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DEATH LINE FOR RADIO PULSARS IN BRANEWORLDS

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Abstract

We investigate the effects of braneworlds on energetic processes in plasma magnetosphere of neutron stars. Obtained that the presence of brane charge causes an increase in the size of the polar cap region. It has shown that the polar capsize of neutron stars with larger rotation period and compactness parameter is also larger. We have studied deathline of radio pulsar in braneworld. Obtained that the deathline shifted down and the pulsars which lying on the deathline become visible in radio band. Moreover, we have extended particle acceleration in polar cap of neutron stars in the presence of the brane charge, obtain that in both curvature radiation ($\gamma \sim 10^6$) and inverse Compton scattering ($50 \lesssim \gamma \lesssim 10^4$) cases the acceleration "rate" of the accelerating charged particle is significantly smaller in the presence of the brane charge.

Keywords: Radio pulsar, Braneworld, Death-line, plasma, magnetosphere.

Physics and Astronomy Classification Scheme:: 04.40.Dg, 04.40.Nr, 04.50.Kd

1 Introduction

Neutron stars are strongly magnetized, rotating gravitational compact objects and mostly observed as radio pulsars and magnetars. The studies of magnetic field configuration of neutron stars developed by several authors [1, 2, 3, 4] solving relativistic Maxwell equation considering that neutron stars are dipolar magnetized compact objects in different theories of gravity.

However, a rotating highly magnetized neutron star can not be surrounded by vacuum due to generation of strong induced electric field pulling out charged particles (generally electrons) from the surface of the neutron star [5]. Namely, due to the rotation of a strongly magnetized and highly conducting neutron star spontaneously creates charged magnetosphere around itself. Strong magnetic field and fast rotation of the neutron star induce very strong radial component of electric field about $E_\parallel \sim 10^{10} - 10^{12}$V/cm on the surface of the neutron star that can force the charged particles pull out from the surface of the neutron star and form plasma magnetosphere around the neutron star. The magnetosphere charge density which will be necessary to complete screening of parallel accelerating electric field $E_\parallel$ is called the co-rotation charge density or the Goldreich-Julian density. One direction of an adequate description of the astrophysical process around neutron stars is to include the
effects of the strong gravitational field. In general relativity, [6] and, independently, [7] was the first to find that the frame-dragging induced by general relativistic effects. Last years the studies of the plasma magnetospheric model have been developed by several authors [8, 9, 10, 11] considering several effects on it.

The theory of brane cosmology is a visible three-dimensional universe which restricted to a brane inside a high dimensional space, so-called the "bulk" (well-known as mathematically "hyperspace"). If the additional space is compact, then the observed universe contains the extra dimension and no reference to the bulk appropriate. The braneworld model investigated by Randall and Sundrum in [12]. In static and spherically symmetric case the exterior vacuum solution of the braneworld model has been firstly proposed in [13]. The effects of braneworlds on particle motion and gravitational energy have been studied in detail and deeply discussed by several authors [14, 15, 16, 17].

Importance of studying particle acceleration that whole radiation processes (we pointed out curvature radiation, inverse Compton scattering, synchrotron/cyclotron radiation processes, but thermal radiation is not included) and plasma magnetosphere formation which strongly depends on secondary pair formation depend on particle acceleration. One more question appears, why in the polar cap region? Because, neutron star magnetosphere consists of two parts: closed and open field lines region and the magnetic field lines from the polar cap are open lines, so the radiation generated by these lines reaches us. Radiation in the region where magnetic field lines are closed reflects into the plasma magnetosphere and it is impossible to observe or in other words, no radiation in this region. Condition for the acceleration of charged particle on the polar cap region of the neutron star and $\gamma$ Lorentz factor in different conditions of plasma electrical currents [10, 18, 19] have been analyzed by several authors.

In this work we extended studying energetic processes such as particle acceleration in polar cap region, deathline - the line that describes critical values for (minimum) magnetic field and (maximum) period for radio pulsars which can not carry on the plasma pair production process and stop shining. In Sec.2 we describe the spacetime of a neutron star in braneworld, Sec.3 devoted to study effects of brane charge on polar cap size, in particular polar angle which is between two last closed field lines. In Sec.4 we have studied deathline for radio pulsars in braneworlds in $P - \dot{P}$ space. Finally we have extended studying particle acceleration in polar cap region of neutron stars in braneworlds in Sec.5.

Throughout this work we use $(-, +, +, +)$ for the space-time signature and system of units where $G = c = 1$. Latin indices run from 1 to 3 and Greek ones from 0 to 3.

2 Briefly about the spacetime of neutron star in braneworld

The geometry of the spacetime of a slowly-rotating magnetized neutron star in braneworlds in spherical coordinates $(t, r, \theta, \phi)$ given the following form [13, 14]
\[ ds^2 = -N^2 dt^2 + N^{-2} dr^2 + r^2 \left[ d\theta^2 + \sin^2 \theta \left( d\phi^2 - 2\omega dt d\phi \right) \right], \] (1)

with the following gravitational metric function
\[ N^2 = 1 - \frac{2M}{r} + \frac{Q}{r^2}, \quad r \geq R, \] (2)

where \( M \) and \( R \) are the total mass and the radius of the a neutron star, respectively. The quantity \( Q \) is brane parameter (or Weyl charge) that arises from the extra dimension [13] and it is negatively defined as \( Q \leq 0 \) and in the equation (1) \( \omega \) is the angular velocity of the dragging of an inertial frame in the braneworld that can be written in the following form [4]
\[ \omega = \frac{2J}{r^3} \left( 1 - \frac{Q}{2Mr} \right), \quad \omega_{LT} = \frac{2J}{r^3}, \] (3)

where \( J = I\Omega \) is the total angular momentum, \( I \) is the moment of inertia and \( \Omega \) is the angular velocity of the neutron star. Actually, in the braneworld, the angular momentum of the star will be different and for simplicity here we assume that it is a conserved quantity [4], where angular momentum in braneworld of the star is the general relativistic value of the angular momentum. To find and measure the moment of inertia of neutron stars is one of the most difficult problems in relativistic astrophysics. Here we will assume that the moment of inertia expresses as \( I = \beta MR^2 \), with the normalization coefficient \( \beta \), the value of \( \beta \) in the Newtonian case is \( \beta = 2/5 \). Here we will introduce standard following normalized notations for radial coordinate \( \eta = r/R \) as a dimensionless radial coordinate and compactness \( \epsilon = 2M/R \) of the neutron star and the parameter \( \kappa \) is defined by \( \kappa = \epsilon \beta \) and \( b = |Q|/M^2 \) is dimensionless brane parameter.

3 The size of the polar cap region of a neutron star in braneworld

Polar cap region is the region at the surface of neutron star where magnetic field lines are open. The importance of studying the area that radiation of the star is strongly depending on it, in the polar cap model. The polar cap may be defined by the locus on the surface of the star of the foot-points of the last closed field lines. All field lines from within the polar cap extend beyond the light cylinder and these cannot be co-rotating because that would violate the special theory of relativity beyond the light cylinder. The magnetic flux in the are of polar cap region at the surface of the neutron star is \( \Psi(r) = S_{pc} B^r \) here \( S_{pc} = \pi \Omega R^3/c \) the area of the neutron star. Conservation of the magnetic field flux gives us to chance to write following relation [6]
\[ \Psi(r) \sin^2 \theta = \Psi(R) \sin^2 \theta_0, \] (4)
Figure 1: The dependence of the dimensionless brane parameter on the polar angle for a neutron star. In the right panel, the dependence is shown with the different value of the period of the neutron star with compactness parameter \( \epsilon = 0.4 \) (mass of the star about \( 1.4M_\odot \) and radius \( R = 10km \)), and in the left one, the same relation but for the different value of compactness of the star with rotation period \( P = 0.1s \).

In Fig.1 we show the effect of dimensionless brane parameter on polar angle using equation (4) in the border of light cylinder \( r = R_{lc} \), here \( R_{lc} = c/\Omega \) is radius of the light cylinder. In the right (left) panel we show polar angle as a function of the dimensionless brane parameter in the different values of the period (compact parameter-mass per radius) of a neutron star. In the case of the same compactness parameter, the dimensionless brane parameter slightly effects on the polar angle and the angle increases with increasing of the value of the dimensionless brane parameter this also indicates that the braneworld acts as additional gravity. An increase of the pulsar rotation period causes to increase due to increasing the radius of the light cylinder. One can see from this figure with increasing of the value of the dimensionless brane parameter the polar angle also increases, this can be explained by the fact that the additional gravity property of the braneworld compresses the spacetime of neutron star spherically, resulting in higher magnetic field lines cross the light cylinder than in the case \( (b = 0) \), and it provides the radiation beam from the polar cap becomes narrow like laser beam, it means the shape of radio signals from radio pulsars becomes more vertical line.

4 Death line for radio pulsars

Deathline is the energy condition for pair production in curvature radiation /inverse Compton scattering processes which separates \( P - \dot{P} \) diagram to two regions that only the radio pulsars which located top the death-line can be observable. Pulsars with enough higher magnetic field and angular momentum can make bigger parallel accelerating electric field and it can pull out electrons and/or ions (some models say that at the surface of neutron stars covered by ionized iron atoms and the binding
energy of the ions and/or electrons is in order of the binding energy of the iron atoms). But pulling out of electrons and/or ions is not enough, they need to accelerate higher energies in short distances and radiate photons with higher energy that can produce electron-positron pair production due to intersecting magnetic field lines $\gamma + B \rightarrow e^+ + e^- + B$. The process called secondary pair production. Then, electrons and positrons start accelerate opposite side and in turns the secondary accelerated particles radiate secondary photon which some of them can scatter with accelerating electrons or positrons (it called inverse Compton scattering) and some of them radiate due to curved trajectory of the charged particles. It is show by several authors that probability of inverse Compton scattering process much more that curvature radiation in $\gamma$ ray producing of pulsars. Due to all radiation processes the neutron star loses its kinetic energy and slows-down (when the value magnetic field is constant). The particle acceleration in the polar cap region strongly depends on the (magnetic field) accelerating parallel electric field and the electric field, in turn, proportional to the surface magnetic field and inverse proportional to the period of neutron star rotation. As the neutron star slows-down, the period increases and the acceleration field value decreases and the particles radiate through curvature radiation / inverse Compton scattering photons with less energy which does not enough produce plasma pairs (even two-photon one pair of electron-positron). No further electron-positron pair ejects in the plasma magnetosphere finally the pulsar stops "shining" in the given (radio) band, and the pulsar dies. The idea of "death of pulsar" (pulsar cannot radiate and create $e^\pm$ pairs) have been suggested by Ruderman & Sutherland in Ref. [20]. We have studied the deathline for radio pulsars in our previously paper in $B - P$ space. Here we will study the deathline of radio pulsar in $P - \dot{P}$ space. In order to find deathline equation for radio pulsar for inverse Compton scattering, we start providing the equation which has obtained in our previously [10]

\[
74 \left( \frac{B}{10^{12} \text{G}} \right)^2 P^{-5/2} = \frac{\eta^2}{1 - \frac{1}{\eta^s} + \frac{\epsilon b}{4} \left( 1 - \frac{1}{\eta^s} \right)} = Y(\eta, \epsilon, b) ,
\]

(5)

Now the questions are in which distance the equation (5) will be minimum. So it means that when $Y(\eta, \epsilon, b) > Y_{min}$ the radio pulsar can radiate otherwise the pulsar can not. The analytic form of the critical distance for the accelerated charge, in particular for electron and positron where $Y(\eta, \epsilon, b) = Y_{min}$ can be found a little bit longer form

\[
\eta_{cr} = \frac{2\sqrt{2}}{\beta} \sqrt{b\epsilon + 4} \left( \beta^* + \sqrt{5\sqrt{2}b\epsilon + 8\beta^3 - \beta^4 + \sqrt{b\epsilon + 4b\epsilon}} \right)
\]

(6)

with $\beta = \frac{1}{4}(b\epsilon + 4) \left( \sqrt{16b^3\epsilon^3(b\epsilon + 4) + 625} + 25 \right)$ and $\beta^* = \sqrt{3^4 - b\epsilon(b\epsilon + 4)}$. The distance was analyzed in [10] in plot form. One can calculate the value of the critical distance at $b = 0$ is $r_{cr} = 1.35721R$ which has shown in [10]. One can easy get
minimum expression for $Y_{\text{min}}$ omitting equation (6) into equation (5) and we have

$$Y_{\text{min}} = \frac{\eta_{cr}^2}{1 - \frac{1}{\eta_{cr}^2} + \frac{\epsilon}{4} \left(1 - \frac{1}{\eta_{cr}^2}\right)} = 74 \left[\left(\frac{B}{10^{12} \text{G}}\right)^2 P^{-5/2}\right]_{\text{deathline}}$$

(7)

In order to get deathline equation in $P$ and $\dot{P}$ we use the surface magnetic field of the dipole like neutron star at the poles is [21]

$$B_d = 6.4 \times 10^{19} \left(\frac{P\dot{P}}{P}\right)^{1/2} \left(\frac{I}{10^{45} \text{g} \cdot \text{sm}^2}\right)^{1/2} \left(\frac{R}{10 \text{km}}\right)^{-3} \text{G}$$

(8)

Here $I$ is inertia momentum of neutron star and it is for typical neutron stars $I = 4.3 \times 10^{45} \text{g} \cdot \text{sm}^2$. We will expand the equation (7) in the order of $\epsilon$ and we have

$$Y_{\text{min}} = 3.06971 - 0.904934b \epsilon = 74 \left[\left(\frac{B}{10^{12} \text{G}}\right)^2 P^{-5/2}\right]_{\text{deathline}}$$

(9)

Using equations (8) and (9) one may immediately obtain equation for deathline in $P - \dot{P}$.

$$\log \dot{P} = \frac{2}{3} \log P + \log(3.06971 - 0.904934b \epsilon)$$

(10)

Now we will analyze the equation (10) in the figure form.

The figure 2 shows the dependence of the rate of slow-down $\dot{P}$ of radio pulsar on its period of rotation $P$. One can see from this figure that the deathline of radio pulsar shifted downward in the presence of the dimensionless brane parameter. We provide more than 20 observed pulsars with their period and slow-down rate and show with a, b, c radio pulsars which lie on the death line calculated in general relativity. In this approach general relativity can not explain the observation of a, b, c radio pulsars. Because the pulsars should not be observed. One could explain that the pulsars observed due to the effect of braneworlds.

5 Charged particle acceleration in the polar cap

In this section we will extend the obtained results in our previously paper [10]. In the paper, the system of equation for $\gamma$ factor of the accelerating particle has been obtained in the following form

$$N_s^2 \frac{d}{ds} \left(s^2 \frac{d\phi}{ds}\right) - \frac{l(l + 1)}{s^2} \phi = \frac{B}{B_0} \left(\frac{j}{V} - \bar{j}\right),$$

(11)

$$\frac{d}{ds} (N_s \gamma) = \frac{1}{N_s^2} \frac{d\phi}{ds},$$

(12)
here, the multipole number $l$ and $\gamma$ is the Lorentz factor which is defined as

$$\gamma \equiv -v^\mu u_\mu = \frac{1}{\sqrt{1 - V^2}},$$

where $u^\mu$ is the four-velocity of fiducial observer, $V^i \equiv v^\mu h^i_\mu/\gamma$ is the relative three-velocity with the projection tensor $h^{\alpha\beta} \equiv g^{\alpha\beta} + u^\alpha u^\beta$, and for convenience we introduce the following normalized variables [18]

$$j \equiv -\frac{2\pi N_*(s) \rho G(s)}{\Omega B(s)}, \quad \phi(s) \equiv \frac{e \Phi(s)}{m},$$

$$\bar{j} \equiv -\frac{2\pi N_*(s) \rho G^1(s)}{\Omega B(s)}, \quad s \equiv \sqrt{\frac{2\Omega B e}{mc^2}} r,$$

where $B \equiv |B| = B^r + O(\theta^2)$, $\Phi$ is the scalar potential of the electromagnetic field and $\rho$ is the charge density at the surface of the neutron star. In order to show the behavior of relations between $j$ and $\bar{j}$ at $\chi = 0$, we expand $j$ in power of small parameter $y = r/R - 1$ up to the first order in the following form [18]

$$\bar{j} \simeq \bar{j}_R \left[ 1 + \frac{3\omega_R}{\Omega} (1 - b) y \right],$$

where $\bar{j}_R = 1 - \frac{\omega B}{\Omega} (1 - b)$, with $\omega_R = \omega(r = R)$ and consequently the brane parameter will provide additional radial dependence to $\bar{j}$.

Fig. 3 illustrates the Lorentz factor as a function of the dimensionless normalized coordinate $r/R - 1$ for the different values of the dimensionless brane charge in both
Figure 3: Numerical solution of equations (11) and (12) for the Lorentz factor $\gamma$ with different values of brane charge parameter for the typical neutron star with mass $M = 2\text{km}$, radius $R = 10\text{km}$ and period $P = 0.1\text{s}$ at the value of multipole number $l = 2$ and $\gamma_R = \gamma(r=R) = 1.000001$. Left panel for the case when $j = 0.99j_{\text{GJ}}$, right panel for the case when $j = 1.01j_{\text{GJ}}$. Solid line corresponds to GR case, blue dot-dashed line for $b = 1$ and red dashed one is for $b = 2$.

case which the Goldreich-Julian current density less (at the right panel) and more (at the left panel) than co-rotation current density. One can see the acceleration "rate" of the accelerating charged particle(s) is significantly smaller in the presence of the brane charge due to additional attracting property of braneworlds [10].

One conclude that in the presence of braneworlds the charged particles reach their maximum energy more distances than general relativistic case. In the case $j < \bar{j}$ (at the left panel in Fig.3) the maximum value of the Lorentz factor reach up to about 60 it means that $(50 \lesssim \gamma \lesssim 10^4)$ the regime is corresponds resonance Compton scattering and its important because the photons blue-shifts into the cyclotron resonance in the electron rest frame, greatly increasing the scattering cross-section and thus the energy loss of the accelerated particle. Then the accelerated electrons/positron radiates $\gamma$-rays though curvature radiation or inverse Compton scattering, in turn, the $\gamma$-rays pair produced by the process $\gamma \rightarrow e^+ + e^-$ in the strong magnetic field. In the negative deformation case, the accelerated electrons from the surface of the star radiate at the distance closer to the surface than it is in general relativity and pair production process starts (at smaller distance than it is in general relativity) "earlier". In the presence of the brane charge, the acceleration distance bigger than general relativity case and the pair production process starts "just later". In the case, $j > \bar{j}$ (at the right panel in figure 3) the value of the Lorentz factor at large distances reaches to more than $10^6$ and the CR dominates in their energy loss which can be the source of (ultra)high energy $\gamma$ photons.

Conclusions

In this work, we have considered the effects of dimensionless brane charge on polar capsize and death line for radio pulsars. The following main results have been
obtained:

- obtained that the polar angle of typical neutron stars slightly increases with increasing of the value of the dimensionless brane parameter $b$
- shown that the deathline of radio pulsar shifted downward in the presence of the dimensionless brane parameter
- obtain that in both curvature radiation ($\gamma \sim 10^6$) and inverse Compton scattering ($50 \lesssim \gamma \lesssim 10^4$) cases the acceleration "rate" of the accelerating charged particle is significantly smaller in the presence of the brane charge

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