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## INFLUENCE OF A STRONG MAGNETIC FIELD ON FERMI ENERGY OSCILLATIONS IN TWO-DIMENSIONAL SEMICONDUCTOR MATERIALS

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# Influence of a strong magnetic field on Fermi energy oscillations in two-dimensional semiconductor materials

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**Abstract:** This article discusses the effect of a quantizing magnetic field and temperature on Fermi energy oscillations in nanoscale semiconductor materials. A generalized mathematical expression is obtained for calculating the dependence of the Fermi energy oscillations on the magnetic field, temperature, and thickness of the quantum well. It is shown that the Fermi energy in nanoscale semiconductor materials under a quantizing magnetic field is quantized. The proposed theory explains the experimental results in two-dimensional semiconductor structures with the help of parabolic dispersion law.

**Keywords:** quantizing magnetic field, temperature, Fermi energy, nanoscale semiconductors, quantum well.

**Аннотация:** В данной статье рассмотрено влияние квантующего магнитного поля и температуры на осцилляции энергии Ферми в наноразмерных полупроводниковых материалах. Получено обобщенное математическое выражение для вычисления зависимости осцилляции энергии Ферми от магнитного поля, температуры и толщины квантовой ямы. Показано, что энергия Ферми в наноразмерных полупроводниковых материалах под влиянием сильного магнитного поля квантована. Предложенная теория объясняет экспериментальные результаты в двумерных полупроводниковых структурах с параболическим законом дисперсии.

**Ключевые слова:** квантующее магнитное поле, температура, энергия Ферми, наноразмерные полупроводники, квантовая яма.

**Аннотация:** Ушбу мақолада нано ўлчамли яримўтказгичли материалларда Ферми энергияси осцилляцияларига квантловчи магнит майдон ва ҳароратнинг таъсири кўрилган. Ферми энергияси осцилляцияларининг магнит майдон, ҳарорат ва квант ўранинг қалинлигига боғлиқлигини ҳисоблаш учун умумлашган математик ифода олинган. Кучли магнит майдон таъсирида наноўлчамли яримўтказгичли материалларда Ферми энергиясининг квантланиши кўрсатилган. Параболик дисперсия қонуни учун икки ўлчовли яримўтказгичли структураларда тавсия этилган назария билан эксперимент натижалари тушунтирилган.

**Калит сўзлар:** квантловчи магнит майдон, ҳарорат, Ферми энергияси, наноўлчамли яримўтказгичлар, квант ўра.

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## I. INTRODUCTION

At present, interest to studying the properties of two-dimensional electronic systems is due to the prospects for their application in nanoscale semiconductor structures. In such systems, the quantum dimensional quantities of the dependence of the characteristics have, as a rule, an oscillating character [1-9]. In two-dimensional semiconductors, macroscopic energy characteristics such as the density of states, effective masses of electrons, and the Fermi energy depend on the thickness of the quantum well. It is assumed that the size of the thickness of the materials  $d$  will be commensurately equal to the de Broglie wavelength of the electron in low-dimensional semiconductors. Let us consider a degenerate (for definiteness - electron) Fermi gas in a magnetic field that is weak compared to

the Fermi energy. In the absence of a field, each state is occupied by two electrons that differ in the projection of the spin to the chosen direction, the energy is determined only by the kinetic energy (momentum) and does not depend on the projection of the spin. If a magnetic field is applied to this system, then one of the states (with a magnetic moment directed along the field) becomes energetically more favorable and there should be more electrons in this state.

As already noted, since the motion in cyclotron orbits is finite, in accordance with the general rules of quantum physics, it must be quantized. Qualitatively, the quantization result can be obtained using the Bohr-Sommerfeld rule. When moving in a cyclotron orbit  $p \cdot 2\pi r = 2\pi m\omega_c r^2 = n\hbar$ . Where is the radius of

the quantized orbit  $r_n = \sqrt{\frac{n\hbar}{2\pi m\omega_c}} = \sqrt{n\frac{c\hbar}{eB}}$ , and the

energy of the quantized motion  $E_n = \frac{p_n^2}{2m} = \frac{n^2\hbar^2}{2mr_n^2} = n\hbar\frac{eH}{2mc} = n\frac{\hbar\omega_c}{2}$ . This answer

gives an idea of the effect in order of magnitude, but, as we will see below, it differs from the exact one by a factor of two. The exact solution to this problem requires the solution of the Schrödinger equation in a magnetic field. Let us consider electrons as non-interacting, then we get a simple one-particle problem. We will use the fact that in a magnetic field the momentum of a particle is

renormalized  $\vec{p} \rightarrow \vec{p} - \frac{e}{c}\vec{A}$ . Where,  $\vec{A}$  is the vector

potential of the magnetic field ( $\vec{B} = \text{rot } \vec{A}$ ), which for this problem is conveniently chosen in the form  $\vec{A} = (0, Bx, 0)$  (the field is directed along Z). The Schrödinger equation for a free electron has the form:

$$\frac{1}{2m}(p_x^2 + (p_y - \frac{e}{c}Bx)^2 + p_z^2)\psi(x, y, z) = E\psi(x, y, z) \quad (1)$$

solution in the form  $\psi(x, y, z) = e^{ik_y y} e^{ik_z z} \psi(x)$ . The variable z immediately separates and gives just free movement along the Z axis with a contribution

to the energy  $\frac{\hbar^2 k_z^2}{2m}$ . For motion in the plane, by

substitution, we obtain

$$\frac{1}{2m}(-\hbar^2 \psi'' + (\hbar^2 k_y^2 - 2k_y \hbar \frac{e}{c} Bx + \frac{e^2}{c^2} B^2 x^2)\psi) = \omega\psi \quad (2)$$

$$-\frac{\hbar^2}{2m}\psi'' + \frac{e^2 B^2}{2mc^2}(x - \frac{\hbar c k_y}{eB})^2 \psi = \omega\psi \quad (3)$$

The equation coincides with the equation of a harmonic oscillator with an equilibrium position at

the point  $x_0 = \frac{\hbar c k_y}{eB}$ , called the position of the leading center. The “hardness” of the oscillator is

$\kappa = \frac{e^2 B^2}{mc^2}$ , respectively, the characteristic frequency that determines the quantum of the oscillator energy

$\omega_c = \sqrt{\frac{\kappa}{m}} = \frac{eB}{mc} = \frac{m_0}{mc} \frac{2\mu_B B}{\hbar}$ , which coincides with the classical cyclotron frequency

( $m_0$  is the free electron mass entering the Bohr magneton). The energy of the n-th level is  $W_n = \hbar\omega_c(n + \frac{1}{2})$ , total energy

$E(n, k_z) = \hbar\omega_c(n + \frac{1}{2}) + \frac{\hbar^2 k_z^2}{2m}$ . Let's introduce the

magnetic length  $l_B = \sqrt{\frac{\hbar c}{eB}}$ , taking this designation

into account  $x_0 = l_B^2 k_y$ . The magnetic

length sets the characteristic scale by which the electron moves away from the leading center; it can be interpreted in the semiclassical approach as a value of the order of the radius of the cyclotron orbit for the first Landau level. In a field of 10 T, the magnetic length is  $0.81 \cdot 10^{-6}$  cm. The magnetic flux penetrating the cyclotron orbit,

$F_0 = \pi l_B^2 B = \frac{\pi \hbar c}{e} = 2.05 \cdot 10^{-7} \text{ KOe} \cdot \text{sm}^2$  called the

flux quantum. The energy levels found are the Landau levels of electrons in an external magnetic field. Note that the energy of levels with different coordinates of the leading center coincides, the Y-component of the wave vector enters only in the coordinate of the leading center.

Thus, to find the equilibrium configuration, we need to minimize the sum of the kinetic energy of electrons of two polarizations and the Zeeman energy, taking into account the constancy of the total number of electrons. If  $n_0$  is the total concentration

of electrons,  $n_{\pm} = \frac{n_0}{2} \pm \delta n$  – concentration of

favorable and unfavorable polarizations (this designation automatically preserves the total number of electrons), then the radii of Fermi spheres for electrons of favorable and unfavorable polarizations

$k_{\pm} = \sqrt[3]{6\pi^2(\frac{n_0}{2} \pm \delta n)}$  (note that under the

root  $6\pi^2$ , not  $3\pi^2$ , since we now consider the Fermi radii without spin degeneracy). Recalling that the average kinetic energy of a degenerate Fermi gas is

$\frac{3}{5}E_F$ , we can write down the total energy, which

is then minimized with respect to  $\delta n$ :

$$E = n_+ \frac{3}{5} \frac{\hbar^2}{2m} k_+^2 + n_- \frac{3}{5} \frac{\hbar^2}{2m} k_-^2 - 2\delta n \mu_B B \quad (4)$$

The g-factor is assumed to be 2. The calculations can be significantly simplified using the concept of the density of states. Indeed, if  $\mu_B \ll E_F$  (which is true for metals in all reasonable fields),

then the redistribution of electrons will occur only near the Fermi surface. Density of states (the number of states in a unit energy range) at the Fermi surface for a degenerate Fermi gas.  $D(E_F) = \frac{3n}{2E_F}$ ,

where  $n$  is the total concentration of electrons. This density of states is total, taking into account both polarizations; for each of the polarizations, the number of states is half. When the field is turned on, the electron energy will change by  $\pm\mu_B B$ , respectively, and the electron concentrations of the corresponding polarizations changed by  $\pm\frac{D}{2}\mu_B B$ .

From where we immediately find the total magnetic moment of a unit volume  $M = D\mu_B^2 B$  and, finally, the susceptibility of a unit of volume

$$\chi_{para} = D\mu_B^2 = \frac{3n\mu_B^2}{2E_F} = \frac{m\mu_B^2}{\hbar^2} \sqrt{\frac{3n}{n^4}}.$$

The resulting magnetic moment is paramagnetic (directed along the applied field), independent of temperature (at  $k_T \ll E_F$ ). This effect is called Pauli paramagnetism. Note that the mass entering into the paramagnetic susceptibility is the effective mass of the electron (it is the effective mass that enters into the density of states on the Fermi surface). Landau levels describe quantized motion in a plane perpendicular to the field. For physical applications, it is essential to check how the density of states will change due to the appearance of this quantization. To find the conditions for quantization, consider a sample in the form a rectangular parallelepiped with edges  $L_x, L_y$  and  $L_z$ , all lengths are considered to be much greater than the magnetic length. The coordinates of the leading center depend on  $k_y$ . At the same time, the center itself must lie in the range from 0 to  $L_x$ ; otherwise, due to the rapid decrease in the wave function of the oscillator, the probability of detecting such a state in the sample will tend to zero. By imposing periodic boundary conditions along  $Y$  and  $Z$ , we obtain the usual condition that one state has an element of the phase volume of the  $k$ -space  $\frac{2\pi}{L_{y,z}}$ . Therefore, in total in the  $XY$  plane

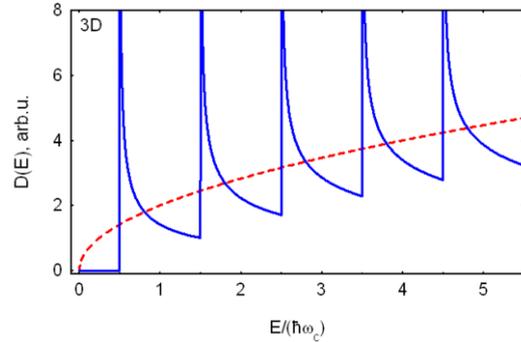
for each value of  $k_z$  there is  $\frac{1}{l_B^2} \frac{L_x L_y}{2\pi}$  states at each

Landau level. And in the interval  $dk_z$  at each Landau level there will be  $dN = \frac{V}{(2\pi)^2} \frac{1}{l_B^2} dk_z$  states.

To find the density of states as a function of energy, consider the  $n$ th Landau level, the electron

energy at this level  $E(n, k_z) = \hbar\omega_c(n + \frac{1}{2}) + \frac{\hbar^2 k_z^2}{2m}$

has a minimum value  $\hbar\omega_c(n + \frac{1}{2})$ , each value is twofold degenerate in the  $k_z$  direction and twofold degenerate in spin (we neglect the interaction of the spin moment with the field, considering the purely orbital motion of the electron).



**Fig. 1.** Schematic representation of the density of states in a three-dimensional Fermi gas with allowance for the quantization of Landau levels. The dotted line is the density of states in the absence of a field.

Whence the contribution of this level to the density of states:

$$D_n(\varepsilon) = 4 \frac{V}{(2\pi)^2} \frac{1}{l_B^2} \frac{d|k_z|}{d\varepsilon} = 4 \frac{V}{(2\pi)^2} \frac{1}{l_B^2} \frac{\sqrt{2m}}{\hbar} \frac{1}{\sqrt[3]{\varepsilon - \hbar\omega_c(n + \frac{1}{2})}} \tag{5}$$

total density of states  $D(\varepsilon) = \sum_{n=0}^{\infty} D_n(\varepsilon)$ . (Fig.1)

Thus, instead of a monotonic dependence of the density of states on energy, periodic sharp maxima appear in it. As we recall, many of the properties of a metal depend on the density of states at the Fermi level - therefore, we can expect that these properties will change dramatically depending on whether the position of the Fermi level falls on the peak of the density of states or on one of the minima.

The presence of a spin in the electron will lead to the splitting of the Landau level into two spin sublevels. If the motion in the cyclotron orbit is independent of the spin degree of freedom, then this will simply lead to a shift of the spin sublevels by  $\pm\frac{g}{2}\mu_B B$  the position of the unsplit level, where  $g$  is the electronic  $g$ -factor. For free electrons  $g = 2$  and  $m = m_0$ , which would lead to the fact that the Zeeman splitting  $\Delta E_{Zeeman} = g\mu_B B$  would coincide with the cyclotron  $\Delta E_c = \hbar\omega_c = 2\mu_B B$ . As a result, the energies of the sublevel of the  $n$ th Lan-

dau level with the Zeeman addition  $+\mu_B B$  and the sublevel of the  $(n+1)$  th Landau level with the Zeeman addition  $(-\mu_B B)$  would coincide. It is necessary, however, to bear in mind that we are not talking about a free electron, but about a quasiparticle in a solid. For such quasiparticles, the cyclotron splitting is determined by the effective mass  $\hbar\omega_c = \frac{m_0}{m} 2\mu_B B$ , a  $g$ - the factor can also change

quite strongly (in problems of assignment there are examples with values of the  $g$ -factor of an electron from  $\approx 1$  to  $\approx 60$ ). Therefore, the relationship between cyclotron and Zeeman splitting can vary within fairly arbitrary limits, and a general approach to constructing the structure of electron levels in a magnetic field with allowance for spin sublevels is impossible. In the literature, the term ‘‘Landau level’’ can be understood as a problem without taking into account spin splitting, which we considered, and a spin sublevel with a certain value of  $S_z$ .

Let us note on a qualitative level that during the orbital motion of electrons in a cyclotron orbit, each electron creates a magnetic field directed against the external one. That is, the orbital motion along cyclotron orbits should lead to a diamagnetic response of the system of electrons - that is, the effect opposite in sign to the Pauli paramagnetism considered above. To find the effect, you need to calculate the total energy. Consider the case of low temperature and weak field  $\mu_B B \ll T \ll \varepsilon_F$ . The smallness of the temperature makes it possible to replace the Fermi distribution with a step, the smallness of the temperature and field makes it possible to neglect the change in the Fermi energy, the smallness of the field in comparison with temperature ‘‘smears out’’ the irregularities in the dependence of the density of states on energy by thermal fluctuations and allows the density of states to be considered a smooth function. We neglect the spin splitting.

For compactness, let us introduce the notation

$\varepsilon_n = \hbar\omega_c (n + \frac{1}{2})$ . Then the energy

$$E = \int_0^{\varepsilon_F} (\varepsilon - \varepsilon_F) D(\varepsilon) d\varepsilon = \int_0^{\varepsilon_F} \sum_n (\varepsilon - \varepsilon_F) D_n(\varepsilon) d\varepsilon \quad (6)$$

For the convenience of subsequent calculations, the Fermi energy has been subtracted, which is not important since we are interested in the field-dependent correction. Taking into account the admissible values, the integration of each term must be carried out on  $\varepsilon_n$  and the summation is limited from above by the condition  $\varepsilon_n < \varepsilon_F$ . We integrate piece by piece:

$$\begin{aligned} f(n) &= \int_{\varepsilon_n}^{\varepsilon_F} (\varepsilon - \varepsilon_F) D_n(\varepsilon) d\varepsilon = \\ &= \frac{V}{\pi^2 l_B^2} \frac{\sqrt{2m}}{\hbar} \int_{\varepsilon_n}^{\varepsilon_F} (\varepsilon - \varepsilon_F) d\sqrt{\varepsilon - \varepsilon_F} = \\ &= -\frac{V}{3\pi^2 l_B^2} \frac{\sqrt{2m}}{\hbar} (\varepsilon_F - \varepsilon_n)^{\frac{3}{2}} \end{aligned} \quad (7)$$

In a small field, the maximum value of  $n$  is large. For large  $n$ , summation can be replaced by integration using a variation of the Euler-Maclaurin formula:

$$\sum_{n=0}^{n_{\max}} f(n) = \int_{-\frac{1}{2}}^{n_{\max}+1/2} f(n) dn - 1/24 \left[ f'(n_{\max} + \frac{1}{2}) - f'(-\frac{1}{2}) \right] \quad (8)$$

Further, we note that during integration it will be necessary to change the variable from  $dn$  to  $d\varepsilon_n$  and, as a result, the integral will not depend on the field  $B$  - hence, it will give (all or partly) the total energy of the Fermi gas in the absence of a field, which is not of interest to us. Derivative

$$\begin{aligned} \frac{df}{dn} &= \frac{df}{d\varepsilon_n} \cdot \frac{d\varepsilon_n}{dn} = \frac{2V}{3\pi^2 l_B^2} \frac{\sqrt{2m}}{\hbar} \frac{3}{2} \sqrt{\varepsilon_F - \varepsilon_n} \hbar\omega_c \quad (9) \\ \varepsilon_{-1/2} &= 0, \quad \varepsilon_{n_{\max}+1/2} \approx \varepsilon_F \end{aligned}$$

Thus, for the field-dependent part of the energy:

$$\begin{aligned} E &= E_0 + \frac{1}{24} \frac{V}{\pi^2} \frac{eB}{\hbar c} k_F \hbar \frac{eB}{mc} = \\ &= E_0 + \frac{V}{24\pi^2} \frac{e^2}{mc^2} \sqrt[3]{3\pi^2 n} \cdot B^2 = E_0 + \frac{V}{6} \frac{m\mu_B^2}{\hbar^2} \sqrt[3]{\frac{3n}{\pi^4}} \cdot B^2 \end{aligned} \quad (10)$$

We see that the energy really grows with the field, that is, there is a diamagnetic contribution. This contribution is called Landau diamagnetism. By double differentiation with respect to the field, we find the diamagnetic susceptibility of a unit volume:

$$\chi_{dia} = -\frac{1}{3} \frac{m\mu_B^2}{\hbar} \sqrt[3]{\frac{3n}{\pi^4}} = -\frac{1}{3} \chi_{para} \quad (11)$$

Thus, the diamagnetic contribution is three times less than the paramagnetic one. It would seem from our conclusion that every electron gas must be paramagnetic. However, it is known from experiment that some metals (copper, gold) are diamagnetic. One of the reasons for the diamagnetism of a metal is that, generally speaking, for a metal with a complex Fermi surface and a non-quadratic spectrum, the effective masses in calculating the paramagnetic susceptibility (‘‘dynamic’’ effective mass at the Fermi surface) and diamagnetic susceptibility (cyclotron mass) can differ markedly both in one and the other direction.

As is known, the energy spectrum of electrons has highly variable properties depending on the relative position of the Fermi level with respect to the Landau levels in two-dimensional semiconductors in the presence of a quantizing magnetic field. All electron gases have a single Fermi level  $\mu$ , which at absolute zero temperature determines the level of filling the energy bands with electrons. As is known from the experimental and theoretical data [10-14], in two-dimensional semiconductors, the Fermi surface at absolute temperature is characterized by rather high amplitudes of the Fermi energy oscillations ( $\mu$ ). But, for a three-dimensional electron gas, the  $\mu$  oscillations will be very weak, even at low temperatures. In three-dimensional semiconductors,  $\mu$  changes only linearly, as in classical magnetic fields.

When studying the electronic and magnetic properties of two-dimensional electronic systems, an important characteristic is the Fermi energy, which determines the main contribution to micro and nanoscale semiconductors. Therefore, the purpose of this work is to analyze the effect of a quantizing magnetic field on dimensional oscillations of the Fermi energy in two-dimensional semiconductor structures.

## II. METHODS

For low-dimensional semiconductor structures and metals, the condition  $\varepsilon_F \gg \hbar\omega_c$  is always satisfied, which means that a large number of Landau levels are always filled for metals. When the field changes, the distance between these levels' changes and when the condition  $\varepsilon_F = \hbar\omega_c(n + \frac{1}{2})$ . The next Landau level crosses the Fermi surface. Due to the large number of occupied levels, the situation when there is  $n \gg 1$  or  $(n-1) \gg 1$  levels under the Fermi surface are almost the same - only the location of the Fermi surface relative to the nearest Landau level is important. Consequently, we can expect the appearance of some oscillations in the properties of the metal when the next Landau level passes through the Fermi surface.

A necessary condition for observing such oscillations is the smallness of the temperature in comparison with the cyclotron splitting of levels - for observing oscillatory effects,  $T \ll \hbar\omega_c$  is required. Before the development of modern techniques such as APRES, oscillation effects, along with cyclotron resonance, were the main methods for studying Fermi surfaces [1]. Now they are used to study the properties of exotic conductors [8] due to the technical simplicity of their implementation

and the possibility of placing the sample in conditions of ultra-low temperatures, strong magnetic fields [9], [10], [11], [12] or high pressures [15].

Conductivity oscillations associated with the passage of Landau levels through the Fermi surface are called the Shubnikov-de Haas effect. This phenomenon was discovered in 1930 on bismuth by the Soviet physicist L.V. Shubnikov and the Dutch physicist W. de Haas working together in Leiden. The physical reason is obvious: the conductivity of the metal  $\sigma = \frac{1}{3}e^2(v^2\tau D(\varepsilon))_\mu$  is determined, among other things, by the density of states at the Fermi surface. Therefore, a periodic change in the density of states (Fig.1) when the Landau level crosses the Fermi surface will lead to resistance oscillations. Conductivity maxima are observed provided that the Fermi energy coincides with the next extremum on the  $D(\varepsilon)$  dependence (Fig.2).

$$\varepsilon_F = \hbar\omega_c(n + \frac{1}{2}) = \hbar \frac{eB}{mc}(n + \frac{1}{2}) \quad (12)$$

That is, the experimentally measured dependence  $\sigma(B)$  should have regular equidistant maxima when plotted as a function of the inverse field  $1/B$ . Oscillations in the coordinates of the reverse field are traditionally characterized by the "frequency"

$$F = \frac{\varepsilon_F mc}{\hbar e}$$

strength (tesla). In the presence of several cyclotron masses, several "frequencies" and "beats" will be observed on the  $\sigma(1/B)$  plot. An example of such data measured on a compound from the "heavy fermion" class. The measurements were carried out in fields up to 20 T at temperatures up to 35 mK. It can be seen that, against the background of a general (monotonic up to almost maximum field) growth of the resistance with the field, resistance oscillations with an amplitude of about 1% arise. The increase in the amplitude of the oscillations with the field is due to the fact that the distance between the Landau levels increases with increasing field, and thermal fluctuations have an ever-smaller effect. For the same reason, the amplitude of the oscillations increases with decreasing temperature. Thus, an analysis of the temperature dependence of the oscillation amplitude (for this purpose, the Lifshitz-Kosevich theory was developed) makes it possible to extract information on the magnitude of the splitting of the Landau levels, that is, on the cyclotron mass. The observed oscillations correspond to an effective electron mass of about 13 masses of a free electron. An example of the observation of Shubnikov oscillations in fields up to 60 T (pulsed magnetic field at the National Laboratory for High

Magnetic Fields in Talahasi, Florida, USA) and at high pressures of more than 10,000 atmospheres, observed in related quasi-two-dimensional organometallic conductors.

When deriving the diamagnetic response of the Fermi gas, we assumed the temperature to be large in comparison with the field, which made it possible to consider the density of states to be smeared by thermal fluctuations to smoothness and to use an approximate formula for summation when calculating the energy. Under conditions when the cyclotron splitting exceeds the temperature, this cannot be done, the exact calculations are rather cumbersome and we will not present them [3]. We only point out that, within the framework of our calculations, small energy oscillations will arise when the difference in the position of the last Landau level from the Fermi energy is taken into account: in the previous calculation, we set  $\varepsilon_{n_{\max}} + 1/2 \approx \varepsilon_F$ , which completely vanished one of the terms. Small oscillations of energy after differentiation can turn into noticeable oscillations of magnetization. The periodicity of these oscillations is determined in the same way as for the resistance oscillations discussed above. For the experimenter, this method is of interest due to the even greater simplicity of measurement than resistance - when measuring under extreme conditions, it is not necessary to lead a wire to the sample. When placed in a magnetic field gradient, the bending of the cantilever is proportional to the magnetization of the sample. In the experiment, field modulation was used to increase the sensitivity; therefore, the observed signal is proportional to the derivative of the magnetic susceptibility. The Fermi surface of gold consists of spheres connected by constrictions at the boundaries of the first Brillouin zone. Therefore, in different orientations of the magnetic field, different extreme cross sections of the Fermi surface contribute to the cyclotron mass. In one of the cases, there is an overlap of two oscillation frequencies. Without going into details of the analysis, we note that this method makes it possible to detect the presence of specific features of the Fermi surface and the presence of carriers with different effective masses, and to determine the effective (cyclotron) mass of carriers and their concentration.

It is known that in  $k$ -space the isoenergy surfaces  $E(k)=const$  are closed and are represented in the form of a sphere. The allowed energy states have a constant density  $V/8\pi^3$  and are distributed in  $k$ -space. Here,  $V$  is the volume of the crystal. Since two opposite orientations of the spin of the electron state are responsible for each value of  $k$ , the wave numbers of all states that will be filled have values

no more than  $k_F$  in the volume of the crystal  $V$ , according to the Pauli principle, and  $k_F$  is determined [15]:

$$\frac{4}{3}\pi k_F^3 \frac{2V}{8\pi^3} = N^{3d} \quad (13)$$

$$\text{From here } k_F = \left( \frac{3\pi^2 N^{3d}}{V} \right)^{1/3} \quad (14)$$

Here,  $N^{3d}$  is the number of electrons for a three-dimensional electron gas.

If the system of electrons is due to the Fermi-Dirac statistics, then the energy in the ground state, i.e. at absolute temperature, called maximum:

$$E_F = \frac{\hbar^2 k_F^2}{2m} \quad (15)$$

$E_F$  - Called the Fermi energy for 3D electron gas. The Fermi surface will have a spherical shape with a radius of  $k_F$  for the isotropic dispersion law. The expressions given above are obtained only for bulk materials and do not consider changes in the Fermi energy oscillations in two-dimensional electron gases.

Now, consider the dependence of the Fermi energy on the quantizing magnetic field in two-dimensional electron gases. In the absence of a magnetic field in two-dimensional electron gases, the electron energy is quantized along the Z-axis, so the electron moves freely only in the XY plane. These quantizations are called dimensional quantization. But, if the magnetic induction  $B$  is directed perpendicular to the XY plane, then the free energy of the electron is also quantized along the XY plane.

The question arises: how will the Fermi energy change in two-dimensional electron gases in the presence of a quantizing magnetic field.

For a 2D electron gas, the allowed energy states have a constant density  $S/4\pi^2$  and are distributed in the XY plane. Here,  $S$  is the surface area crystal. Then, using formulas (1) and (2), we determine the electron concentration for a two-dimensional electron gas:

$$N^{2d} = 4\pi k_F^2 \frac{2L^2}{4\pi^2} \quad (16)$$

$$\text{Hence: } k_F^{2d} = \frac{1}{L} \left( \frac{\pi N^{2d}}{2} \right)^{1/2} \quad (17)$$

Now, we calculate the Fermi energies for a two-dimensional electron gas with a parabolic dispersion law. Substituting (5) to (6), one can determine the Fermi energies in two-dimensional electron gases in the absence of a magnetic field:

$$\mu^{2d} = \frac{p_\mu^2}{2m} = \frac{\pi \hbar^2 N^{2d}}{4mL^2} \quad (18)$$

Here,  $N^{2d}$  is the concentration of electrons for a two-dimensional electron gas,  $L^2$  is the surface of the plane of motion,  $p_\mu$  is the Fermi momentum.

In the motion of a plane perpendicular to the magnetic field, the classical trajectories of electrons are circles. In quantum physics, such trajectories of electrons (periodic rotation of an electron) are equidistant discrete Landau levels:

$$E_n = \hbar\omega_c \left( n_L + \frac{1}{2} \right) \quad (19)$$

Where,  $n_L$  is the number of Landau levels.  $\omega_c = \frac{eH}{mc}$  - cyclotron frequency.

It is known that in three-dimensional semiconductors, a continuous quadratic energy spectrum  $\frac{p_z^2}{2m}$  is added to the energy spectrum of formula (20). However, in two-dimensional semiconductors, the movement of electrons along the Z-axis is quantized:

Indeed, the thickness of the quantum well  $d$  is covered by the dimensional quantization condition, in other words, the thickness is relatively close to the de Broglie wavelength of the electron in the crystal. The motion of an electron along the Z axis is calculated from the potential  $V_z$ :

$$V(z) = \begin{cases} 0, & 0 < z < d, \\ \infty, & z \leq 0, z \geq d \end{cases} \quad (21)$$

In the absence of a magnetic field in two-dimensional electron gases, the normalized wave functions of particles have the following form [15]:

$$\begin{aligned} \psi_{k_{fx}, k_{fy}, n_{fz}}(x, y, z) &= \\ &= \frac{1}{\sqrt{L_{f1}}} \exp(ik_{fx}x) \frac{1}{\sqrt{L_{f2}}} \exp(ik_{fy}y) \varphi_{n_z}(z) \end{aligned} \quad (22)$$

Where,  $k_{fx}$ ,  $k_{fy}$  are the wave numbers for the Fermi energy of electrons,  $n_{fz}$  is the number of dimensional quantizers along the Z axis.

In formula (9), the normalized functions  $\varphi_{n_z}(z)$  in accordance with (8) are written in the following form:

$$\varphi_{n_z}(z) = \sqrt{\frac{2}{d}} \sin \frac{\pi n_z z}{d}, \quad n = 1, 2, 3, \dots \quad (23)$$

The Fermi energies of electrons corresponding to states (9) will be

$$E(k_{fx}, k_{fy}, n_{fz}) = \frac{\hbar^2}{2m} (k_{fx}^2 + k_{fy}^2) + \frac{\pi^2 \hbar^2 n_{fz}^2}{2md^2} \quad (24)$$

Substituting expressions (7), (11) into (6), we obtain the following formula in the presence of a magnetic field:

$$\mu_F(H) = \frac{\pi \hbar^2 N^{2d}(H)}{4mL^2} + \frac{\pi^2 \hbar^2 n_{fz}^2}{2md^2} \quad (25)$$

For an area equal to one ( $L_x L_y = 1$ ) of formulas (12), the following is calculated:

$$\begin{aligned} \mu_F(H) &= \frac{\pi \hbar^2 N^{2d}(H)}{4m} + \frac{\pi^2 \hbar^2 n_{fz}^2}{2md^2} = \\ &= \frac{1}{8} \hbar \frac{eH}{mc} \cdot \frac{2\pi \hbar N^{2d}(H)c}{eH} + \frac{\pi^2 \hbar^2 n_{fz}^2}{2md^2} = \frac{1}{8} \hbar \omega_c \nu + \frac{\pi^2 \hbar^2 n_{fz}^2}{2md^2} \end{aligned} \quad (26)$$

Here,  $\nu = \frac{2\pi \hbar c N^{2d}(H)}{eH}$  - filling factor [16].

This is the number of Landau levels, taking into account their spin splitting, in a quantizing magnetic field, at absolute zero temperature, completely filled with electrons. This dimensionless parameter is used for convenience in discussing quantum oscillatory effects in 2D electron gases.

As can be seen from formula (13), the Fermi energies are quantized if the filling factor is an integer, then the minimum energy quantum will be  $\frac{1}{8} \hbar \omega_c$ , that is, formula (13) gives the exact value of the energy for the first level corresponding to  $\nu = 1$

$$\mu_1(H) = \frac{1}{8} \hbar \omega_c + \frac{\pi^2 \hbar^2 n_{fz}^2}{2md^2} \quad (27)$$

For all other levels, the rigorous theory gives the expression

$$\mu(H) = \hbar \omega_c \left( \nu + \frac{1}{8} \right) + \frac{\pi^2 \hbar^2 n_{fz}^2}{2md^2} \quad (28)$$

Here, the fill factor is an integer,  $\nu = 0, 1, 2, 3, \dots$

In addition, in two-dimensional semiconductors, in the presence of a quantizing magnetic field, the energy spectrum of electrons is purely discrete. A purely discrete energy spectrum, in this case the Fermi energy, is usually characteristic of a quantum dot. In this case, the magnetic induction vector will be directed along the Z axis and perpendicularly along the plane of the transverse two-dimensional layer. In a transverse quantizing magnetic field, quantum wells become analogous to a quantum dot, in which motion is limited in all three directions.

### III. RESULTS

It can be seen from the obtained formulas (15) that the Fermi energies depend strongly on the magnetic field, on the concentration of electrons, and on the thickness of the quantum well. In a quantizing magnetic field, the concentration of electrons in the considered two-dimensional semi-

conductors is determined in the following form [17]:

$$n^{2d} = \int_0^\infty \sum_{n_L} N_s^{2d}(E, H, n_L) f_0(E, E_F(H=0), T) dE \quad (29)$$

Here,  $N_s^{2d}(E, H, n_L)$  - density of states of two-dimensional electronic systems when exposed to a quantizing magnetic field;  $f_0(E, E_F(H=0), T)$  - Fermi-Dirac distribution function in the absence of a magnetic field.

In two-dimensional electronic systems, the energy density of states is taken as the sum of Gaussian peaks in the presence of a magnetic field, disregarding spin splittings [17]:

$$N_s^{2d}(E, H, n_L) = \frac{eH}{2\pi c} \sum_{n_L} \sqrt{\frac{2}{\pi}} \frac{1}{G} \exp \left[ -2 \left( \frac{E - \hbar\omega_c \left( n_L + \frac{1}{2} \right)}{G} \right)^2 \right] \quad (30)$$

$G$  is the broadening parameter taken constant. Here we consider two-dimensional electronic systems of noninteracting electrons according to the parabolic dispersion law at a finite temperature  $T$ , in the presence of a quantizing magnetic field  $B$  parallel to the growth direction.

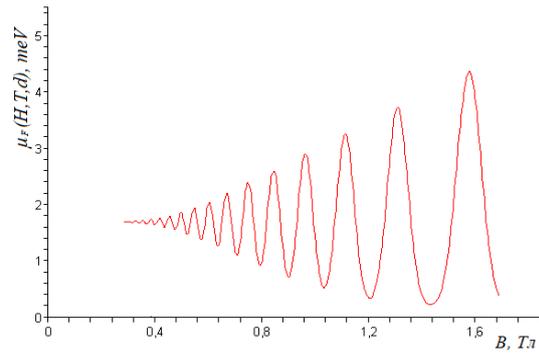
And also, two features should be highlighted here. First, in addition to the Gaussian peak in the density of states, at each Landau level there is a common magnetic field factor  $B$  ahead of the total energy density of states. This means that with increasing magnetic field  $B$ , each Landau level can contain more and more electrons. Secondly, according to the form taken in the formula. According to (17), there is no density of states between Landau levels if their  $\hbar\omega_c$  distance is noticeably larger than  $G$ .

Using expressions (15), (16), and (17), one can determine the dependence of the Fermi energy oscillations on the magnetic field, temperature, and thickness of the quantum well in two-dimensional semiconductors with a parabolic dispersion law without taking into account the spin per unit surface of the plane of motion:

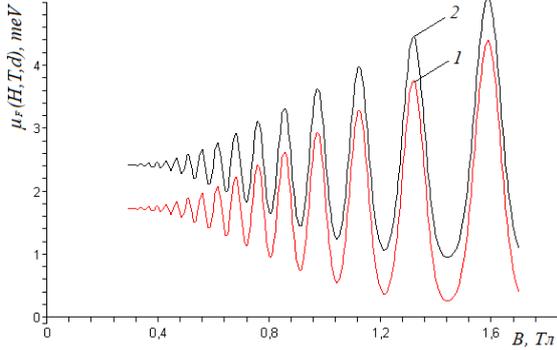
$$\mu_F(H, T, d) = \frac{2\pi\hbar^2}{m} \int_0^\infty \sum_{n_L} N_s^{2d}(E, H, n_L) f_0(E, E_F(H=0), T) dE + \frac{\pi^2\hbar^2 n_F^2}{2md^2} \quad (31)$$

Thus, using formula (18), one can calculate the dependence of the Fermi energy oscillations on the magnetic field, temperature, and thickness of a quantum well with a quadratic dispersion law. As can be seen from formula (18), oscillations of the

density of energy states strongly affect the Fermi energies for two-dimensional electronic systems.



**Fig.2.** Dependence of the Fermi energy oscillations on the quantizing magnetic field with *InAs/GaSb/AlSb* quantum wells at  $T=4.2$  K,  $d=8$  nm. Calculated by the formula (18)



**Fig.3.** Influence of the thickness of the quantum well on the oscillations of the Fermi energy in a quantizing magnetic field. Here,  $T = 4.2$  K, is calculated by formula (18) for *InAs/GaSb/AlSb* quantum wells. 1)  $d=8$  nm, 2)  $d=5$  nm.

Let's analyze the Fermi energy oscillations for two-dimensional semiconductors. Fig.2 shows the dependence of the Fermi energy oscillations on the quantizing magnetic field for *InAs/GaSb/AlSb* quantum wells at a constant temperature and at a constant thickness of the quantum well. Here, temperatures are  $T = 4.2$  K, the thickness of the *InAs/GaSb/AlSb* quantum well is  $d = 8$  nm, the number of Landau levels is  $n_L = 10$ ,  $G = 0.6$  meV,  $E_F = 94$  meV [18]. In this case, doped with Mn with a concentration of  $5 \cdot 10^{16} \text{ cm}^{-3}$  on an *n-InAs* substrate and two quantum wells 12.5 nm in size (*InAs*) and 8 nm (*GaSb*) bounded by two *AlSb* barriers 30 nm thick [18]. As can be seen from the figure, with an increase in the magnetic field, the amplitude of the oscillation of the Fermi energy will increase.

$\mu_F(H, T, d)$  the graph (Fig.2) was constructed using formula (18). In addition, using formula (18), one can also obtain graphs  $\mu_F(H, T, d)$  at different temperatures and at different thicknesses of quantum wells.

We now turn to calculating the dependence of the Fermi energy oscillations on the thickness  $d$  of the quantum well in a quantizing magnetic field with a parabolic dispersion law. We will be interested in the changes in the oscillation of the Fermi energy  $\mu_F(H, T, d)$  at different  $d$  and at constant temperature. It is seen that the formula (18),  $\mu_F(H, T, d)$  is inversely proportional to  $d^2$  with another values constant. Fig.3 shows the oscillations of the Fermi energy in a quantizing magnetic field at different thicknesses of the quantum well  $d$ .

As can be seen from the figure, a decrease in the QW thickness  $d$  leads to an upward motion of the Fermi oscillations. Modern scientific literature indicates that in the absence of a magnetic field, the amplitude of the Fermi energy oscillations strongly depends on the thickness of the quantum well  $d$ .

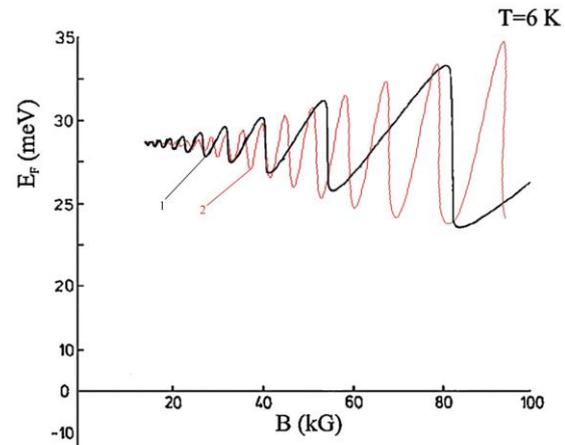
But, as can be seen from Fig.2., the increase in amplitude depends only on the value of the magnetic field, and the thickness of the quantum well  $d$  leads to its motion along the axis  $\mu_F(H, T, d)$ .

#### IV. DISCUSSION

In recent years, two-dimensional semiconductor materials have been the subject of intense theoretical and experimental studies and represent a dynamically developing field of semiconductor physics. The application of a strong magnetic field to two-dimensional semiconductor materials is a powerful tool for experimentally determining the basic parameters of the material, that is, their effective mass, Fermi energy and electron concentration. In quantizing magnetic fields, these parameters determine the relevance of experimental and theoretical studies of magneto-optical and electronic properties of nanoscale semiconductor devices and heterostructures based on them.

Now, let us analyze the oscillations of the Fermi energy of specific low-dimensional materials in a quantizing magnetic field. Fig.4 shows the oscillations of the Fermi energy when measuring  $m=0,0665m_0$ ,  $N=8 \cdot 10^{11} \text{ sm}^{-2}$ ,  $G=0,5 \text{ meV}$  и  $T=6 \text{ K}$  for two-dimensional electron gases in quantum wells (quantum wells, mainly GaAs/GaAlAs heterostructures) [17]. Let us calculate this graph of the quantized Fermi energy by  $\mu_F(H, T, d)$  – functions. When calculating, take the ideal  $\mu_F(H, T, d)$  by formula (18). A comparison of theory with experiment is shown in Fig.4 at various magnetic fields and constant temperatures. Using formulas (18), you can plot graphs  $\mu_F(H, T, d)$  at high temperatures and at different thicknesses of the quantum

well for quantum wells, mainly GaAs/GaAlAs heterostructures. It can be seen that the Fermi energy at a constant electron density is quantized rather strongly as a function of  $B$  in the theoretical and experimental plots in Fig.4.



**Fig.4.** Oscillations of the Fermi energy when measuring  $m=0.0665m_0$ ,  $N=8 \cdot 10^{11} \text{ sm}^{-2}$ ,  $G=0.5 \text{ meV}$  and  $T=6 \text{ K}$  for two-dimensional electron gases in quantum wells (quantum wells, mainly GaAs/GaAlAs heterostructures). 1) experiment [17]; 2) theory calculated by formula (18)

#### V. CONCLUSION

Based on the study, the following conclusions can be drawn: It is shown that the Fermi levels of a nanoscale semiconductor in a quantizing magnetic field are quantized. A method is proposed for calculating the Fermi energy oscillations for a two-dimensional electron gas at different magnetic fields and temperatures. The proposed formula is used to study the experimental results in nanoscale semiconductor structures. Using formula (18), the Fermi energy oscillations are explained for two-dimensional electron gases in quantum wells (quantum wells, mainly GaAs/GaAlAs heterostructures) with a parabolic dispersion law.

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