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Tolkun Mirakhmedov
National University of Uzbekistan, mirakhmedov_td@nuu.uz

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CREATION OF A NUMERICAL MATHEMATICAL MODEL OF GEOFILTRATION PROCESSES AND ITS ADAPTATION FOR SOLVING EPIGNOSIC AND FORECAST PROBLEMS (IN TERMS OF GURLEN DISTRICT OF KHOREZM REGION)

MIRAKHMEDOV T. D.
National University of Uzbekistan, Tashkent, Uzbekistan
e-mail: mirakhmedov_td@nuu.uz

Abstract

The article considers the creation of a mathematical model of geofiltration processes in Gurlen District. Methods for calculating and developing a mathematical model and algorithms were carried out, and applied software was developed, and the developed geofiltration mathematical model was adapted to solve the epignosis and forecast problems of the area under consideration.

Keywords: module, hydrogeology, groundwater, hydrogeological monitoring, filtration, boundary conditions, algorithm, mathematical model, numerical model, geofiltration process, hydrochemical processes, filtration coefficient, reserve, resource, migration, groundwater deposit, adaptation, boundary conditions, boundary conditions, groundwater level, river, canal, drainage, variance, hydrology, geosystem, GIS technology, database.

1 Introduction

In order to compile a mathematical model, it is necessary to know what parameters and main factors influence on the filtration area in order to create a hydrogeological (physical) model and use the laws of change in hydrodynamic and migration processes. For a single-valued solution of forecasting problems, it is necessary to introduce additional conditions characterizing specific objects, which are called boundary conditions, including [1, 2, 3]:

- Geometrical dimensions of the object (external contours, position of the water-resistant underlaying bed and top, change in thickness and bedding of the formation, contours of engineering structures);
- Geological structure and physical properties (filtrational and capacitive, the law of their change in area and section);
- Boundary conditions that determine the relationship of the object with the external environment, the distribution and interaction of internal and external sources of disturbance (the degree of relationship between groundwaters and surface waters - feed, drainage; the law of the change in groundwater level (GWL), flow rate, flow velocity on the external and internal borders);
- The initial conditions characterizing the initial state of the object and the process under study (the state of GWL at a certain date, which is taken as the beginning of unsteady processes, feed and drainage conditions);
The boundary conditions can be of the first kind (these include watercourses, reservoirs of hydraulic connection with groundwaters), on the border of the filtration area, the pressure values are set:

\[ H(x, y, t) = f(x, y, t) \text{ or } H = \text{const}, \quad (x, y) \in \Gamma, \quad t \geq t_0. \]  

(1)

At the boundary of the second kind, the flow rate is set: where \( \Gamma \) is the boundary of the filtration area,

\[ \frac{\partial H}{\partial n} = f(x, y, t), \quad (x, y) \in \Gamma, \quad t \geq t_0, \]  

(2)

where \( n \) is the normal to the boundary or \( Q = \text{const} \) (with the inflow or outflow of GW) and \( Q = 0 \) (with an impenetrable boundary). At the boundary of the third kind, a functional dependence of the flow rate and flow strength is specified

\[ \gamma \frac{\partial H}{\partial n} + \beta H = f(x, y, t), \quad (x, y) \in \Gamma, \quad t \geq t_0 \]  

(3)

or \( Q = f(H) \).

The boundary conditions are determined by the geological (and tectonic) structure and hydrogeological processes: feeding conditions for drainage, dispersion - evaporation, underground drain - local and regional, the degree of interconnection of ground and surface waters, etc.

The initial conditions are hydroisolohyps for June 2015.

The boundary conditions are accepted as follows:

- In a section - clays and siltstones of the Upper Pliocene at a depth of 40-50 meters are taken for water resistance; border of the second kind \( Q = f(H) \);

- On the layout, external and internal borders are heterogeneous: in the northeastern (NE) part of the object the Amu Darya River flows - the border of the first kind \( H = \text{const} \); regional inflow and outflow of groundwaters, respectively, in the eastern and western parts of the page - kind 2 borders \( Q = \text{const} \) (for hydroisolohyps), large channels (internal borders) for Klychniyazbay, Turangisak, Oktyabr-Arna and Gurlen branch - boundaries of the first kind \( H = \text{const} \), areal infiltration (on irrigated fields and small irrigators) - the boundary of the second kind \( Q = \text{const} \), CDS (collector - drainage system) - the boundary of the second kind \( Q = \text{const} \) (or \( Q = f(t) \)), apparently it is necessary to additionally evaluate the role of large collectors (Daryalyk, Divankul) by the degree of their relationship with the subsurface waters - and creating piezominimums.

An indicator of the degree of relationship between groundwater and surface water is the hydraulic resistance (\( \Delta L \)) of the bottom sediments of watercourses, with an increase in resistance, the hydraulic connection (banked-up filtering mode) is broken and a free filtration mode (“overhead irrigation”) occurs, at which the filtration losses \( (q_{ef}) \) are maximum at GWL below the bottom of the watercourse and are completely determined by the filtration coefficient \( (K_k) \), power \( (m) \) of the ground layer (bed), its area - \( F_k \) (width and length of the watercourse) and power (“column”) - \( h_B \), layer of surface water in a watercourse and is determined by the formula:

\[ q_{ef} = K_k F I = K_k F_k \frac{h_B + m_k}{m_k}. \]  

(4)
Consequently, free filtration occurs at the maximum possible throughput $C_n$ of the features of bottom sediments under the conditions of deep occurrence of the GWL.

The hydraulic connection between groundwaters and surface waters may be disrupted under the conditions of groundwaters usage and the creation of a regional cone of depression within the channel (GWL decrease below the bottom of the channel), which can occur in the case of linear operational selection ($q_{e,l}$), exceeding the maximum possible filtration losses from watercourse ($Q_{k,l,max}$), i.e. at $q_{e,l} > Q_{k,l,max}$.

This position must be taken into account when substantiating the boundary conditions ($H = const - Q = const$) and their change during groundwater operation. The initial and boundary conditions are displayed in the form of one-dimension (profile models), two-dimensions (planar models) and three-dimensions (volumetric models).

Then, using modeling, the inverse and epignosis problems are solved in order to determine the adequacy of the mathematical model of the object of research: if it is not adequate, then clarification is introduced on the initial hydrogeological information (and model), including boundary conditions.

A schematic map of the initial and boundary conditions of Gurlen District with the application of observation wells, groundwater hydroisohypses, isolines of the power of alluvial deposits, water intakes, etc. was compiled.

To create a mathematical model, the problem of a numerical model of geofiltration processes, determining the change in groundwater level and the relationship with surface waters is considered.

We take for the reference plane a random non-horizontal and impermeable (relatively) aquiclude. To simplify the linearization calculation, we consider a one-dimensional equation. The xot reference plane does not change much throughout the hard filtering area.

The mathematical model of groundwater movements in such aquifers is described by a quasilinear partial differential equation in parabolic type partial derivatives, which has the following form:

$$\mu \frac{\partial H}{\partial t} = \frac{\partial}{\partial x} \left( k m \frac{\partial H}{\partial x} \right) + f - \delta Q_e, \quad (5)$$

where $\mu$ - the coefficient of water loss of the aquifer (dimensionless quantity); $H = H(x,t)$ - groundwater level, m; $k$ is the filtration coefficient, m/day; $M = H(x,t)$ - $b(x)$ is the power of the aquifer; $b(x)$ is the mark of the aquiclude; $f(x,t) = Q_v - Q_d - Q_{isp}$ infiltration feed of groundwaters, consists of part of the atmospheric precipitation and irrigation water (from the river, canals) seeping into the aquifer. It changes significantly over time and in area, so it can be represented as a function of coordinates and time, here;

$Q_{isp}$ is evaporation from the groundwater level, has an areal character and is a function of spatial and temporal coordinates;

$Q_v$ is groundwater supply from canals, rivers and irrigated fields;

$Q_d$ is value, drained by groundwater, $Q_{skv}$ is wellflow rate;
\(Q_{sv} = Q(t)\delta(x - x_0), \ t \geq t_0, \ \delta - \text{Dirac delta function;}
\)
\[
\delta = \begin{cases} 
\frac{1}{\sqrt{\pi}}, & x = x_0, \\
0, & x \neq x_0 
\end{cases}
\]

\(X\) is the spatial and \(t\) is time coordinates. \(t_0\) is the initial calculation time.

We study the change in the groundwater level, i.e., solve equation (5) in \(G\) area. This area will be called the filtration area and we assume that it is bounded by a sufficiently smooth curve \(\Gamma\) when we consider the section, it is a continuous line.

The general solution of equation (5) depends on random functions. Therefore, for an unambiguous solution, additional conditions are necessary, the nature of which depends on the specific problem associated with the study of changes in the level of ground and surface waters.

One of the additional conditions is the so-called “initial”, which is the value of the groundwater level at a certain “initial” \(t_0\). We can assume that \(t_0 = 0\) the initial condition is written as:
\[
H(x, t) = F_1(x, t); \ x \in G; \ t \geq t_0.
\]  \hfill (6)

Boundary conditions are determined based on the natural hydrogeological and hydrological conditions. At the boundaries of the filtration area, either groundwater level markers, or water flow rate, or a linear relationship between flow rate and level should be specified, i.e., they can be called boundary conditions of the 1st, 2nd and 3rd kind.

If boundary \(\Gamma\) passes along hydroisohypse or watercourses that have good hydraulic connection with groundwaters, i.e. at any point in time the levels of groundwater are known, then a boundary condition of the first kind is set.
\[
H(x, t) = F_2(x, t); \ x \in \Gamma; \ t \geq t_0
\]  \hfill (7)

If there is an underground inflow and outflow, or an impenetrable boundary on the section of the border \(\Gamma\), then the flow of groundwater, or a boundary condition of the second kind, is set.
\[
kH \frac{\partial H}{\partial n} = F_3(x, t), \ x \in \Gamma, \ t \geq t_0
\]  \hfill (8)

If the border passes through an imperfect watercourse or body of water, which, due to fluctuations in the flow rate of the water, leads to a change in the level of groundwater, it is necessary to set a linear relationship between the flow rates of water in reservoirs with the level of groundwater, i.e. boundary condition of the third kind.
\[
kH \frac{\partial H}{\partial n} = \gamma(H_d - H), \ x \in \Gamma, \ t \geq t_0.
\]  \hfill (9)

The water exchange of streams with an aquifer can be represented as follows (Luckner, Shestakov, 1976) when fed from a river or canal:
\[
Q_r = k \frac{H_r - H}{\phi},
\]
during groundwater drainage

\[ Q_d = k \frac{H_d - H}{\phi}, \]

where \( G \) is the filtration area, \( \Gamma \) is the boundary area; \( F_1, F_2, F_3 \) are defined functions - \( \gamma \) characterizes the hydraulic conditions of the relationship between groundwater and surface waters; \( Q_r \) is filtration losses from the river; \( Q_d \) is the value of groundwater drainage; \( \phi - L + \Delta L \) where \( L \) is the distance of the wells from the watercourse, \( \Delta L \) is the bed resistance (bottom of the canal, drain) - the filtration resistance of the watercourses, \( H_r \) is the water level in the river, in the canal, \( H_d \) is the water level in the drain, \( k \) is the filtration coefficient, \( H \) is the level of hot water (in the well).

The above equations allow us to simulate the processes of groundwater filtration and the relationship of surface and ground waters in irrigated massifs, riverine and channel water intakes, etc.

To solve the boundary value problem of equation (5) - (9), according to the methodology of F.B. Abutaliev, U.U. Umarov and others they use a numerical method. To convert the original differential equation to a system of algebraic equations, you can choose the finite-difference method.

2 Preliminary results

We cover the filtration area \( G \) with the boundary \( \Gamma \) with a uniform grid area; it is replaced by the original uniform grid with steps \( \Delta x \) and \( \Delta t \). Then the values of the function \( H \) and the arguments \( x \), time \( t \) are replaced by mesh analogues:

\[ H_{i,j} = H(x_i, t_j); \quad x_i = \Delta x \cdot i; \quad t_j = \Delta t \cdot j; \quad i = 1, N; \quad j = 1, M \]

where \( N, M \) are the number of rows and columns of the mesh area.

The sizes of steps \( \Delta t, \Delta x \) as well as the meaning of the subindex \( i, j \) are indicated in Fig. 1. For each moment of time, we determine the state of \( H_{i,j} \) and then, using the formula, we calculate the filtration rates.

\[ V_{i,j} = k_{i,j} \frac{H_{i+1,j} - H_{i,j}}{\Delta x} \]  \hfill (10)

and moving paths \( S = V_{i,T} \). If \( S/\Delta x \geq k \), then the boundary will move “\( k \)” steps along the \( x \) coordinate.

![Fig.1. Discrete grid mesh](image-url)
Thus, we obtain the distance of the change of section borders over time $\Delta t$.

Hydrochemical processes are very inert, they characterize a complex set of hydrodynamic and physico-chemical interactions, and therefore the forecast of changes that occur in hydrogeochemical processes is the most important component of hydrogeological forecasts.

N.N. Verigin (1969) proposes to study the movement of fresh filtration waters and the relationship with surface water through mineralization or saline soils. These preliminary studies include the collection of some hydrological, hydrogeological and hydrochemical data on the type of existing and expected groundwater pollution. In addition, from the statement of goals it is clear that this problem is multifaceted. Therefore, it is most likely that both models - piston displacement and dispersion - will be necessary, and the sequence of their decisions should be included in the programme.

Dispersion at small and sometimes at large scales of research, and a purely convective piston displacement model - at large scales.

The dispersion of the fluid flow in a porous medium is the formation and development over time of the transition zone between the propagation areas of two phases of different chemical composition.

Dispersion is the result of the simultaneous action of both purely mechanical and physicochemical processes.

The formation and development of the transition zone between two moving miscible liquids can be explained by the tendency to equalize the concentration in the solution, although we will also see that the dispersion mechanisms are complex and cannot be considered from one point of view.

Mineralization is a change in the physical, chemical, and biological properties of water, limiting or excluding their use in various directions, where water usually plays a significant role.

When addressing the quality of watermarks (groundwater) of aquifers, molecular diffusion and dispersion must be taken into account in order to study the movement of fresh groundwater and the migration of mineralized water, as well as the relationship with surface waters. For this purpose, it is necessary to solve the following system of equations:

\[
\begin{align*}
\frac{\partial H}{\partial t} &= (k \frac{\partial H}{\partial x}) + W_n - W_o \\
\frac{\partial (mc)}{\partial t} &= \frac{\partial}{\partial x} \left(Dm \frac{\partial c}{\partial x} - mv_x C\right) + W_n C_n - W_o C 
\end{align*}
\]

with boundary conditions

\[
\begin{align*}
H(x, t) &= F_1(x, t); \quad x \in \Gamma_1; \quad t \geq t_0, \\
C(x, t) &= F_2(x, t); \quad x \in \Gamma_2; \quad t \geq t_0 \\
-km \frac{\partial H}{\partial n} &= F_3(x, t); \quad x \in \Gamma_3; \quad t \geq t_0, \\
-Dm \frac{\partial c}{\partial n} + m \vartheta_n c &= m \vartheta_n c; \quad x \in \Gamma_4; \quad t \geq t_0 \\
-km \frac{\partial H}{\partial \varphi} &= km \frac{H_{n \varphi}}{\varphi}; \quad x \in \Gamma_5; \quad t \geq t_0, \\
-Dm \frac{\partial c}{\partial \varphi} + m \vartheta_n c &= m \vartheta_n c; \quad x \in \Gamma_6; \quad t \geq t_0
\end{align*}
\]
and initial conditions

\[ \begin{align*}
  H(x, t_0) &= F_4(x); \ x \in \Gamma_7 \\
  C(x, t_0) &= F_5(x); \ x \in \Gamma_8
\end{align*} \]  

(15)

where \( \mu \) - free water loss; \( W_n = \sum_{i=1}^{n} W_i^n(x, t) \) is the intensity of various groundwater supply factors (including mineralized) with a concentration of \( C_n(x, t) \); \( W_0 = \sum_{i=1}^{n} W_i^0(x, t) \) is groundwater flow rate (withdrawal, drain, evaporation) from a certain concentration of \( C_0(x, t) \); \( D = D_m + \lambda \nu \) is the convective diffusion coefficient; \( D_m \) is the molecular diffusion coefficient - \( \lambda \), the dispersion coefficient.

The dispersion of the fluid flow in a porous medium is the formation and development over time of the transition zone between the propagation areas of two phases of different chemical compositions.

\[ G = \sum_{i=1}^{n} \Gamma_i, \]

\( \nu(x, t) \) is the filtration rate determined by Darcy’s law; \( F_1(x, t), \ldots, F_5(x, t) \) given functions. A detailed solution of system (11) + (15) is given below.

Numerical implementation of the model. In system (11) and boundary conditions, we pass to dimensionless variables using formulas (6,7,11).

\[ \begin{align*}
  H^# &= \frac{H}{H_0}; \ m^# &= \frac{m}{H_0}; \ k^# &= \frac{k}{T_0}; \ C^# &= \frac{C}{C_0}; \ D^# &= \frac{D}{D_0}; \ \xi &= \frac{x}{L}; \ \tau &= \frac{T_0 \nu}{\mu L^2 t}
\end{align*} \]

where \( H_0, C_0, D_0, T_0 \) are some characteristic values of the head function, concentration, convective diffusion and filtration coefficient, \( L \) is the maximum extent in the \( G \) area. Since in what follows we will deal with equations in dimensionless form, we will omit \((\#)\) stars in dimensionless variables (9,12). Get the system

\[ \begin{align*}
  \mu \frac{T_0 H_0^2}{L^2} \frac{\partial H}{\partial \xi} &= \frac{T_0 H_0^2}{L^2} \frac{\partial H}{\partial \tau} \left( km \frac{\partial H}{\partial \xi} \right) + \frac{T_0 H_0^2}{L^2} (W_n - W_0), \\
  \mu \frac{T_0 H_0^2 C_0}{\mu L^2} \frac{\partial (mc)}{\partial \tau} &= \frac{\partial}{\partial \xi} \left( \frac{T_0 H_0^2 C_0}{L^2} D m \frac{\partial C}{\partial \xi} - \frac{T_0 H_0^2 C_0}{L^2} m \frac{\partial C}{\partial \xi} \right) + \frac{T_0 H_0^2 C_0}{L^2} (W_n C_n - W_0 C),
\end{align*} \]

(16)

In what follows, we will deal with the equation in a dimensionless form, transforming the system (16), we obtain

\[ \begin{align*}
  \frac{\partial H}{\partial \xi} &= \frac{\partial}{\partial \tau} \left( km \frac{\partial H}{\partial \xi} \right) + W_n - W_0, \\
  \frac{\partial (mc)}{\partial \tau} &= \frac{\partial}{\partial \xi} \left( D m \frac{\partial C}{\partial \xi} - m \frac{\partial C}{\partial \xi} \right) + W_n C_n - W_0 C.
\end{align*} \]

(17)

Solutions of the equation describing the filtration processes and geochemical problems are carried out sequentially using implicit schemes on a uniform rectangular mesh (Fig.1). We introduce indices \( i, j \) and steps \( \Delta \xi, \Delta \tau \) along the \( x, t \) axes, respectively. Parts of the boundary with a condition of the first kind can be random, but with conditions of the second or third kind they are taken as stepped lines with segments parallel to the coordinate axes.
In the $\xi_{i-0.5} \leq \xi \leq \xi_{i+0.5}$, $\tau_j \leq \tau \leq \tau_{j+1}$ rectangle, we will write the balance equation for system (13).

\[
\begin{align*}
\left\{ \begin{array}{l}
\frac{H_i - R_i}{\Delta \tau} \Delta \xi = (m \dot{v})_{i-0.5} - (m \dot{v})_{i+0.5} + (W_{ni} C_{ni} - W_{0i} C_i) \Delta \xi, \\
\frac{H_i - R_i}{\Delta \tau} C_i + \frac{C_i}{\Delta \tau} m_i \Delta \xi = q_{i-0.5} - q_{i+0.5} + (W_{ni} C_{ni} - W_{0i} C_i) \Delta \xi
\end{array} \right.
\end{align*}
\]

(18)

Here

\[
(m \dot{v})_{i-0.5} = -(km)_{i-0.5} \frac{H_i - H_{i-1}}{\Delta \xi}, \quad (m \dot{v})_{i+0.5} = -(km)_{i+0.5} \frac{H_{i+1} - H_i}{\Delta \xi}
\]

(19)

\[
\left\{ \begin{array}{l}
q_{i-0.5} = -(Dm)_{i-0.5} \frac{C_i - C_{i-1}}{\Delta \xi} - (m \dot{v})_{i-0.5} C_{Li} \\
q_{i+0.5} = -(Dm)_{i+0.5} \frac{C_{i+1} - C_i}{\Delta \xi} - (m \dot{v})_{i+0.5} C_{Ri}
\end{array} \right.
\]

(20)

The concentration values of $CL_i$ and $CR_i$ at the flow points are calculated through the concentration values in neighbouring nodes depending on the direction of the velocity between these nodes, using the counter current method (9).

\[
CR_i = \begin{cases} 
C_{i-1}, & \text{if } -(m \dot{v})_{i-0.5} > 0, \\
C_{i+1}, & \text{if } -(m \dot{v})_{i+0.5} < 0,
\end{cases} \quad CL_i = \begin{cases} 
C_{i-1}, & \text{if } -(m \dot{v})_{i-0.5} > 0, \\
C_i, & \text{if } -(m \dot{v})_{i+0.5} < 0,
\end{cases}
\]

After some conversions we get the standard system

\[
\begin{align*}
\begin{cases}
a_i H_{i-1} - b_i H_i + C_i H_{i+1} = -d_i \\
a_i^1 C_{i-1} - b_i^1 C_i + C_i^1 C_{i+1} = -d_i^1
\end{cases}
\end{align*}
\]

(21)

Here

\[
a_i = (km)_{i-0.5}, \quad C_i = (km)_{i+0.5}, \quad b_i = a_i + c_i + \rho, \quad d_i = \rho \cdot H_i + W_{ni} - W_{0i}, \quad \rho = \frac{\mu DX^2}{T_0 H_0 DT}
\]

\[
a_i^1 = \begin{cases} 
(Dm)_{i-0.5} + UL_i, & \text{if } UL_i > 0, \\
(Dm)_{i-0.5}, & \text{if } UL_i < 0,
\end{cases} \quad C_i^1 = \begin{cases} 
(Dm)_{i+0.5} - UR_i, & \text{if } UR_i > 0, \\
(Dm)_{i+0.5}, & \text{if } UR_i < 0,
\end{cases}
\]

\[
b_i^1 = a_i^1 + c_i^1 + \gamma \tilde{m}_i + w_{ri}, \quad d_i^1 = \gamma \tilde{m}_i C_i, \quad Uh_i = (m \dot{v})_{i-0.5} \cdot \Delta \xi, \quad UR_i = (m \dot{v})_{i+0.5} \Delta \xi.
\]

We consider the approximation of the boundary conditions of the second and third kind separately. We write an approximation for the boundary condition of the second kind - and - for

\[
km \frac{\partial H}{\partial n} = F_2 \quad Dm \frac{\partial C}{\partial n} + m \vartheta_n C = F_2 C_n \quad \xi = 0.
\]

To do this, we use the balance equation in the range $0 \leq \xi \leq \xi_{0.5} = 0.5 \cdot \Delta \xi$

\[
\begin{align*}
\left\{ \begin{array}{l}
0.5 \Delta \xi \frac{H_0 - R_0}{\Delta \tau} = C_0 \frac{H_0 - H_0}{\Delta \xi} + P_2 \Delta \xi \\
0.5 \Delta \xi \left( \frac{H_0 - R_0}{\Delta \tau} C_0 + \frac{C_0 - C_0}{\Delta \xi} \tilde{m}_0 \right) = (Dm)_{0.5} \frac{C_i - C_0}{\Delta \xi} - UR_0 - CR_0 + F_2 C_n \Delta \xi
\end{array} \right.
\end{align*}
\]

or

\[
\begin{align*}
H_0 = C_0 \frac{C_0 + \gamma \tilde{m}_0 + S_i}{C_0 + \gamma \tilde{m}_0 + S_i} C_1 + \frac{C_0^2}{C_0^2 + \gamma \tilde{m}_0 + S_i} C_1 + \frac{\gamma \tilde{m}_0 C_0 + S_i C_n}{C_0^2 + \gamma \tilde{m}_0 + S_i},
\end{align*}
\]

(22)
Here
\[ S_1 = \begin{cases} F_2^1, & \text{if } F_2^1 > 0, \\ 0, & \text{if } F_2^1 < 0, \end{cases} \]
\[ F_2^1 = \Delta \xi^2 \cdot F_2, \quad \gamma = 0, 5\gamma. \]

We approximate similarly for \( \xi = 1 \)
\[ \begin{align*}
H_N &= \frac{a_N}{a_N + \gamma} H_{N-1} + \frac{\gamma H_N + F_2^1}{a_N + \gamma}, \\
C_N &= \frac{a_N}{a_N + \gamma m_N + S_2} C_{N-1} + \frac{\gamma m_N C_N + S_2 C_N}{a_N + \gamma m_N + S_2},
\end{align*} \]

Here
\[ S_2 = \begin{cases} F_2^1, & \text{if } F_2^1 > 0 \\ 0, & \text{if } F_2 < 0 \end{cases} \]

In the case of a boundary condition of the third kind, we obtain the following ratios: at \( \xi = 0 \)
\[ \begin{align*}
H_0 &= \frac{c_0}{c_0 + \gamma + \gamma_1} H_1 + \frac{\gamma H_0 + \gamma_1 H_e}{c_0 + \gamma + \gamma_1}, \\
C_0 &= \frac{c_0}{c_0 + \gamma_0} C_1 + \frac{\gamma \alpha_0 C_0 + p_1 C_n}{c_0 + \gamma_0},
\end{align*} \]
and at \( \xi = 1 \)
\[ \begin{align*}
H_N &= \frac{a_N}{a_N + \gamma + \gamma_2} H_{N-2} + \frac{\gamma \beta N + \gamma_2 H_e}{a_N + \gamma + \gamma_2}, \\
C_N &= \frac{a_N}{a_N + \gamma m_N + p_2} C_{N-1} + \frac{\gamma \alpha_3 C_N + p_2 C_N}{a_N + \gamma m_N + p_2},
\end{align*} \]

Here
\[ P_1 = \begin{cases} \gamma_1 (H_B - H_0), & \text{if } H_B > H_0, \\ 0, & \text{if } H_B \leq H_0, \end{cases} \]
\[ \gamma_1 = \frac{DXK_0 \gamma_0}{DNO - \phi_0}, \quad \gamma_2 = \frac{DXK_0 \gamma_2 \gamma_0}{DNO - \phi_0}. \]

To solve (21), we apply the sweep method (Samara). The solution for the difference system of equations (21) will be sought in the form:
\[ H_i = d_{i+1} + \beta_{i+1}, \quad C_i = d_{i+1} C_{i+1} + \beta_{i+1} \]
\[ \begin{align*}
\alpha_{i+1} &= \frac{c_1}{a_1 - a_i b_1}, \\
\beta_{i+1} &= \frac{d_i + a_i b_1}{b_1^2 - a_i d_1^2} \beta_{i+1}.
\end{align*} \]
\[ \begin{align*}
\alpha_{i+1} &= \frac{c_1}{a_1 - a_i b_1}, \\
\beta_{i+1} &= \frac{d_i + a_i b_1}{b_1^2 - a_i d_1^2} \beta_{i+1}.
\end{align*} \]
Solving together (23) and (26), we obtain

\[ H_N = \frac{\alpha_N \cdot \beta_N + \gamma \cdot \tilde{H}_N + F_2^1}{\gamma + a_N (1 - \alpha_N)}, \quad C_N = \frac{\alpha_N \cdot \beta_N + \gamma \cdot \tilde{m}_N C_N + S_2 C_n}{\gamma \cdot \tilde{m}_N + a_N (1 - \alpha_N) + S_2} \quad (28) \]

In the case of boundary conditions of the second kind, from (28) we find

\[ \alpha_1 = \frac{c_0}{c_0 + \tilde{\gamma} + \gamma_1}, \quad \beta_1 = \frac{\tilde{\gamma} H_0 + \gamma_1 H_B}{C_0 + \tilde{\gamma} + \gamma_1}, \quad \alpha_1^1 = \frac{C_0^1}{C_0^1 + \tilde{\gamma} m_0 + \gamma_1}, \quad \beta_1^1 = \frac{\tilde{\gamma} m_0 C_0 + P_1 C_n}{C_0^1 + \tilde{\gamma} m_0 + P_1} \]

Solving together (25) and (26), we define

\[ H_N = \frac{\alpha_N \cdot \beta_N + \gamma \cdot H_N + \gamma_2 H_B}{a_N (1 - \alpha_N) + \gamma + \gamma_2}, \quad C_N = \frac{\alpha_N \cdot \beta_N^1 + \gamma \cdot \tilde{m}_N C_N + P_2 C_n}{a_N^1 (1 - \alpha_N^1) + \gamma \cdot \tilde{m}_N + P_2} \]

Knowing \( \alpha_1, \beta_1, \alpha_1^1, \beta_1^1 \) and moving from \( i \) to \( i + 1 \) in formulas (2.31), we define \( \alpha_1, \beta_1, \alpha_1^1, P_1 \) for all \( i = 2, 3, \ldots \); knowing \( H_N, C_N \) by the formula we find all \( H_i \) and \( C_i \) in the opposite direction for \( i = N - 1, N - 2, \ldots, 0 \).

### 3 Main results

Thus, using the above schemes, it is possible to study the processes of groundwater movements and the relationship with surface waters and changes in the concentration of fresh water pollution both in space and in time.

Based on the above numerical schemes and algorithm, one-dimensional and two-dimensional programmes for solving by the finite difference method have been compiled (Fig. 2)
Based on the hydrogeological conditions for modelling, a piecewise homogeneous single-layer scheme of the structure of the aquifer with horizontal aquiclude was adopted. For the initial hydrodynamic conditions of the filtration flow, the distribution of GWL 2015 was taken. A discrete grid mesh model based on schematization, carrying information about the initial boundary conditions and calculation parameters, consists of \(114 \times 52 = 5,928\) nodal points (scale 1:50 000) with grid mesh spacing \(\Delta x = \Delta y = 500\,\text{m}\).

At the nodal points, the interpolated absolute values of the elevations of the earth’s surface and the absolute elevations of the groundwater level and aquiclude, as well as the initial values of the coefficient of transmissibility (filtration coefficient) are set. On the external borders, boundary conditions are set in the north-eastern part of the Amu Darya river, drainage - 3 kinds, 2 kinds - in the north and south, \(Q = \text{const.}\) accordingly: inflow - 17,280 m\(^3\)/day, outflow -30,240 m\(^3\)/day; in the west, the impenetrable border \(Q = f(H)\), in the east, the border of the first kind, \(H = \text{const.}\). Internal boundary conditions: area infiltration is uniform but areal (within the allocated areas).

Based on geofiltration schematization and hydrogeological conditions, the filtration area is reduced to the following calculation scheme:
- The filtration area is single-layer, conditionally uniform;
- It is assumed that the aquiclude of the groundwater aquifer is not horizontal, random aquiclude (in terms of power) and, accordingly, boundary conditions of the second kind \(Q = f(H)\) are set on its boundary;
- Infiltration, selection, pinching out and evaporation are set on the upper boundary of the flow;
- The relationship of groundwater with surface waterways is determined by the amount of infiltration or drainage;
- The selection of groundwater is determined by the boundary conditions of the second kind \((Q_c = \text{const.} = \text{const.})\).

Thus, a hydrogeological formalization of the research object was carried out.

To confirm the developed methodology, we consider a hydrodynamic model of a fresh groundwater site.

In further studies, it is necessary to conduct a series of computational experiments to determine the geofiltration and geomigration parameters, primarily, the speed and direction of flow, as well as forecasting the status of aquifers in order to clarify the relationship of groundwaters with surface watercourses during groundwater withdrawal for irrigation. Based on the geological and hydrogeological conditions and the results of geofiltration schematization, a mathematical model of hydrodynamic conditions is selected. It is described by a system of differential equations for partial derivatives of parabolic type.

The solution of inverse and epignosic problems for a system of differential equations for partial derivatives is due to the fact that in many cases they are the key to the successful solution of the investigated direct problem and to obtain a more complete and correct idea of the hydrogeological and physical processes that are actually taking place. One of the necessary conditions for such conformity is the correct setting
of the coefficients of the differential equation, which are the physical and mechanical parameters of the object under study. These parameters can be static parameters (water transmissibility coefficient, water loss coefficient and other filtration parameters of the aquifer), as well as dynamic values - infiltration, evaporation, withdrawal, etc. The process of solving problems is carried out according to this scheme (see Fig. 3).

![Diagram](image)

**Fig. 3. Problem solving process**

The solution is considered known at some fixed point in space at all instants of time, and the desired parameter is some parameter of the equation. We solve this problem: according to observations at one point, the distribution in space is determined, for example, the filtration coefficient and the coefficient of water loss or balance elements that take into account the relationship between surface and ground waters.

From the point of view of hydrogeology, we determined the hydrogeological structure and hydrodynamic conditions, then, in order to clarify the initial (known) parameters, we performed calculations of balance items - numerical modelling calculations.

Based on the numerical schemes and the algorithm, a one-dimensional and two-dimensional programme has been compiled. The plan includes stages that reflect the need for careful selection of various one-dimensional transverse and longitudinal models, as well as a consistent transition from local to regional scales.

Based on preliminary studies, the area is divided into homogeneous, one-dimensional, linear sections, and geologically, zones. Each zone is divided into experimental sections, including one or more wells. To present the order of pollution, we note that experiments in aquifers were determined at experimental plots with a length of 1.0 to 3.0 km, in two-dimensional models with an area of 3 to 10 km².

In the general case, the area models are two-dimensional. The full matrix (additionally longitudinal and transverse and diagonal linear shape) in rectangular coordinates is used.

The results are models of the current state of groundwater, layouts of zones prone to pollution, and isolines of groundwater of various concentrations (mineralization and pollution map).

When developing a hydrogeological model, there are always errors that gradually accumulate as the model is developed, a transition is made from natural conditions...
sequentially to a filtration model, and then to a materialized model, on which forecast or other problems are solved. The issue is how to evaluate the correctness of developing a model and reliability by modelling the results. In addition, at what stages of developing the model and solving the problem as a whole do such errors occur, i.e. what is their origin, how to determine their size and is it always possible? It is also natural to find out whether these errors can be reduced, and in what way.

All the above questions determine the problem associated with the quantitative assessment of the accuracy of the data obtained by calculation. The accuracy of problem solving is primarily of great practical importance in engineering calculations. Here, practice requires that the results of solved hydrogeological problems must have high accuracy and ensure the reliability of practical recommendations made for them. Solving the problems of the research direction requires obtaining reliable results in terms of hydrogeological content. It should be noted that in hydrogeology there is no single point of view on the content of concepts about the accuracy, reliability and credibility of a solution (Gvich I.K., 1980). The concepts used in hydrogeology, to some extent, differ in content from those used in technology. An engineering solution in technology must have accuracy within acceptable limits. The latter are established for specific tasks, depending on the type of problem being solved, its importance and the quality of the initial information. In technical practice, the permissible limits are found by probabilistic and statistical methods and determine a certain probability of a solution, which characterizes the degree of its reliability.

In hydrogeology, changes in natural conditions are studied and evaluated, which depend on the effect of so many factors. The latter are not always quantitatively characterized to the extent necessary for the application of probabilistic-statistical estimates. In these cases, the quality of the results obtained is judged using other methods, for example, epignosic modelling is performed and the coincidence of the modelling results with the observation data of the groundwater mode collected during the survey is checked. Having received a satisfactory coincidence, they conclude that the model “works” well, and the forecast problems are solved based on this result. However, as shown above, the accuracy of solving inverse problems requires appropriate justification and evaluation. In addition, the concept of “modelling accuracy” in the evaluation of hydrogeological results has a dual meaning. On the one hand, we mean the correspondence of the model to the object in terms of geological and hydrogeological content, and on the other hand, the meaning of purely quantitative coincidence of the calculation results and observations at individual points of the model and the object under study is embedded in this concept. Often, any coincidence is regarded as reliable. Such a different understanding of terms does not contribute to solving the problem.

To facilitate the subsequent presentation of the material, we define the concepts of “accuracy”, “reliability” and “credibility”. The concepts of “accuracy” and “reliability” of a modelling solution are currently under discussion. In many works, accuracy is understood as the resulting modelling error, including in it all possible types of errors. However, the influence of errors on the final result of the solution is not the same, since the resulting modelling error is determined by the influence of a significant number
of various factors of different activity. The influence of some points can be estimated relatively simply and taken into account during the modelling. These mainly include metrological and random errors associated with the accuracy of measuring quantities, the accuracy of the machine, etc. Others, despite the high accuracy of the decisions made on the machine, continue to influence the final result of forecast calculations. These include mainly methodological, systematic errors associated with the choice of calculation method, schematization of hydrogeological conditions, unequal reliability of the adopted calculation parameters as a result of incomplete surveys and varying degrees of knowledge of the territory. Many of them are complex and may contain both random and systematic errors, for example, errors in constructing water transmissibility maps, etc.

We will consider a reliable model that is closest in all elements (i.e., accepted effective factors and parameters) to the hydrogeological object (natural conditions) and provides a solution to the problem with a given accuracy. According to the degree of reliability, two types of models can be built: 1) adequate to nature; 2) engineering model.

In the first case, the model with high reliability coincides with nature in all elements of its structure and in response to disturbances. In the second, the model is basically identical to nature and is characterized by the presence of a well-known engineering margin in model parameters and boundary conditions. An engineering model may contain generalized parameters (generalized values of capacity, power, water transmissibility) or correspond to such a rigid design filtration scheme that obviously provides a certain engineering margin in the final modelling results. In those cases when the engineering model contains generalized parameters, it is called functional. This emphasizes the main feature of the model: being not very similar in structure to the studied object, it reacts to all changes in the level and flow rate in the same way as a real natural object. Usually, such a similarity in changes in the level and flow rate is not observed for a very long time; therefore, a functional model can be used for a limited period.

An adequate model reproduces all the features of the geological structure and hydrogeological conditions of the study area. In this case, the action of each boundary condition, infiltration supply or evaporation and deep overflow are reproduced on the model separately, by independent elements.

In an engineering model, the action of some, even important factors can be neglected, but so that the model contains an engineering supply. Sometimes they do the same when performing analytical calculations, choosing the so-called rigid calculation scheme.

In this statement, the concept of "reliable model" includes the concept of "credible", since both adequate and engineering models must guarantee the solution of the problem with a given accuracy. To confirm the developed methodology, we will consider a hydrodynamic model of a fresh groundwater site. In further studies, it is necessary to conduct a series of computational experiments to determine the geofiltration parameters, primarily, the speed and direction of flow, as well as forecasting the status of aquifers in order to clarify the relationship of groundwater with surface
watercourses. Based on the geological and hydrogeological conditions and the results of geofiltration schematization, a mathematical model of hydrodynamic conditions is selected. It is described by a system of differential equations in partial derivatives of parabolic type [4].

The solution of inverse problems for a system of differential equations for partial derivatives is due to the fact that in many cases they are the key to the successful solution of the investigated direct problem and to obtain a more complete and correct idea of the hydrogeological and physical processes that are actually taking place. One of the necessary conditions for such conformity is the correct setting of the coefficients of the differential equation, which are the physical and mechanical parameters of the object under study. These parameters can be static parameters (water transmissibility coefficient, water loss coefficient and other filtration parameters of the aquifer), as well as dynamic values - infiltration, evaporation, withdrawal, etc.

The solution of the inverse problem. The results of these solutions and the results obtained during the hydrogeological studies of Khorezm hydrogeological station made it possible to proceed to the solution of the inverse problem under the conditions of operation of the water intake.

![Fig. 4. Absolute groundwater elevation and hydroisohypse acceptance for modelling](image)

However, before this, a number of profile tasks were solved in order to clarify the parameters of the aquifer and determine the quantitative relationship of the surface water and groundwater.

The main goal of solving fragmented profile problems was to clarify the relationship of groundwater with surface watercourses, to determine filtration losses from them, as well as to clarify some characteristics of the filtration properties of the flow,
namely, to clarify the filtration coefficient and water loss coefficient and balance sheet items. The adopted model diagram was refined and adjusted in the course of solving the inverse problem. The inverse problem was solved in a stationary mode and consisted of selecting parameters for the model corresponding to the field conditions on the map of hydroisohypses presented as of 2015 (see Fig. 4).

To compile a numerical model describing the natural hydrodynamic conditions of the field, an inverse stationary problem has been solved, the purpose of which is to clarify the values of the coefficient of water loss and the area distribution of the filtration coefficient and balance elements. Here we apply the method of factor-range estimation of parameters (trial and error method). First, to clarify the direction of groundwater flow, the area distribution of the filtration coefficient is selected. At the same time, the balance of groundwater and hydroisohypses compiled for 2015 are taken as initial data, the coefficient of water loss \( \mu = 0.15 \).

Hydroisohypses reflect hydrogeological conditions (feed, drainage) and the main directions and velocities of the movement of ground waters. The regional flow of groundwater movement from the southwest to the northeast is subparallel to the riverbed of the Amudarya river, except for a small strip on the right-bank part of the Daryalyka river in the valley (along the border), where there is a sharp change in the direction of movement from the side to the center of the valley.

As a result of solving a series of stationary problems on the model, the coefficient of water loss and water balance is selected. According to field studies, the water loss coefficient varies from 0, 10 to 0, 18 (Fig. 5).

![Graph of Changes (Discrepancies with nature) h at control points at different values of water loss.](image)

**Fig.5. Graphs of Changes (Discrepancies with nature) h at control points at different values of water loss.**

Within this interval, the most optimal parameter values are selected \( \mu \).

Computational experiments at three selected characteristic points with coordinates (15, 14); (45, 20); (15, 100); (25, 75); (38, 96) with the value \( = 0.10 - 0.12 \), showed the minimum deviations of groundwater levels and stable fluctuations of the difference \( h \) in time. \( \mu \Delta \) Stabilization of \( h \) occurs already after 60 days. \( \Delta \) With a value of < 0, 20 the difference in levels during the iteration increases rapidly, and with a value of \( = 0, 10 \) – 0, 12 it remains practically unchanged (Fig.6).
Thus, for the model of the site of the Gurlen underground water field, the optimal value of water loss is 0.12. Further, with different values of the filtration coefficients, the corresponding values of the desired parameter are selected, ensuring the coincidence of the general direction of the flow.

When clarifying the values of the filtration coefficients, hydroisohypses (initial groundwater level) and absolute elevations of the earth’s surface as of July-August 2015 were taken as the basis (Fig. 7). Balance items: inflow $Q = const$ of groundwater in the southern part of the site, the upper boundary of the model; outflow in the north-eastern part of the site, the lower boundary of the model; infiltration and water withdrawal by existing water intakes within the site; in the eastern and western parts of the site, connection with surface watercourses, the permeable boundary $Q_k$ or $Q_p$.
Infiltration areal and linear. In a multivariate computational experiment, the most optimal model values of the filtration coefficient and the value of filtration losses and drainage (5-10 l/s per running km.) were obtained. The reliability of the numerical model was verified by solving the forecast problem for a period of one year. The hydrogeological conditions of 2014 were taken as the initial state and a comparison was made with the results of field measurements at the observation points of 2015. The result of the strength problem reflects the hydrogeological conditions of the object under study for the state of 2015 (Fig. 7).

The solution for this problem was reduced in the case of discrepancy between nature and model data, to the selection of the coefficient of water transmissibility and the coefficient of water loss so as to achieve this coincidence. Here we apply the method of factor-range estimation of parameters.

The solution of the inverse problem was considered complete if the maximum discrepancy between nature and model potentials at individual points ranged from 1 to 3.

Here, the accuracy of calculating the iteration \( \varepsilon = 0.01 \). First, to clarify the direction of movement of the groundwater flow (hydroisohypses map), the area distribution of the water transmissibility coefficient is selected. At the same time, the groundwater balance is taken according to the data of the Khorezm hydrogeological station for 2015, the coefficient of water loss \( \mu = 0,10 - 0,12 \) (see Fig.8).

![Graphs of groundwater level changes.](image)

As follows from the analysis of the map of filtration parameters and boundary conditions obtained as a result of modeling on a computer, the largest percentage of mismatch in the water transmissibility values obtained from the results of experimental work and the modeling method, it is noted in the water intake area and in the upper part in the northwest, where the groundwater inflow coming from the channel of the Gurlen’s branch is set.

With different values of the water transmissibility coefficient, the corresponding values of the desired parameter were matched, ensuring the coincidence of the general flow direction, the most optimal model values were obtained, and a comparison was made with the results of field measurements at observation points.

In a multivariate computational experiment at five selected characteristic points in four corners and one in the middle with coordinates (15, 14); (45, 20); (15, 100);
(25, 75); (38, 96) with a value of the coefficient of water loss \( \mu = 0.12 \) they showed minimal deviations of the levels and stable fluctuations of the difference with the initial time.

Based on the results of the solved inverse stationary problem, maps of water transmissibility and hydroisohypses are constructed, combined with actual values and the difference between the initial and calculated information.

In the developed software package, it became possible to set a random underlying aquiclude and the thickness of the aquifer at each mesh node. Then, determining the power of the aquifer, the area distribution of the filtration coefficient was selected.

A comparison of nature data with model data obtained as a result of solving the inverse problem allows us to draw conclusions about the sufficient reliability of the adopted geofiltration scheme in its main features. The solution of the inverse problem to clarify \( K_f \) was considered complete.

The methodology for solving the problem was as follows.

The entire filtration area was divided by a uniform and orthogonal mesh. The continuous area is replaced by a discrete one. The obtained discrete area was divided into subareas in which the maximum and minimum values of the filtration coefficients \( (K_f) \) were specified. For each node of each subarea, \( T \) was calculated as a linear combination of the maximum and minimum values of \( T \).

For the obtained distribution of \( K_f \) in a discrete rectangular area, a numerical model for a pressureless aquifer was solved and the difference (\( \Delta \)) between the nature and obtained values were calculated.

For other linear combinations, the same calculations were performed. As a result, numerous values were obtained. The distribution \( T \), at which the minimum difference is the groundwater level with the initial values, was taken as the final result.

The initial values and boundary conditions were taken to be the same values as in the modelling, they were considered optimal and the difference with the initial one was minimal, as well as a comparison of the results with field measurements at the observation points.

The solution was obtained in such a way that the level value in the observation wells coincided with the level in the model. The occurrence of the error was attributed to the incorrect account of the layer transmissibility. To determine the value of \( L \), a method was used that uses the relationship between the levels in the river or reservoir, in the observation well closest to the river. \( \Delta \) In this case, considering the known conductivity quantity \( (T) \), the value was determined

\[ \Delta L = \Phi_c \cdot T \]

where \( T \) is the conductivity of bed, \( \Phi_c \) is the filtration resistance.

As a result of solving the inverse problem:
1) A map of the distribution of filtering coefficients at each nodal point of the discrete area is obtained;
2) conditions of ) A map of the distribution of model GWLs was constructed in comparison with a nature map of hydroisohypses (Fig. 9);
3) Comparison of model and natural levels at reference points proves the reliability.
of the adopted filtration scheme;
4) A comparative map of nature and model values of groundwater levels for the object under study was compiled (Fig.9).

![Fig.9. Scheme of hydroisohypses modelling results](image)

The results of solving inverse problems confirmed the basic laws of the distribution of the filtration properties of alluvial pebbles of modern and upper Quaternary age.

The problem was solved in a stationary mode. In the modelling, the location of the wells and the pumpings for them is grouped near the lying mesh node, on one experimental node (group pumping from wells 36, 25e, 4e), etc.

Results of numerical studies

The developed mathematical model of geofiltration processes and operation of water intake systems was tested in terms of Gurlen site of the groundwater field.

The relationship between surface waters and groundwaters located in the north-eastern part of the Levoberejniy groundwater deposit within Gurlen district of Khorezm region was studied and determined. The adequacy and correctness of the model is clarified.

The solution of the direct problem. First, with the help of section models, the compatibility of conditions at the block boundary is established, especially the continuity of flows. Then, an averaged equivalent model is constructed that includes the boundary conditions of the blocks and using the new ones - zonal boundary conditions.

On the site, then, experiments were performed to determine the hydrogeological parameters.

Then the zones are used as elementary blocks in the area model - first by combining them with compatibility conditions at their boundaries, and then within the model representing the area with equivalent coefficients.

The reliability of the numerical model was verified by solving the epignosis problem for a period of 1 year. A comparison of these results (groundwater levels) with
the results of field measurements at observation points of 2015 according to the data of the Khorezm HGS (hydrogeological station) was verified.

The values of geofiltration parameters, balance items and hydroisohypses obtained as a result of solving the inverse problem are taken as the initial ones for the selected mathematical model, which makes it possible to create a numerical model of the hydrogeological conditions of the site of the Gurlen groundwater field.

The solution for this issue can be approached in a slightly different way, in addition to the software package, one module was added, which is compared in each mesh node three numbers across and one longitudinally, according to the algorithm it is established that if the average number is greater than the first and less than the third, then the arrow is to the left, otherwise the direction of the arrow is to the right, if the average number is larger than the rest, then the arrows are towards it, if less the arrows are from it, if both conditions are not met, then the three numbers are equal, there will be no arrow at this point.

For the first question of the model, let us imagine the flow of groundwater at a constant speed if the groundwater level decreases by more than 3m, shuts off production wells, during a computational experiment, a final time of 360 days was set with printing every 30 days.

The distance travelled $L$ is proportional to time according to the formula $L = ut$. Thus obtained between 60 - 90 days is movement by one step along the mesh forward, and on 120-150 days another step moved forward. The distance between the nodes of the mesh, that is, the mesh spacing is 500 m. Let us determine the speed of movement of $500 \text{ m} / 75 \text{ day} = 6, 3 \text{ m/day}$.

![Fig. 10. Comparative drilling map and filter interval for the installation of the working part of the filter and its type of nature and model values of groundwater levels by solving epignosis and direct problems, as well as a series of computational experiments.](image-url)
4 Conclusions

It should be noted that, according to the solution of epignosis and forecasting problems, the filtration and balance characteristics in nature and on the model were clarified, forecasted problems were solved and recommendations were made on the rational (and optimal) use of groundwater for irrigation with the achievement of the reclamation effect and the conservation (and improvement) of environmental conditions (see fig.10):

- Solving forecast problems by the method of mathematical modeling made it possible to scientifically substantiate the optimal module of operational (and drainage) resources equal to the average annual renewable power sources, the value of which is proposed to be selected (extracted) by intensive operation during the growing season (with operation of GWL up to 2-3 m) followed by replenishment;

- Well location scheme: a) Linear along the channels at distances of 300-500 meters from the channel, in order to achieve minimal damage to surface drain. Well discharges \( 15 \rightarrow 20 \, l/s \). The distance between the wells is 700-1,100 meters. b) Away from the channel - location scheme - areal. The distances between the wells are determined by the modules of operating resources and well debits and are 700-1,000 meters;

- estimated reserves can be selected forcibly during the growing season in the amount of \( 14 \rightarrow 15 \, m^3/s \), by means of the depletion of capacitive reserves with their subsequent replenishment; basic elements of well construction are its depth, the diameter of the column.

The depth of the wells should be \( 40 \rightarrow 45 \, m \). For this, airlift blast pipes must be loaded at a sufficient depth so that the condition is met: the ratio of the latter to the dynamic level should be 2.5-3.0. The diameter of drilling depends on the design of the filter and the technical capabilities of the drilling equipment. Since at the discharge of wells reaching \( 20 \rightarrow 25 \, l/s \), computer type pumps and others are used as a water-lifting mechanism, the minimum diameter of the filter string should be 12 inches. So, the diameter of well drilling taking into account the necessary thickness of gravel \((0,20 \rightarrow 0,30 \, m)\) will be 1.0 m; - during groundwater withdrawal for irrigation (with the proposed arrangement of wells), damage to surface drain (main canals, inter and intra farm irrigation network) will be practically insignificant. Significant damage will be done to drainage;

- For the widespread use of groundwater for irrigation, it is necessary to conduct field studies in key areas. In hydrochemically favourable areas (along the Amu Darya River, old river Daryalyka), limited use of groundwater for irrigation is possible without additional research. The control of the hydrodynamic (hydrochemical) mode will be performed on the model by assessing the allowable decrease and the corresponding wells discharge (using developed application software for operating GWL) according to monitoring studies.

- Farms in isolated cases (favourable hydrochemical conditions) can use groundwater for irrigation under the control of Oblselvodkhoz and Oblgoskomriroda.
References


