... & d_{1n}=Z(A_{1n},B_{1n}) \\ d_{21}=Z(A_{21},B_{21}) & d_{22}=Z(A_{22},B_{22}) & ... & d_{2n}=Z(A_{2n},B_{2n}) \\ ... & ... & ... & ... \\ d_{m1}=Z(A_{m1},B_{m1}) & d_{m2}=Z(A_{m2},B_{m2}) & ... & d_{mn}=Z(A_{mn},B_{mn}) \end{bmatrix} \quad (1)$$

The linguistic terms for comparison of criteria (alternatives) are shown in Table 1.

After determination of the averaged decision maker`s preferences (if there is more than one decision maker) and the comparison of criteria and alternatives the decision making procedure will be as follows.

Step 1. The geometric mean Z_{r_i} and total the geometric means are computed for each criterion C_i to aggregate Z-number-valued comparison

$$Z_{r_i} = \left(\prod_{j=1}^{n} Z(A_{ij}, B_{ij}) \right)^{1/n}, i = 1, 2, ..., m, \quad j = 1, 2, ..., n. \quad (2)$$

$$Z_{r_i}^{Total} = \sum_{i=1}^{m} Z_{r_i}, \; i = 1, 2, ..., m. \quad (3)$$

Step 2. Z-number valued importance weights of criteria are computed normalizing each of the geometric means by dividing by the total just computed:

$$Z_{w_i} = Z_{r_i} \otimes (Z_{r_1} \oplus Z_{r_2} \oplus ... \oplus Z_{r_m})^{-1} \quad (4)$$

After these operations, we do not apply the converting of Z-number valued importance weights of criteria to fuzzy and crisp, since this implies a significant loss of information in this case. We use the analogous procedure for pairwise comparison of alternatives with respect to the corresponding criterion and calculate the values of criteria C_i for

Table 2. Z-number valued pairwise comparison of the choice criteria

C	C_1	C_2	C_3	C_4	C_5	C_6	C_7
C_1	((1,1,1), (0.2,0.3,0.4))	((1/6,1/5,1/4), (0.3,0.4,0.5))	((1/6,1/5,1/4), (0.3,0.4,0.5))	((2,3,4), (0.2,0.3,0.4))	((1/6,1/5,1/4), (0.4,0.5,0.6))	((6,7,8), (0.4,0.5,0.6))	((2,3,4), (0.4,0.5,0.6))
C_2	((4,5,6), (0.3,0.4,0.5))	((1,1,1), (0.2,0.3,0.4))	((1/8,1/7,1/6), (0.4,0.5,0.6))	((1/6,1/5,1/4), (0.3,0.4,0.5))	((2,3,4), (0.2,0.3,0.4))	((1/8,1/7,1/6), (0.4,0.5,0.6))	((1/4,1/3,1/2), (0.2,0.3,0.4))
C_3	((4,5,6), (0.3,0.4,0.5))	((6,7,8), (0.4,0.5,0.6))	((1,1,1), (0.2,0.3,0.4))	((6,7,8), (0.4,0.5,0.6))	((1/4,1/3,1/2), (0.4,0.5,0.6))	((4,5,6), (0.2,0.3,0.4))	((1/4,1/3,1/2), (0.3,0.4,0.5))
C_4	((1/4,1/3,1/2), (0.2,0.3,0.4))	((4,5,6), (0.3,0.4,0.5))	((1/8,1/7,1/6), (0.4,0.5,0.6))	((1,1,1), (0.2,0.3,0.4))	((4,5,6), (0.6,0.7,0.8))	((1/6,1/5,1/4), (0.6,0.7,0.8))	((6,7,8), (0.4,0.5,0.6))
C_5	((4,5,6), (0.4,0.5,0.6))	((1/4,1/3,1/2), (0.2,0.3,0.4))	((2,3,4), (0.4,0.5,0.6))	((1/6,1/5,1/4), (0.6,0.7,0.8))	((1,1,1), (0.2,0.3,0.4))	((6,7,8), (0.4,0.5,0.6))	((2,3,4), (0.2,0.3,0.4))
C_6	((1/8,1/7,1/6), (0.4,0.5,0.6))	((6,7,8), (0.4,0.5,0.6))	((1/6,1/5,1/4), (0.2,0.3,0.4))	((4,5,6), (0.6,0.7,0.8))	((1/8,1/7,1/6), (0.4,0.5,0.6))	((1,1,1), (0.2,0.3,0.4))	((6,7,8), (0.6,0.7,0.8))
C_7	((1/4,1/3,1/2), (0.4,0.5,0.6))	((2,3,4), (0.2,0.3,0.4))	((2,3,4), (0.3,0.4,0.5))	((1/8,1/7,1/6), (0.4,0.5,0.6))	((1/4,1/3,1/2), (0.2,0.3,0.4))	((1/8,1/7,1/6), (0.6,0.7,0.8))	((1,1,1), (0.2,0.3,0.4))

Table 3. Comparison of alternatives for criteria C_1 and C_2 d

C_1	A_1	A_2	A_3	C_2	A_1	A_2	A_3
A_1	((1,1,1), (0.2,0.3,0.4))	((1/4,1/3,1/2), (0.2,0.3,0.4))	((4,5,6), (0.2,0.3,0.4))	A_1	((1,1,1), (0.2,0.3,0.4))	((1/4,1/3,1/2), (0.2,0.3,0.4))	((6,7,8), (0.4,0.5,0.6))
A_2	((2,3,4), (0.2,0.3,0.4))	((1,1,1), (0.2,0.3,0.4))	((1/8,1/7,1/6), (0.4,0.5,0.6))	A_2	((2,3,4), (0.2,0.3,0.4))	((1,1,1), (0.2,0.3,0.4))	((1/8,1/7,1/6), (0.4,0.5,0.6))
A_3	((1/6,1/5,1/4), (0.2,0.3,0.3))	((6,7,8), (0.4,0.5,0.6))	((1,1,1), (0.2,0.3,0.4))	A_3	((1/8,1/7,1/6), (0.4,0.5,0.6))	((6,7,8), (0.4,0.5,0.6))	((1,1,1), (0.2,0.3,0.4))

Table 4. Comparison of alternatives for criteria C_3 and C_4

C_3	A_1	A_2	A_3	C_4	A_1	A_2	A_3
A_1	((1,1,1), (0.2,0.3,0.4))	((4,5,6), (0.6,0.7,0.8))	((6,7,8), (0.4,0.5,0.6))	A_1	((1,1,1), (0.2,0.3,0.4))	((4,5,6), (0.6,0.7,0.8))	((2,3,4), (0.4,0.5,0.6))
A_2	((1/6,1/5,1/4), (0.6,0.7,0.8))	((1,1,1), (0.2,0.3,0.4))	((1/8,1/7,1/6), (0.4,0.5,0.6))	A_2	((1/6,1/5,1/4), (0.6,0.7,0.8))	((1,1,1), (0.2,0.3,0.4))	((1/8,1/7,1/6), (0.4,0.5,0.6))
A_3	((1/8,1/7,1/6), (0.4,0.5,0.6))	((6,7,8), (0.4,0.5,0.6))	((1,1,1), (0.2,0.3,0.4))	A_3	((1/4,1/3,1/2), (0.4,0.5,0.6))	((6,7,8), (0.4,0.5,0.6))	((1,1,1), (0.2,0.3,0.4))

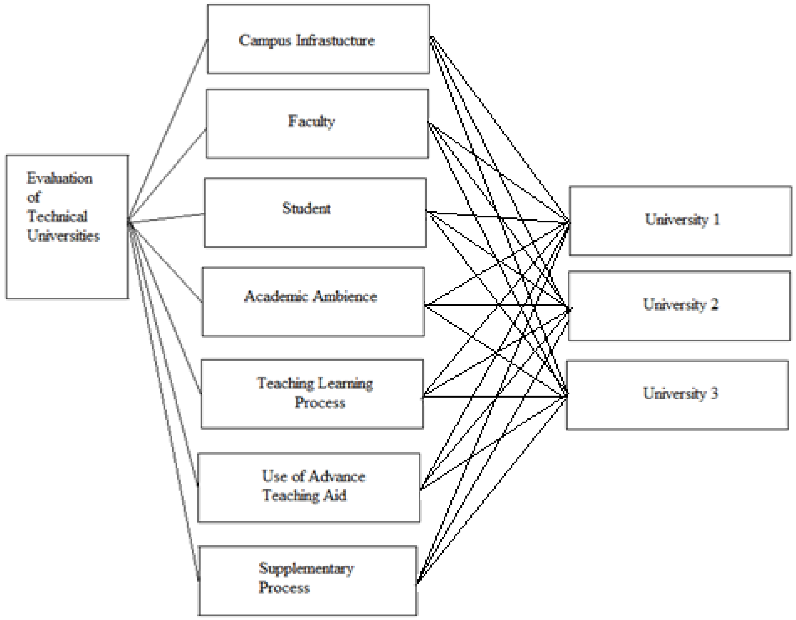

Figure 1. ***Detailed hierarchy of the problem***

alternatives a_j $: Z_{x_{ji}}$ After these procedures we determine Z-number valued utilities for each alternative a_j as a weighted average:

$$Z_{U(a_j)} = \sum_{i=1, j=1}^{m,n} Z_{w_i} Z_{x_{ji}},\ i = 1, 2, ..., m,\ \ j = 1, 2, ..., n. \quad (5)$$

A pairwise comparison of Z-number valued utilities is realized according to Definition 7. We choose the preferred alternative with the maximal Z-number valued utility.

3.An Example

Let's consider a case of estimation of technical institutions [15,16]. Detailed hierarchy of the problem is shown in Fig. 1. Thus, we have 7 criteria: campus infrastructure, faculty, students, academic ambience, teaching learning process, use of advance teaching aid, supplementary process and 3 alternatives: technical institutions. At first we consider a Z-number valued pairwise comparison of the choice criteria conducted by decision maker (Table 2). We suppose that a subjective evaluation is consistent. The geometric means values of Z-number valued criteria are calculated by (2). For example, the geometric mean value for the first criteria is calculated according to Definitions 4 and 6 and (2) as:

$$Z_{r_1} = (Z(A_{11}, B_{11}) \times Z(A_{12}, B_{12}) \times Z(A_{13}, B_{13}) \times Z(A_{14}, B_{14}) \times$$
$$\times Z(A_{15}, B_{15}) \times Z(A_{16}, B_{16}) \times Z(A_{17}, B_{17}))^{1/7} =$$

$=(((1,1,1),(0.2,0.3,0.4)) \times ((1/6,1/5,1/4),(0.3,0.4,0.5)) \times ((1/6,1/5,1/4),(0.3,0.4,0.5)) \times ((2,3,4),(0.2,0.3,0.4)) \times ((1/6,1/5,1/4),(0.4,0.5,0.6)) \times ((6,7,8),(0.4,0.5,0.6)) \times ((2,3,4),(0.4,0.5,0.6)))^{1/7}=((0.703,\ 0.906,\ 1.104),\ (0.203,\ 0.239,\ 0.282))$.

As it is seen it is not needed to carry out computation of B part calculating degree value of Z-number, it is the same as the initial value if the boundaries of the fuzzy number A are located on the positive axis. We find the summation for geometric mean for each criterion according to Definition 2 and (3):

$$Z_{r_i}^{Total} = Z_{r_1} + Z_{r_2} + Z_{r_3} + Z_{r_4} + Z_{r_5} + Z_{r_6} + Z_{r_7} =$$

$=((6.811,\ 8.546,\ 9.786),(0.59,0.78,\ 0.81))$.

In the third step Z-number valued importance weights of criteria are determined, following (4). For example, an importance value for the first criteria is calculated as:

$Z_{w_1} = Z_{r_1} / Z_{r_i}^{Total} = ((0.071,\ 0.111,\ 0.162),\ (0.2816,\ 0.3127,\ 0.3444))$.

The similar methodology is applied to find the corresponding values for alternatives (Tables 3-6). We calculate the geometric mean of each row $Z_{y_{ji}}$ in the matrixes for criteria C_i and alternatives a_j and determine the weights $Z_{x_{ji}}$. Thus, we construct the following decision matrix for alternatives.

For example, Z-number value of geometric mean $Z_{y_{11}}$ for the first alternative and the first criteria is determined as follows:

$Z_{y_{11}} = (Z(A_{11},B_{11}) \times Z(A_{21},B_{21}) \times Z(A_{31},B_{31}))^{1/3} =$

$=(((1,1,1),(0.2,0.3,0.4)) \times ((1/4,1/3,1/2),(0.2,0.3,0.4)) \times ((4,5,6),(0.2,0.3,0.4)))^{1/3}$

$Z^{Total}_{y_1} = Z_{y_{11}} + Z_{y_{21}} + Z_{y_{31}} =$

$=((2.62,3.05,3.69),(0.2,0.22,0.25))$

$Z_{x_{11}} = Z_{y_{11}} / Z^{Total}_{11} = (0.99,1.18,1.4),(0.17,0.21,0.26))$ /((2.62, 3.05, 3.69), (0.2, 0.22, 0.25))=((0.27, 0.39, 0.57), (0.17, 0.18, 0.2))

Thus, we determine Z-number values of utilities, following (5):

$$Z_{U(a_1)} = Z_{w_1} \times Z_{x_{11}} + Z_{w_2} \times Z_{x_{12}} + Z_{w_3} \times Z_{x_{13}} + Z_{w_4} \times Z_{x_{14}} + Z_{w_5} \times Z_{x_{15}} + Z_{w_6} \times Z_{x_{16}} + Z_{w_7} \times Z_{x_{17}}$$

$Z_{U(a_1)} =$ ((0.43, 0.92, 1.84), (0.062, 0.065, 0.068)),

$$Z_{U(a_2)} = Z_{w_1} \times Z_{x_{21}} + Z_{w_2} \times Z_{x_{22}} + Z_{w_3} \times Z_{x_{23}} + Z_{w_4} \times Z_{x_{24}} + Z_{w_5} \times Z_{x_{25}} + Z_{w_6} \times Z_{x_{26}} + Z_{w_7} \times Z_{x_{27}}$$

$Z_{U(a_2)} =$ ((0.088, 0.16, 0.34), (0.077, 0.079, 0.08)),

$$Z_{U(a_3)} = Z_{w_1} \times Z_{x_{31}} + Z_{w_2} \times Z_{x_{32}} + Z_{w_3} \times Z_{x_{33}} + Z_{w_4} \times Z_{x_{34}} + Z_{w_5} \times Z_{x_{35}} + Z_{w_6} \times Z_{x_{36}} + Z_{w_7} \times Z_{x_{37}}$$

$Z_{U(a_3)} =$ ((0.31, 0.59, 1.24), (0.1, 0.11, 0.118)).

For ranking of Z-numbers we apply approach suggested in [17]. Thus, the first alternative will be an optimal one.

Conclusion

This paper investigates the possibility of technical university choice applying a Z-number valued Analytical Hierarchical Process analysis based on imperfect information. Direct computation with Z-numbers is very important solving the problems with imprecise, uncertain and non-reliable information in decision analysis, economics, management, forecasting, optimization and etc. fields.

Table 5. Comparison of alternatives for criteria C_5 and C_6

C_5	A_1	A_2	A_3	C_6	A_1	A_2	A_3
A_1	((1,1,1), (0.2,0.3,0.4))	((4,5,6), (0.6,0.7,0.8))	((1/4,1/3,1/2), (0.4,0.5,0.6)	A_1	((1,1,1), (0.2,0.3,0.4))	((4,5,6), (0.3,0.4,0.5))	((1/4,1/3,1/2), (0.4,0.5,0.6))
A_2	((1/6,1/5,1/4), (0.6,0.7,0.8))	((1,1,1), (0.2,0.3,0.4))	((1/8,1/7,1/6), (0.4,0.5,0.6))	A_2	((1/6,1/5,1/4), (0.3,0.4,0.5))	((1,1,1), (0.2,0.3,0.4))	((2,3,4), (0.2,0.3,0.4))
A_3	((2,3,4), (0.4,0.5,0.6))	((6,7,8), (0.4,0.5,0.6))	((1,1,1), (0.2,0.3,0.4))	A_3	((2,3,4), (0.4,0.5,0.6))	((1/4,1/3,1/2), (0.2,0.3,0.4))	((1,1,1), (0.2,0.3,0.4))

Table 6. Comparison of alternatives for criterion C_7

C_7	A_1	A_2	A_3
A_1	((1,1,1), (0.2,0.3,0.4))	((4,5,6), (0.3,0.4,0.5))	((1/4,1/3,1/2), (0.4,0.5,0.6))
A_2	((1/6,1/5,1/4), (0.3,0.4,0.5))	((1,1,1), (0.2,0.3,0.4))	((1/6,1/5,1/4), (0.6,0.7,0.8))
A_3	((2,3,4), (0.4,0.5,0.6))	((4,5,6), (0.6,0.7,0.8))	((1,1,1), (0.2,0.3,0.4))

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