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DECOMPOSITIONAL METHOD FOR MODELLING AND STUDYING PULSE-WIDTH AUTOMATIC CONTROL SYSTEMS BASED ON DYNAMIC GRAPHS

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Abstract

Systems with pulse-width modulation are essentially non-linear automatic control systems. The complexity factors of pulse-width systems include multivariable, the multirate nature of the pulse-width modulators work, and the nonstationarity of control objects. Such systems have been known for a long time and are now widely used. Various exact and approximate methods have been proposed for the analysis and synthesis of PWM systems. The field of the practical application of known methods is limited to single-variable systems because classical approaches provide for the consideration of the initial structures as a whole. Hence, the root cause of the fundamental difficulties arising in the study of such systems.This article proposes a decompositional method for modelling and studying multivariable pulse-width automatic control systems based on the dynamic graph models. One of the key factor when create the one approach for mathematical formulation, analysis and synthesis of discrete dynamic systems is the maximum consideration of general physical special features in terms of these systems. The general fundamental singularity of systems concerned is the natural decomposition (structure discretization) on simple subsystems or structural states of $\mathbf{S_i}$*. In the multivariable pulse-width systems, the model of each separate or cross channel is a single-variable impulse system graph. Decomposition into processes in separate and cross channels allows to change the parameters of certain channels and to carry out interval correction of dynamic processes occurring in transmission channels. This method can be used for analysis and synthesis of both single-variable and multivariable systems.*

Key words: *pulse-width modulation, pulse-width system, nonlinear system, decompositional method, dynamic graph.*

Systems with pulse-width modulation (PWM) are essentially nonlinear pulse systems [1, 2]. Depending on the nonlinear modulation characteristic kind and the error signal magnitude, the pulse duration for each period varies in accordance with the expression

$$\tau_n = \begin{cases} \varphi[e(nT)], & at\ \ \varphi[e(nT)] < T \\ T \quad\quad\quad\ , & at\ \ \varphi[e(nT)] \geq T \end{cases}, \tag{1}$$

where T – pulse repetition period; τ_n – pulse duration in the n-th repetition period; φ – nonlinear modulation characteristic.

The structural scheme of a single-variable pulse-width system (PWS) can be represented in the form (Figure 1a).

The continuous part of the system is linear and is represented by the transfer function $W(p)$. In the case of the nonlinear continuous part (NCP), we have a system with double nonlinearity (Figure 2). All other things being equal, the pulse sequence, as shown in Figure 2b will already have a different look.

Classical general and particular approaches always provide for the consideration of the initial structures as a whole [3-7]. This gives rise to the root cause of fundamental difficulties in the study of pulse-width systems [8-18]. In contrast to the well-known approaches, in this paper, we develop a decomposition method based on the physics of the pulsed systems work.

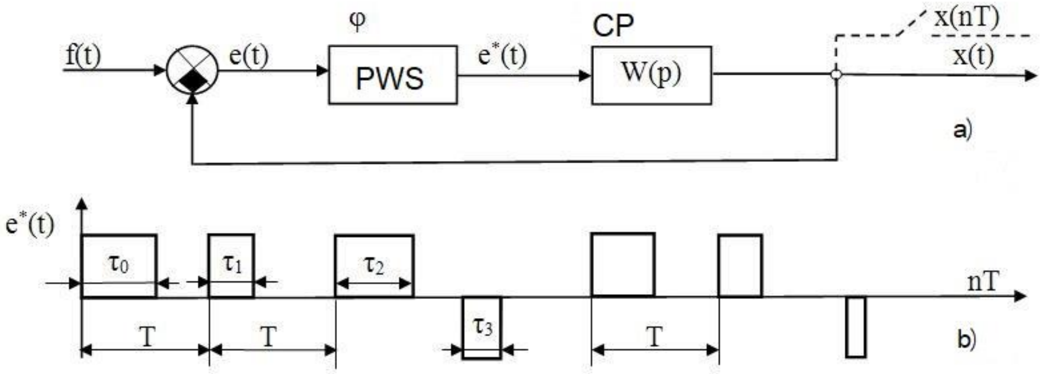

Fig. 1. *Structural scheme of PWS (a), pulse sequence at the output of PWM (b)*

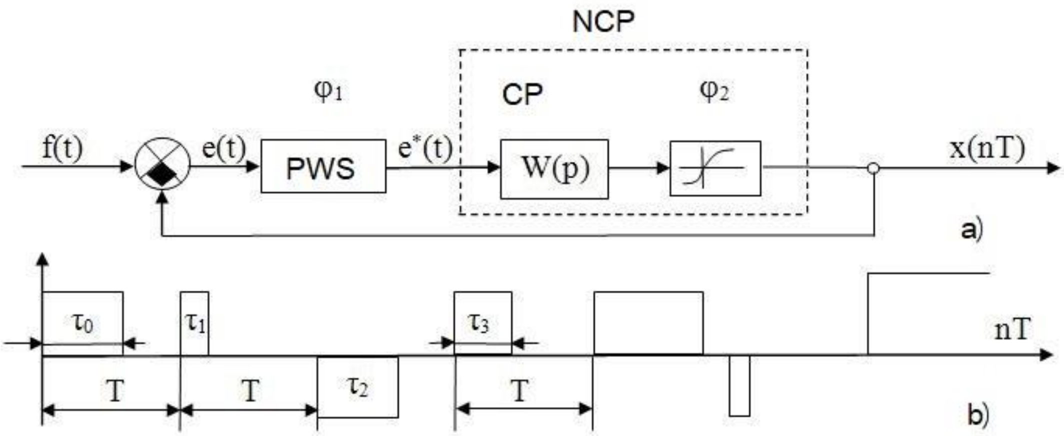

Fig. 2. *Structural scheme of a pulse-width system with a nonlinear continuous part (a), the pulses sequence at the output of PWM (b)*

Analyzing at the macro-level two pulse-width systems presented in Figures 1 and 2, we can see the overall picture for them. Namely, during each period of operation, at the moments of the presence of pulses, both systems belong to the category of closed-loop systems, and with pauses between pulses, they belong to open-loop systems. That is, at the macro level, we can represent both systems in the form of successive structural states S_1 and S_2, the first of which correspond to PWS at intervals of the presence of pulses, and the second – to intervals of the absence of pulses (Figure 3).

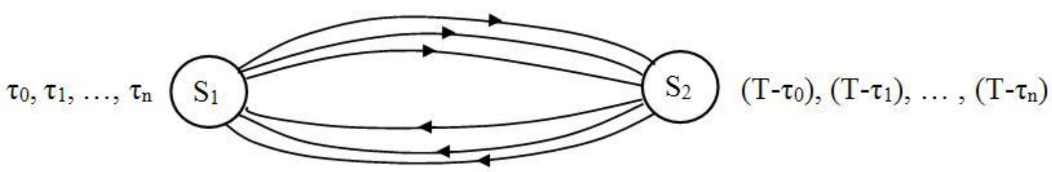

Fig. 3. *Graph of structural states of pulse-width systems shown in Fig. 1, 2*

We turn again to the system (Figure 1). Analyzing the picture of the sequence of pulses arising at the output of the modulator, we can conclude that single pulses can be replaced by unit pulses of the $\delta(t)$ type, and the pulses real form can be taken into account by introducing a forming unit into the continuous part of the system.

However, it should be noted that in the case of PWS, the forming unit will have nonlinear and nonstationary properties due to the variability of the parameter τ_n in each repetition period and its determination in accordance with the nonlinear modulation characteristic φ, i.e.

$$W_{fu}(p) = \frac{1 - e^{-p\tau_n}}{p} \tag{2}$$

With that said, the system (Figure 1) we can represent the following equivalent system (Figure 4).

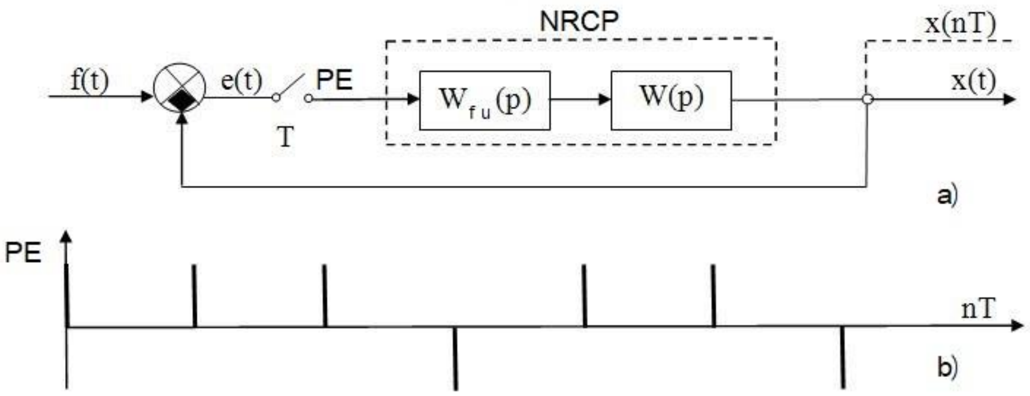

Fig. 4. *PWS with a nonlinear reduced continuous part (a), pulse sequence at the output of the pulse element (b)*

Figure 4b shows that we can represent the original pulse-width system as a system with an instantaneous nature of the pulse element closure. In turn, this allows us to consider PWS (Fig. 4a) as an open-loop system on the base of the following statement.

Statement

Any multivariable (single-variable) nonlinear (linear) discrete system with an instantaneous nature of the pulse elements closure is open relative to the signals flowing in it, regardless of the position of the keys, if each simple loop contains at least one pulse element.

The proof follows directly from the relation

$$e^i(jT^+) = f^i(jT) - x^i(jT), \tag{3}$$

which is a condition for the closure of the i-th loop of a multivariable discrete system and, at the same time, shows that the signals $f^i(jT)$, $x^i(jT)$ preceding the closure time are used to calculate the signals at the moments (jT^+) system closures.

It is the consideration of the real physical picture of phenomena that proves the statement.

Above (Figure 3), we showed that the proposed approach allows representing pulse-width systems at the macro level as a set of simpler closed-loop and open-loop systems interacting with each other. The value of the proved statement lies in the fact that it allows considering all subsystems, and therefore, the system as a whole, as open-loop at the signal level (microlevel). Hence, the features and advantages of the proposed decompositional approach are clear.

To modelling pulse-width systems at the micro-level or the level of dynamic processes, it is necessary to represent each structural state with its dynamic graph of the form

$$G_t = \langle X_t, (V_t, \Omega_t) \rangle, \tag{4}$$

where x_t, v_t, Ω_t – respectively, the set of vertices, arcs and weights of arcs:

$$X_t : t_* \to X,\ \ V_t : t \to V,\ \ \Omega_t : t \to \Omega,\ \ t_* = (t_1, t_2, \dots, t_n)$$

a linearly ordered finite set of points in time.

A general dynamic graph describing the processes on time intervals, equal to the pulse repetition period T, will represent a union of graphs G_t^1, G_t^2, то есть,

$$G_t = G_t^1 \cup G_t^2, \tag{5}$$

where G_t^1 – simulates the system in the area of the presence of pulses, G_t^2 – in the areas of the absence of pulses.

We will illustrate the application of the decompositional method of graph modelling and the calculation of transients with a specific example.

Example

It is required to calculate the transient process $x_1(t)$ for the PWS, shown in Figure 1, with the following characteristics.

Input action $f(t) = 1(t)$, pulse duration:

$$\tau_n = \begin{cases} \varphi[e(n\mathrm{T}^+)] & at\ \ \varphi[e(n\mathrm{T}^+)] < 1 \\ T & at\ \ \varphi[e(n\mathrm{T}^+)] \geq 1 \end{cases}.$$

To be more precise, let

$$\tau_n = |\,e(n\mathrm{T}^+)\,| \ \text{ for } \ |e(n\mathrm{T}^+)| < 1; \quad \tau_n = \mathrm{T} \ \text{ for } \ |e(n\mathrm{T}^+)| \geq 1;$$

$$\mathrm{T} = 1; \quad x_1(0) = 0; \quad x_2(0) = 0.$$

In the interval $n\mathrm{T} < t \leq n\mathrm{T}+\tau_n$ the height of the pulse is equal to 1 for $e(n\mathrm{T}^+) > 0$; (–1) for $e(n\mathrm{T}^+) < 0$. The transfer function of the control object $W(p)=\frac{1}{p(p+0{,}5)}$; the transfer function of the forming unit $W_{fu}(p)=\frac{1-e^{-p\tau_n}}{p}$; the transfer function of the reduced continuous part $W_{rcp}=W_{fu}(p)\,W(p)$.

Decision

Taking into account the system characteristics, we construct a graph model (Figure 5).

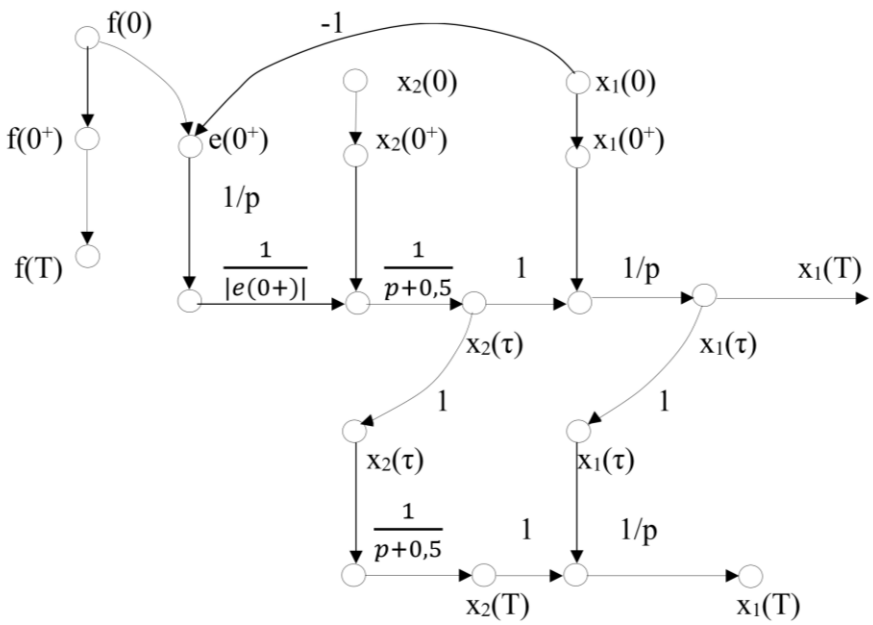

Fig. 5. *The first version of the dynamic graph model of a PWS*

Given the above ***Statement***, the graph model can be represented in the form (Figure 6). Directly on the graph, we define the transfers of arcs:

$\frac{x_1(\tau)}{x_1(0)}$ = L^{-1} $\{1/p\}$ = 1; $\frac{x_1(\tau)}{x_2(0)}$ = L^{-1} $\left\{\frac{1}{p(p+0,5)}\right\}$ = 2(1 – e$^{-\tau/2}$);

$\frac{x_1(\tau)}{e(0^+)}$ = L^{-1} $\left\{\frac{1}{p^2(p+0,5)}\right\}$ = 2(τ- 2 + 2 e$^{-\tau/2}$);

$\frac{x_2(\tau)}{e(0^+)}$ = L^{-1} $\left\{\frac{1}{p(p+0,5)}\right\}$ = 2(1 - e$^{-\tau/2}$);

$\frac{x_2(\tau)}{x_2(0)}$ = L^{-1} $\left\{\frac{1}{p+0,5}\right\}$ = e$^{-\tau/2}$;

$\frac{x_1(T)}{x_1(\tau)}$ = L^{-1} $\{1/p\}$ = 1;

$\frac{x_1(T)}{x_2(\tau)}$ = L^{-1} $\left\{\frac{1}{p(p+0,5)}\right\}$ = 2(1–$e^{-\frac{T-\tau}{2}}$);

$\frac{x_2(T)}{x_2(\tau)}$ = L^{-1}$\left\{\frac{1}{p+0,5}\right\}$=$e^{-\frac{T-\tau}{2}}$.

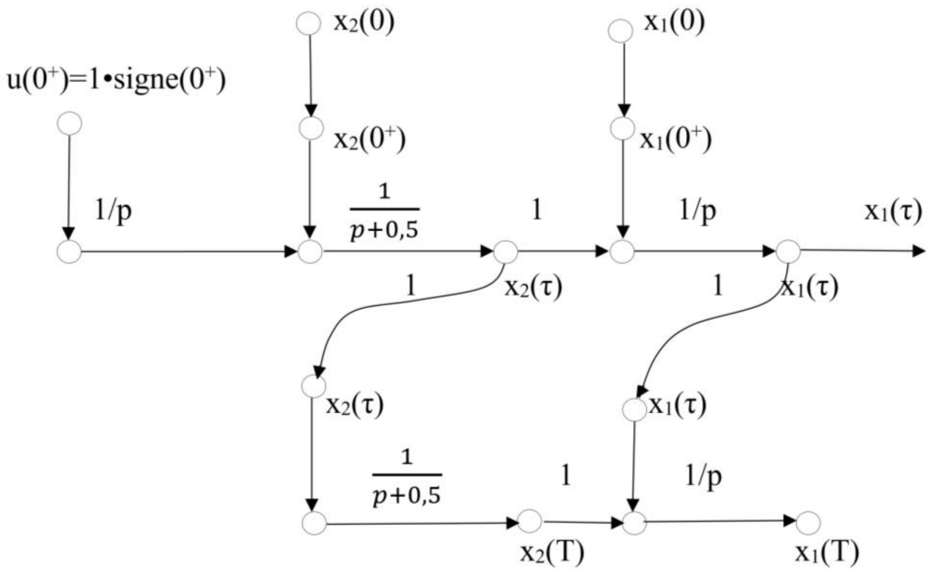

Fig. 6. *The second version of the dynamic graph model of a PWS*

After determining the transfers of the graph, we can proceed to the bipartite form of its presentation (Figure 7).

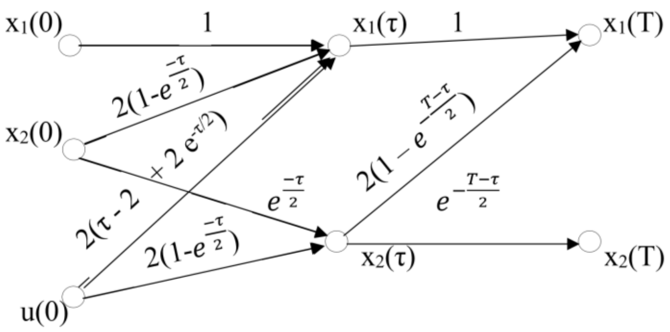

Fig. 7. *Bipartite dynamic graph*

Using a bipartite graph, we can directly calculate the processes at the system output. We will show this.

Step 1

We determine the error

$$e(0^+) = f(0) - x_1(0) = 1.$$

We determine the pulse duration τ_0: $\tau_0 = T$, since $e(0^+) | = 1$.
We determine the input signal: $u_0 = \text{sign } e(0^+) = 1$.

Step 2

From the graph, we calculate the values of the variables taking into account

$$x_1(T) = x_1(0) + 2(1 - e^{-\frac{\tau}{2}}) \bullet x_2(0) + 2(\tau - 2 + 2\,e^{-\frac{\tau}{2}}) \bullet u(0) = 0{,}424\ ;$$
$$x_2(T) = x_2(0) \bullet e^{-\frac{\tau}{2}} + u(0) \bullet 2(1 - e^{-\frac{\tau}{2}}) = 0{,}788;$$
$$x_1(T) = 0 + 2(1 - 0{,}606) \bullet x_2(0) + 2(1\text{-}2\text{+}2\bullet 0{,}606) = 0{,}424;$$
$$x_2(T) = 1\bullet 2(1 - 0{,}606) = 0{,}788.$$

We calculate the error

$$e(T^+) = f(T) - x_1(T) = 1 - 0{,}424 = 0{,}576;$$

determine the pulse duration: $\tau_1 = | e(T^+) | = 0{,}576$; the height of the pulse is equal to 1.

Step 3

$$x_1(T+\tau) = x_1(T) + x_2(T)\bullet 2(1 - e^{-\frac{\tau}{2}}) + u(T)\bullet 2(\tau - 2 + 2e^{-\frac{\tau}{2}}) = 0{,}97;$$
$$x_2(T+\tau) = x_2(T)\bullet e^{-\frac{\tau}{2}} + u(T)\bullet 2(1 - e^{-\frac{\tau}{2}}) = 1{,}091;$$
$$x_1(2T) = x_1(T+\tau) + x_2(T+\tau)\bullet 2(1 - e^{-\frac{T-\tau}{2}}) = 1{,}406;$$
$$x_2(2T) = x_2(T+\tau)\bullet e^{-\frac{T-\tau}{2}} = 0{,}873.$$

The error $e(2T^+) = f(2T) - x_1(2T) = 1 - 1{,}406 = -0{,}406$; the pulse duration $\tau_2 = 0{,}406$; the height of the pulse is equal to (-1).

Step 4

$$x_1(2T+\tau) = x_1(2T) + x_2(2T)\bullet 2(1 - e^{-\frac{\tau}{2}}) + u(2T)\bullet 2(\tau - 2 + 2e^{-\frac{\tau}{2}}) =$$
$$=1{,}406 + 0{,}873\bullet 0{,}37 - 0{,}072 = 1{,}66;$$
$$x_2(2T+\tau) = x_2(2T)\bullet e^{-\frac{\tau}{2}} + u(2T)\bullet 2(1 - e^{-\frac{\tau}{2}}) = 0{,}873\bullet 0{,}815 - 0{,}37 = 0{,}34;$$
$$x_1(3T) = x_1(2T+\tau) + x_2(2T+\tau)\bullet 2(1 - e^{-\frac{T-\tau}{2}}) = 1{,}66 + 0{,}34\bullet 0{,}514 = 1{,}83;$$
$$x_2(3T) = x_2(2T+\tau)\bullet e^{-\frac{T-\tau}{2}} = 0{,}34\bullet 0{,}92 = 0{,}313.$$

The error $e(3T) = f(3T) - x_1(3T) = 1\text{–}1{,}83 = -0{,}83$; the pulse duration $\tau_3 = 0{,}83$; the height of the pulse is equal to (-1).

Step 5

$$x_1(3T+\tau) = x_1(3T) + x_2(3T)\bullet 2(1 - e^{-\frac{\tau}{2}}) + u(3T)\bullet 2(\tau - 2 + 2e^{-\frac{\tau}{2}}) =$$
$$=1{,}83 + 0{,}313\bullet 0{,}68 + (-1)\bullet 0{,}3 = 1{,}74$$
$$x_2(3T+\tau) = x_2(3T)\bullet e^{-\frac{\tau}{2}} + (-1)\bullet 2(1 - e^{-\frac{\tau}{2}}) = 0{,}313\bullet 0{,}66 - 0{,}68 = -0{,}47$$

$$x_1(4T) = x_1(3T+\tau) + x_2(3T+\tau)\bullet 2(1 - e^{-\frac{T-\tau}{2}}) = 1{,}74 - 0{,}47\bullet 0{,}16 = 1{,}66$$

$$x_2(4T) = x_2(3T+\tau)\bullet e^{-\frac{T-\tau}{2}} = -0{,}47\bullet 0{,}92 = -0{,}43$$

The error $e(4T^{+}) = f(4T) - x_1(4T) = 1-1{,}66 = -0{,}66$; the pulse duration $\tau_4 = 0{,}66$; the height of the pulse is equal to (–1).

Similarly, we can calculate the subsequent values of the transient. Figure 8 shows the pulse sequence at the pulse-width modulator output and the transient curves.

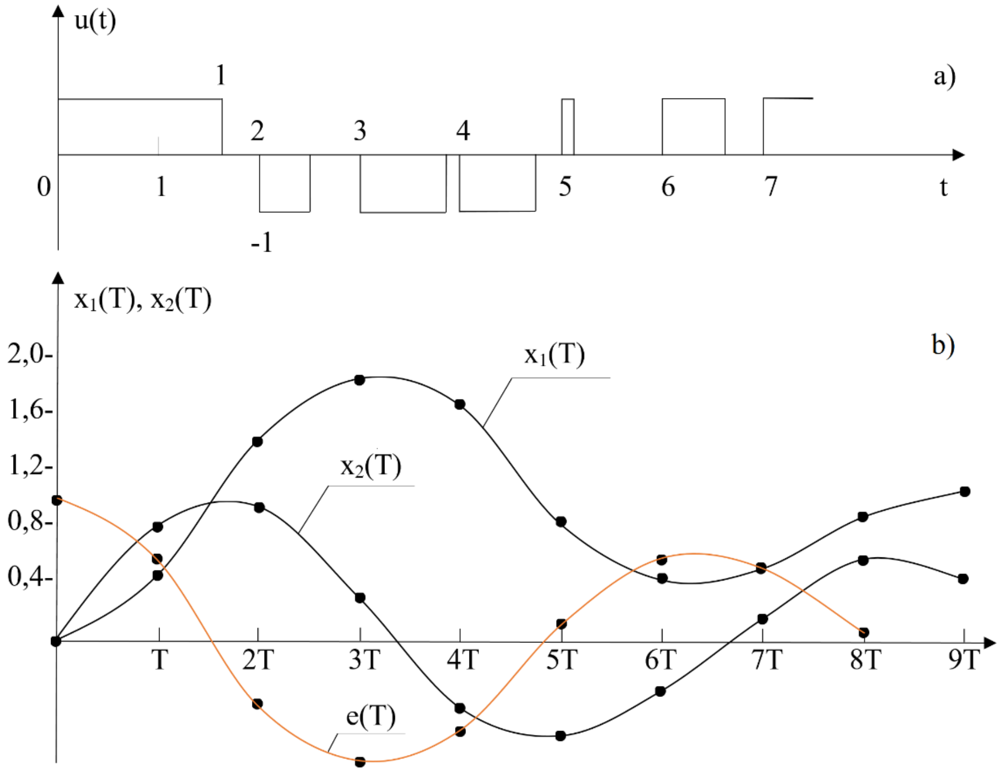

Fig. 8. *The sequence of pulses at the modulator output (a), the curves of the system processes $x_1(nT)$, $x_2(nT)$, $e(nT)$ (b)*

In conclusion, we note that the article outlines the decomposition method based on dynamic graphs. The proposed method can be used for modelling and studying pulse-width automatic control systems. The simplicity and effectiveness of the method are confirmed by an example of calculating transient processes in a pulse-width system.

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