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## THE PROBLEM OF RAMCHUNDRA FOR A PROBLEM OF -CAPTURE

Bahrom Samatov

*NamSU professor and masters at the Department of differential equation and mathematics physic*

Saboxat Uralova

*NamSU professor and masters at the Department of differential equation and mathematics-physic*

Umidjon Mirzamaxmudov

*NamSU professor and masters at the Department of differential equation and mathematics-physic*

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## THE PROBLEM OF RAMCHUNDRA FOR A PROBLEM OF -CAPTURE

Cover Page Footnote

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Erratum

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### ***l*-TUTISH MASALASI UCHUN RAMCHANDR MASALASI**

Samatov Bahrom Tajiahmedovich, Uralova Saboxat Ismoiljon qizi, Mirzamaxmudov Umidjon Alijon o'g'li  
NamDU Differensial tenglamalar va matematik fizika kafedrası proffesori va magistrantlari

*Annatatsiya.* Biz bu maqolada Ramchandr masalasini *l*-tutish differensial o'yini yordamida o'rgandik. Ramchandr masalasini geometrik chegaralangan hol uchun o'rgandik. Qayiqning strategiyasi faqat geometrik chegaralanishga bog'liq holda qurilgan. Bu maqolada biz qayiqning kemaga yaqinlashish strategiyasini qurdik.

*Kalit so'zlar:* Ramchandr, *l*-tutish, geometrik chegaralanish, differensial tenglamalar, differensial o'yin

### **ЗАДАЧА РАМЧАНДРА ДЛЯ ЗАДАЧЫ *l*-ПОИМКИ**

Саматов Бахром Тажиахмедович, Уралова Сабохат Исмоилжон кизи, Мирзмахмудов Умиджон Алижон угли  
НамГУ кафедрасы Дифференциальная уравнения и математической физики профессор и магистранты.

*Аннотация.* В статье, мы изучаем задачу Рамчандра для *l*-поимки дифференциальной игры. Мы изучаем проблему Рамчандра с геометрическим ограничением. Мы выстроили стратегию подхода корабля к лодке

*Ключевые слова:* Рамчандра, *l*-поимки, дифференциальной игры, геометрическим ограничением, дифференциальная уравнения.

### **THE PROBLEM OF RAMCHUNDRA FOR A PROBLEM OF *l*-CAPTURE**

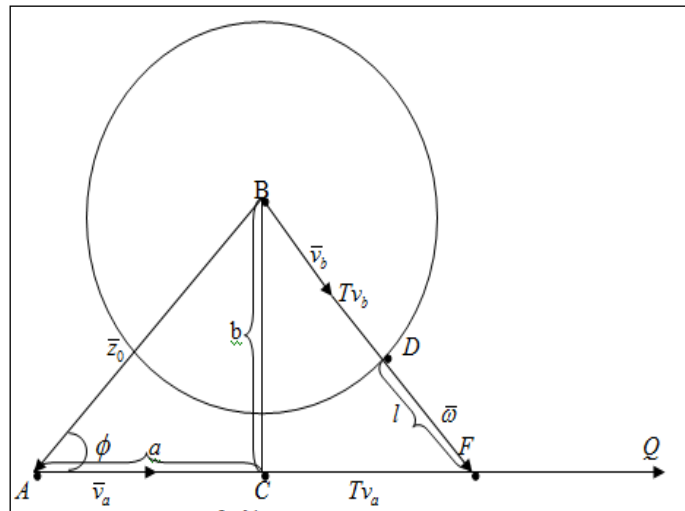
Samatov Bahrom Tajiahmedovich, Uralova Saboxat Ismoiljon qizi, Mirzamaxmudov Umidjon Alijon o'g'li,  
NamSU professor and masters at the Department of differential equation and mathematics-physics

*Abstract.* We study the problem for a differential game of *l*-capture. We study the problem of Ramchundra with geometric constraint. A strategy of the boat is constructed depending only with geometric constraint. In this article, we constructed a strategy for approaching the ship to the boat.

*Key words:* Ramchundra, *l*-capture, geometric constraint, differential equations, differential game

One of the problems [2] discussed by Ramchundra (Indian mathematician Nesudas Ramchundra (1821-1880)) is the intercept problem for a slow pursuer versus a fast target. He wrote: Supposing a ship to sail from given place **A**, in a given place, at the same time that a boat from another place **B** sets out in order (if possible) to come up with her and supposing the rate at which each vessel progresses to be given it is

required to find in what direction the letter must proceed, so that if a cannot up with the former, it may however approach it as near as possible.



picture1

We learn the problem of Ramchundra in the case  $l$ -interception. Let in the space  $R^n$  the controlled object  $B$  (the boat), intercepts another object  $A$  (the ship). Suppose  $x$  and  $y$  are the locations of the boat and the ship respectively, and  $x_0, y_0$  ( $x_0 \neq y_0$ ) are their initial locations. The motions of the objects are described by the equations

$$B: \dot{y}(t) = v_b, y(0) = y_0; \quad A: \dot{x}(t) = v_a, x(0) = x_0. \quad (1)$$

where  $x, y, v_b, v_a \in R^n, n \geq 1$ ,

$|v_a| \leq \beta$  is the control functions of the object  $A$  (the ship) and  $|v_b| \leq \alpha$  is that of the object  $B$  (the boat),  $\alpha$  and  $\beta$  are given positive numbers.

If we say  $z(t) = x(t) - y(t)$ , then from (1) we will get the equation:

$$\dot{z} = v_a - v_b, \quad z_0 = z(0) \quad (2)$$

where  $z_0 = x_0 - y_0$ ,

The goal of the boat  $B$  is achievement of the inequalities  $|x(\bar{t}) - y(\bar{t})| \leq l$  if at some moment  $\bar{t} > 0$ . And according to the equation (2) we have  $|z(\bar{t})| < l$ . We know constructing strategy for boat (see [1], [3-4]). According to the figure we have

$$Tv_b + w = Tv_a - z_0 \quad (3)$$

Squaring both sides (2) of these equalities and we obtain a quadratic equation with respect to  $T$

$$T^2(\beta^2 - \alpha^2) - 2T(l\alpha + \langle v_\beta, z_0 \rangle) + |z_0|^2 - l^2 = 0$$

$$\begin{aligned} T_{1,2} &= \frac{1}{\beta^2 - \alpha^2} \left[ l\alpha + \langle v_\beta, z_0 \rangle \pm \sqrt{(l\alpha + \langle v_\beta, z_0 \rangle)^2 - (\beta^2 - \alpha^2)(|z_0|^2 - l^2)} \right] = \\ &= \frac{1}{\beta^2 \left(1 - \left(\frac{\alpha}{\beta}\right)^2\right)} \left[ |z_0| \cdot \beta \left(\frac{l}{|z_0|} \cdot \frac{\alpha}{\beta}\right) + \langle v_\beta, z_0 \rangle \pm \sqrt{\left(|z_0| \cdot \beta \cdot \frac{l}{|z_0|} \cdot \frac{\alpha}{\beta} + \langle v_\beta, z_0 \rangle\right)^2 - \beta^2 \cdot z_0^2 \cdot \left(1 - \left(\frac{\alpha}{\beta}\right)^2\right) \left(1 - \left(\frac{l}{z_0}\right)^2\right)} \right] = \\ &= \frac{1}{\beta^2 \left(1 - \left(\frac{\alpha}{\beta}\right)^2\right)} \left[ |z_0| \cdot \beta \left(\frac{l}{|z_0|} \cdot \frac{\alpha}{\beta} + \langle \hat{v}_\beta, \hat{z}_0 \rangle\right) \pm |z_0| \cdot \beta \sqrt{\left(\frac{l}{|z_0|} \cdot \frac{\alpha}{\beta} + \langle \hat{v}_\beta, \hat{z}_0 \rangle\right)^2 - \left(1 - \left(\frac{\alpha}{\beta}\right)^2\right) \left(1 - \left(\frac{l}{z_0}\right)^2\right)} \right] = \\ &= \frac{|z_0| \cdot \beta}{\beta^2 \left(1 - \left(\frac{\alpha}{\beta}\right)^2\right)} \left[ \frac{l}{|z_0|} \cdot \frac{\alpha}{\beta} + \langle \hat{v}_\beta, \hat{z}_0 \rangle \pm \sqrt{\left(\frac{l}{|z_0|} \cdot \frac{\alpha}{\beta} + \langle \hat{v}_\beta, \hat{z}_0 \rangle\right)^2 - \left(1 - \left(\frac{\alpha}{\beta}\right)^2\right) \left(1 - \left(\frac{l}{z_0}\right)^2\right)} \right] = \\ &= \frac{|z_0|}{\beta(1-k^2)} \left[ mk + \xi \pm \sqrt{(mk + \xi)^2 - (1-k^2)(1-m^2)} \right] \end{aligned}$$

and we have

$$T(\xi) = \frac{|z_0|}{\beta(1-k^2)} \left[ mk + \xi - \sqrt{(mk + \xi)^2 - (1-k^2)(1-m^2)} \right],$$

where

$$\xi = \langle \hat{v}_a, \hat{z}_0 \rangle, \quad k = \frac{\alpha}{\beta}, \quad m = \frac{l}{|z_0|}, \quad \hat{v}_a = \frac{v_a}{|v_a|}, \quad \hat{z}_0 = \frac{z_0}{|z_0|}.$$

**Lemma.** If  $1 \geq \xi \geq -mk + \sqrt{(1-k^2)(1-m^2)}$  then the function is  $T(\xi) > 0$ .

It is not difficult to show that this function is positive.

We now substituting  $T(\xi)$  into relation (3) find the strategy for boat in form

$$v_b = v_a - F(\xi)(mv_a + \alpha \hat{z}_0) \quad (4)$$

where

$$F(\xi) = \frac{mk + \xi + \sqrt{(mk + \xi)^2 - (1-k^2)(1-m^2)}}{\xi m + k + m \sqrt{(mk + \xi)^2 - (1-k^2)(1-m^2)}}$$

**Theorem.** If  $-mk + \sqrt{(1-k^2)(1-m^2)} \leq \xi \leq 1$  then the boat using of the strategy (3) realizes  $l$ -interception with the ship.

Proof:

Obviously

$$z(t) = z_0 + \int_0^t (v_a - v_b) d\tau$$

Squaring both sides of this equality and we find

$$\begin{aligned}
 |z(t)|^2 &= |z_0|^2 - 2 \left\langle z_0, \int_0^t F(\xi(\tau))(mv_b(\tau) + \alpha \hat{z}_0) d\tau \right\rangle + \left| \int_0^t F(\xi(\tau))(mv_b(\tau) + \alpha \hat{z}_0) d\tau \right|^2 \leq \\
 &\leq |z_0|^2 - 2 \int_0^t F \xi(\tau) \langle z_0, mv_b(\tau) + \alpha \hat{z}_0 \rangle d\tau + \left( \int_0^t F \xi(\tau) |mv_b(\tau) + \alpha \hat{z}_0| d\tau \right)^2 = \\
 &= |z_0|^2 - 2\beta |z_0| \int_0^t N(\xi(\tau)) d\tau + \beta^2 \left( \int_0^t M(\xi(\tau)) d\tau \right)^2,
 \end{aligned}$$

where

$$N(\xi) = F(\xi)(m\xi + k), \quad M(\xi) = F(\xi)\sqrt{m^2 + 2km\xi + k^2}$$

And hence

$$N(p) \leq N(\xi) \leq N(1), \quad M(p) \leq M(\xi) \leq M(1) \quad , \text{where } p = -mk + \sqrt{(1-k^2)(1-m^2)}$$

Thus

$$\begin{aligned}
 |z(t)|^2 - l^2 &\leq |z_0|^2 - l^2 - 2\beta |z_0| \int_0^t N(\xi(\tau)) d\tau + \beta^2 \left( \int_0^t M(\xi(\tau)) d\tau \right)^2 = \\
 &= |z_0|^2 (1-m^2) - 2\beta |z_0| \sqrt{1-m^2} \int_0^t \frac{N(\xi(\tau))}{\sqrt{1-m^2}} d\tau + \beta^2 \left( \int_0^t \frac{N(\xi(\tau))}{\sqrt{1-m^2}} d\tau \right)^2 + \\
 &+ \beta^2 \left( \int_0^t M(\xi(\tau)) d\tau \right)^2 - \beta^2 \left( \int_0^t \frac{N(\xi(\tau))}{\sqrt{1-m^2}} d\tau \right)^2 = \\
 &= \left[ |z_0| \sqrt{1-m^2} - \beta \left( \int_0^t \frac{N(\xi(\tau))}{\sqrt{1-m^2}} d\tau \right) \right]^2 - \beta^2 \left[ \left( \int_0^t \frac{N(\xi(\tau))}{\sqrt{1-m^2}} d\tau \right)^2 - \left( \int_0^t M(\xi(\tau)) d\tau \right)^2 \right]
 \end{aligned}$$

We show that

$$\begin{aligned}
 N(\xi) &\geq \sqrt{1-m^2} M(\xi) \\
 F(\xi)(m\xi + k) &\geq \sqrt{1-m^2} \cdot F(\xi)\sqrt{m^2 + 2km\xi + k^2} \\
 m^2 \xi^2 + 2km\xi + k^2 &\geq (1-m^2)(m^2 + 2km\xi + k^2) \\
 \xi^2 &\geq 1 - (m^2 \xi^2 + 2km\xi + k^2) \\
 \xi^2 + 2km\xi - 1 + k^2 + m^2 &\geq 0,
 \end{aligned}$$

From here we have

$$|z(t)|^2 - l^2 \leq \left( |z_0| \sqrt{1-m^2} - \beta \int_0^t \frac{N(\xi(\tau))}{\sqrt{1-m^2}} d\tau \right)^2$$

The last function  $N(\xi)$  is limited below with  $\sqrt{(1-k^2)(1-m^2)}$

From here, we can say  $|z(\bar{t})| < l$  if at some time  $\bar{t} < \infty$

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