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FACILITY LOCATION PROBLEM BY USING FUZZY TOPSIS METHOD

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Abstract. Facility location problems are long-term decision making problems for selection best geographical location to begin the operations of a new facility or for expansion of existing facilities. These are strategic investment decision including many factors that may be inconsistent in nature. To solve this problems, some alternatives based on different criteria’s need to be selected. To make a decision on such fuzzy problems, fuzzy multiple criteria decision making can be applied. In this paper, fuzzy multiple criteria decision making with TOPSIS and weighted product (WP) methods is used to select the best location of company facility. The results of solving represent that TOPSIS and Weighted Product fuzzy multiple criteria decision making methods can be used to select the most suitable place for new facility or for expansion of existing facilities.

Key words: Type-2 Fuzzy Sets, TOPSIS, Fuzzy Numbers, Multicriteria decision making, Uncertainty.

Introduction

A facility location problem consists in defining the position of a set of facilities within a given location area on the basis of the distribution of demand to be allocated to the facilities. In the practical applications either in private or in public sector, these problems deal with strategic and long term decisions involving huge investment costs. Selecting the right place with some common factors exist that influence facility location method. The Fuzzy TOPSIS method represents relatively favorable practice samples, especially in realistic problems where individual opinions are defined by linguistic data. For this goals a fuzzy multi factor decision making method, which is an extension of the Fuzzy Technique for Order Preference by Similarity to Ideal Solution (FTOPSIS) approach is used. TOPSIS was suggested by Hwang and Yoon in 1981 [1]. In this method, the basic concept is that the most preferred alternative should have the shortest distance from the Positive Ideal Solution (PIS) and the longest distance from the Negative Ideal Solution (NIS) [2]. Based on Wang and Elhag [3], positive ideal personal is the one that maximizes the benefit criteria and minimizes the cost criteria, while the negative ideal personal functions in the opposite way. As distinction to the original supplementation, of TOPSIS where the weight of the attribute and the ratings of alternatives are known exactly, many decision problems are compared with unquantifiable, imperfect and unapproachable information [4] that make precise judgment impossible. This is when fuzzy TOPSIS comes into play where the criteria weights and alternative ratings are given by linguistic variables, expressed by fuzzy numbers. TOPSIS was extended by Chen [5] to fuzzy environments, which used a fuzzy linguistic value as a substitute for the directly given crisp value in the grade assessment. From our results, the business climate, living conditions, transportation, infrastructure, supplies are the most important in facility location. Fuzzy TOPSIS, that used by this paper presents a solution for decision makers when dealing with real data that are usually multi attributes and involves a complex decision making process. In
this work, using this method is demonstrated in the facility location problem.

1. Preliminaries

Definition 1. Decision-makers define some alternatives that will be selected following several attributes or criteria. A fuzzy set \( \tilde{A} \) in variable \( X \) is determined by a membership function \( \mu_{\tilde{A}}(x) \) which each element \( x \) in \( X \) a real number in the interval \([0,1]\). The value \( \mu_{\tilde{A}}(x) \) is called the grade of membership function of \( x \) in \( \tilde{A} \) [6].

Definition 2. In TOPSIS, the realization of each alternative requires to be sorted with \( x \)-decision matrix; \( i=1,2,\ldots,m; \) and \( j=1,2,\ldots,n \). An element \( r_{ij} \) of the normalized decision matrix \( R \) can be rated as follows [5]

\[
r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}}
\]

Definition 3. Input data are defined in the decision matrix format. A configuration of weights \( W = (w_1, w_2, w_3, \ldots, w_N) \), (where: \( \sum w_i = 1 \)) determined by the managers is provided to the decision matrix to create the weighted normalized matrix \( V \) as follows [5]:

\[
V = \begin{bmatrix}
w_{1}r_{11} & w_{1}r_{12} & \cdots & w_{1}r_{1n} \\
\vdots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
w_{N}r_{N1} & w_{N}r_{N2} & \cdots & w_{N}r_{Nn}
\end{bmatrix}
\]

Definition 4. The weighted normalized fuzzy decision matrix positive ideal solution \( A^* \) and negative ideal solution \( A^- \) can be defined on base of the weighted normalized rating. Positive ideal solution matrix is calculated with function (3), where the negative ideal solution matrix based on function (4):

\[
A^* = \left\{ (\max v_{ij}, j \in J), (\min v_{ij}, j \in J) \right\}
\]

\[
|j = 1,2,3,\ldots,M| = \{v_{i1}, v_{i2}, \ldots, v_{iN}\}
\]

and the negative-ideal \( A^- \) solutions are determined as follows:

\[
A^- = \left\{ (\min v_{ij}, j \in J), (\max v_{ij}, j \in J) \right\}
\]

\[
|j = 1,2,3,\ldots,M| = \{v_{i1}, v_{i2}, \ldots, v_{iN}\}
\]

For this aim a fuzzy multi factor decision making method, which is an evolution of the fuzzy technique for order preference by similarity to ideal solution (FTOPSIS) method is used.

Definition 5. The Euclidean distance method is used to grade the separation distances of each alternative to the positive ideal solution and negative-ideal solution [5].

\[
S_i = \sqrt{\sum(v_{ij} - v_{j}^*)^2}, \ i=1,2,3,\ldots,M,
\]

Definition 6. The relative closeness to the ideal solution of an alternative \( A_i \) with respect to the ideal solution \( A^* \) is defined as follows where \( 0 \leq C_i \leq 1 \), that is, alternative \( i \) is closer to the fuzzy positive ideal reference point and far from the fuzzy negative ideal reference point as \( C_i \) approaches [5]:

Evidently, \( C_i = 1 \) if \( A_i = A^* \) and \( C_i = 0 \) if \( A_i = A^- \)

2. Statement of the problem

Suppose that an multi attribute decision problem involves 5 criteria - \( C_1, C_2, C_3, C_4 \) and 4 alternatives - \( A_1, A_2, A_3, A_4 \). \( C_1 \) - Business climate ; \( C_2 \) - Living conditions ; \( C_3 \) - Transportation ; \( C_4 \) - Infrastructure; \( C_5 \) - Supplies (Table 1). The relative weights of the 5 criteria were determined to be

\( W_1=0.35, \ W_2=0.25, \ W_3=0.20, \ W_4=0.10, \ W_5=0.10 \)

Decision matrix gives the linguistic performance in terms of Type-2 fuzzy numbers. These linguistic performance rating are presented in Table 2.

\[
\begin{array}{c}
\text{Very high } = \{0.8, 0.9, 1.1\} \\
\text{High } = \{0.6, 0.7, 0.8, 0.9\} \\
\text{Average } = \{0.4, 0.5, 0.6, 0.7\} \\
\text{Low } = \{0.2, 0.3, 0.4, 0.5\}
\end{array}
\]

![Fig. 1. Interval-valued approximation to fuzzy number.](image)
3. Solution of the problem

Step 1: We determine the decision matrix of fuzzy ratings of alternatives with respect to criteria and the weights of criteria.

Step 2: In this step we construct the normalized decision matrix. A set of weights $W = (w_1, w_2, w_3, ..., w_N)$, (where: $\sum w_i = 1$) determined by the decision maker to create the weighted normalized matrix $V$ as follows (Table 3):

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Location A1</th>
<th>Location A2</th>
<th>Location A3</th>
<th>Location A4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$ -(Business climate)</td>
<td>Very high</td>
<td>High</td>
<td>Average</td>
<td>High</td>
</tr>
<tr>
<td>$C_2$ -(Living conditions)</td>
<td>Average</td>
<td>High</td>
<td>Very high</td>
<td>High</td>
</tr>
<tr>
<td>$C_3$ -(Transportation)</td>
<td>High</td>
<td>Average</td>
<td>Average</td>
<td>Average</td>
</tr>
<tr>
<td>$C_4$ -(Infrastructure)</td>
<td>High</td>
<td>High</td>
<td>High</td>
<td>Very high</td>
</tr>
<tr>
<td>C5- (supplies)</td>
<td>Low</td>
<td>Average</td>
<td>High</td>
<td>Low</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Linguistic performance rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attribute</td>
</tr>
<tr>
<td>$C_1$ -(Business climate)</td>
</tr>
<tr>
<td>$C_2$ -(Living conditions)</td>
</tr>
<tr>
<td>$C_3$ -(Transportation)</td>
</tr>
<tr>
<td>$C_4$ -(Infrastructure)</td>
</tr>
<tr>
<td>C5- (supplies)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fuzzy decision matrix and fuzzy weight of four candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>W</td>
</tr>
<tr>
<td>A1</td>
</tr>
<tr>
<td>A2</td>
</tr>
<tr>
<td>A3</td>
</tr>
<tr>
<td>A4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fuzzy normalized weighted decision matrix of four candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>A1</td>
</tr>
<tr>
<td>A2</td>
</tr>
<tr>
<td>A3</td>
</tr>
<tr>
<td>A4</td>
</tr>
</tbody>
</table>

Step 3: In this step we determine the positive and the negative ideal solutions. The ideal $A^*$ and the negative ideal $A^-$ solutions are determined as follows:

$$A^* = \left\{ \max_{j \in J} v_j \left| j \in J \right| i = 1,2,3, ..., M \right\}$$

$$A^- = \left\{ \min_{j \in J} v_j \left| j \in J \right| i = 1,2,3, ..., M \right\}$$

For example , $0.28, 0.21, 0.14, 0.21$ - ideal is 0.28, negative is 0.14.

$0.31, 0.24, 0.17, 0.24$ - ideal is 0.31, negative is 0.17.

$0.35, 0.28, 0.21, 0.28$ - ideal is 0.35, negative is 0.21.

$0.35, 0.31, 0.24, 0.31$ - ideal is 0.35, negative is 0.24 (Table 4)
The Ideal and the Negative-ideal Solutions

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A*</td>
<td>0.28,0.31, 0.35</td>
<td>0.2,0.22, 0.25</td>
<td>0.12,0.14, 0.16</td>
<td>0.08,0.09, 0.1</td>
<td>0.08,0.09, 0.1</td>
</tr>
<tr>
<td>A'</td>
<td>0.14,0.17, 0.21</td>
<td>0.1,0.12, 0.15</td>
<td>0.08,0.1</td>
<td>0.06,0.07, 0.08</td>
<td>0.02,0.03, 0.04</td>
</tr>
</tbody>
</table>

**Step 4.** The Euclidean distance method is next applied to measure the separation.

For example, the separation distances of each alternative to the positive ideal solution and negative-ideal solution are given by

$$ S_{ij} = \left( \sum_{i=1}^{n} (v_{ij} - v^*_j)^2 \right)^{1/2}, \quad i = 1, 2, 3, \ldots, n; \quad j = 1, 2, 3, \ldots, m $$

where $v_{ij}$ is the evaluation of the $i$-th attribute of the $j$-th alternative, and $v^*_j$ is the average value of the $j$-th attribute.

For example, the separation distances are:

- $S_{12} = ((0.21 - 0.14)^2 + (0.24 - 0.17)^2 + (0.28 - 0.21)^2 + (0.31 - 0.24)^2) = 0.13$
- $S_{13} = ((0.14 - 0.28)^2 + (0.17 - 0.31)^2 + (0.21 - 0.28)^2 + (0.24 - 0.24)^2) = 0.21$
- $S_{14} = ((0.14 - 0.14)^2 + (0.17 - 0.17)^2 + (0.21 - 0.21)^2 + (0.24 - 0.24)^2) = 0.14$
- $S_{15} = ((0.14 - 0.14)^2 + (0.17 - 0.17)^2 + (0.21 - 0.21)^2 + (0.24 - 0.24)^2) = 0.13$
- $S_{16} = ((0.14 - 0.14)^2 + (0.17 - 0.17)^2 + (0.21 - 0.21)^2 + (0.24 - 0.24)^2) = 0.12$

**Step 5.** Calculate the Relative Closeness to the Ideal Solution. The relative closeness of an alternative $A_i$ with respect to the ideal solution $A^*$ is defined as follows:

$$ C_i = \frac{S_i}{S_i + S_{-i}}, 0 \leq C_i \leq 1 $$

Table 4.
The separation distances of each alternative to the positive ideal solution and negative-ideal solution.

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A*</td>
<td>0.26</td>
<td>0.12</td>
<td>0.21</td>
<td>0.14</td>
<td>0.19</td>
</tr>
<tr>
<td>A'</td>
<td>0.26</td>
<td>0.13</td>
<td>0</td>
<td>0.13</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A*</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>A'</td>
<td>0.08</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A*</td>
<td>0.11</td>
<td>0.075</td>
<td>0</td>
<td>0.11</td>
<td>0</td>
</tr>
<tr>
<td>A'</td>
<td>0.11</td>
<td>0.075</td>
<td>0</td>
<td>0.11</td>
<td>0</td>
</tr>
</tbody>
</table>
For example,  
\[ C_i = S_i / (S_1 + S_i) \]
\[ = (0.26/(0.26+0)+0/(0.19+0)+0.08/(0+0.08)+  
+0/(0.03+0)+0/(0.11+0))/4=0.5 \]
\[ C_1 = (0.13/0.25+0.11/0.19+0/0.8+0/0.3  
+0.04/0.115) =  
[0.52+0.57+0.35]/4=0.36 \]
\[ C_3 = (0+1+0+0+1)/4=0.5 \]
\[ C_4 = (0.48+0.578+0+1+0)/4=0.52 \]
From this calculations we get that  
\[ c_i = (0.5, 0.36, 0.5, 0.52) \]

**Step 6.** Next step is ranking the precedence order. The best place for facility can be determined by using preference rank order of \( c_i \). The best place is the one that has the smallest distance to the ideal solution. The relationship of alternatives represents that any alternative which has the smallest distance to the ideal solution is guaranteed to have the longest distance to the negative-ideal solution (Table 6).

<table>
<thead>
<tr>
<th>C_i</th>
<th>A_1</th>
<th>A_2</th>
<th>A_3</th>
<th>A_4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.36</td>
<td>0.5</td>
<td>0.52</td>
<td></td>
</tr>
</tbody>
</table>

By using fuzzy TOPSIS method the order ranking we determine that  
\[ A_4 > A_1 = A_3 > A_2 \]
The result shows that place (A_4) is the best location and (A_2) is the poor place for facility location.

### 3. Conclusion

In this paper, fuzzy TOPSIS was used in the selection of the best place according to five criteria’s for facility location. First criteria is business climate, second criteria is living conditions, third criteria is transportation, fourth criteria is infrastructure and fifth criteria is supplies. Results determined from the relative closeness to the ideal solutions were used to rank the preference order in the selection of place for facility location.

### REFERENCES