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THERMAL EVOLUTION MODIFICATION DUE TO RADIO-WAVE PRODUCTION INSIDE ROTATING MAGNETIZED NS

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Abstract

We study effect of additional cooling mechanism on the thermal evolution of rotating magnetized neutron star (NS). The influence of general relativistic effects on charge distribution inside a NS leads to the qualitative distinction of space charge distribution inside the conducting crust from that inside the superconducting core which may play a source of a possible mechanism of radio-wave radiation production in the intermediate medium inside a rotating neutron star. This radiation may interact the matter of the neutron star and energy of the electromagnetic radiation can be transformed into the heating energy. The intensity of radio-wave radiation produced inside NS and its relation for the relaxation time of cooling of the rotating magnetized neutron star in the curved space-time are estimated. The relaxation time of cooling of the rotating magnetized neutron star is essentially increased by heating arisen by the interaction of the generated electromagnetic radiation with the conducting matter of the neutron star crust. New suggested physical mechanism for the heating energy of the radio-wave radiation is proposed and some estimates on.

Keywords: radio waves, neutron stars, cooling.

Physics and Astronomy Classification Scheme: 04.40.Dg, 04.40.Nr, 04.50.Kd

Introduction

Neutron stars are natural astrophysical laboratories of superdense matter. In their cores, at densities above the nuclear matter density $\rho_0 = 2.8 \times 10^{14} \text{g} \cdot \text{cm}^{-3}$, the properties of matter, such as equation of state and even composition, are largely unknown [1]. In the absence of an exact theory of superdense matter, different theoretical models predict different equations of state and compositions (neutrons, protons and electrons; hyperons; pion or kaon condensates; deconfined quarks).

One of the potentially powerful methods used to probe the internal structure of isolated NSs is modelling their cooling [2]. The theoretical cooling curves depend on the adopted physical models of the stellar interior, especially the neutrino emission and heat capacity, as well as the superfluidity of neutrons and superconductivity of protons in the core. Confronting theory and observations allows one, for example, to constrain the range of the critical temperatures of the superfluidity [3].

Observing the thermal emission of the very young NSs, with $t \leq 100 \text{yr}$, opens a possibility of studying the properties of the NS crust. Soon after a supernova explosion, the young star has large temperature gradients in the inner parts of the

crust. While the powerful neutrino emission quickly cools the core, the crust stays hot. The heat gradually flows inward on a conduction time-scale, and the whole process can be thought of as a cooling wave propagation from the centre towards the surface. During this thermal relaxation the effective temperature stays almost constant at about 250eV. When the cooling wave reaches the surface, the effective temperature drops sharply by as much as an order of magnitude in the fast-cooling scenario, and by a factor of 2 – 3 in the slow-cooling scenario. The duration of the relaxation epoch depends mainly on the heat capacity and the thermal conductivity of the inner crust [4], [5].

The purpose of this work is to study thermal evolution of rotating magnetized neutron star (NS) which includes new heating mechanism. The paper is organized as follows. In Section 1 we study the general relativistic electric charge density inside rotating magnetized relativistic neutron star with the constant magnetic and matter densities. Then we will show that the influence of general relativistic effects on charge distribution inside a NS may lead to the qualitative distinction of space charge distribution inside the conducting crust from that inside the superconducting core which may play a source of a possible mechanism of radio-wave radiation production in the intermediate medium inside a rotating neutron star. Then we formulate the radio-wave intensity inside rotating magnetized NS and find the estimated expressions for the electromagnetic radiation produced. In the Section 2 we introduce heat balance equation which includes interaction of the matter of the neutron star with electromagnetic radiation produced i.e. the heating energy transformed from the energy of the electromagnetic radiation. Then the intensity of radio-wave radiation produced inside NS and its relation for the relaxation time of heating of the rotating magnetized neutron star in the curved space-time are estimated. The relaxation time of cooling of the rotating magnetized neutron star is increased by heating arisen by the interaction of the generated electromagnetic radiation with the conducting matter of the neutron star crust. We include new suggested physical mechanism for the heating energy of the radio-wave radiation into the coefficient of conductivity. Finally in the Section 4 we summarize and discuss our results.

Throughout this paper we use a system of units in which $c = 1 = G$, a space-like signature $(-, +, +, +)$, a spherical coordinate system (t, r, θ, ϕ) , and Greek letters (running from 0 to 3) for four-dimensional space-time tensor components, while Latin letters (running from 1 to 3) will be employed for three-dimensional spatial tensor components.

1 Radio-Wave Radiation inside Rotating Magnetized Neutron Star

In a coordinate system (t, r, θ, ϕ) , the slow rotation spacetime metric for a rotating relativistic NS is [6, 7, 8, 9]

$$ds^2 = -e^{2\Phi(r)} dt^2 + e^{2\Lambda(r)} dr^2 - 2\omega_{LT}(r)r^2 \sin^2 \theta dt d\phi + r^2(d\theta^2 + \sin^2 \theta d\phi^2) , \quad (1)$$

where $\Phi(r)$ and $\Lambda(r)$ are unknown metric functions to be found as solutions of the field equations, $\omega_{LT}(r)$ is the Lense-Thirring angular velocity, which represents the angular velocity of dragging of inertial frames. The radial dependence of $\omega_{LT}(r)$ in the region of space-time internal to the NS has to be found as the solution of the differential equation

$$\frac{1}{r^3} \frac{d}{dr} (r^4 \bar{j} \frac{d\bar{\omega}}{dr}) + 4 \frac{d\bar{j}}{dr} \bar{\omega} = 0, \quad (2)$$

where

$$\bar{j} = e^{-(\Phi+\Lambda)}, \quad (3)$$

and

$$\bar{\omega} = \Omega - \omega_{LT}(r) \quad (4)$$

is the angular velocity of the internal fluid as measured from the local free-falling (inertial) frame, Ω is the angular velocity of the NS. In the vacuum region of space-time external to the NS, $\omega_{LT}(r)$ is known and given by

$$\omega_{LT}(r) = -\frac{g_{t\phi}}{g_{\phi\phi}} = \frac{2J}{r^3}, \quad (5)$$

where $J = I(M, R_s)\Omega$ is the total angular momentum of the gravitational source as measured from infinity and $I(M, R_s)$ is its momentum of inertia.

The simplest analytic interior solution to the Einstein field equations is the Schwarzschild one for an incompressible fluid with the constant mass density $\rho = \text{constant}$ when the metric functions take the form (see, e.g. [10])

$$e^{\Phi(r)} = \frac{3}{2}(1 - \varepsilon)^{1/2} - \frac{1}{2}e^{-\Lambda}, \quad e^{-2\Lambda(r)} = 1 - \varepsilon\bar{r}^2, \quad r < R_s, \quad (6)$$

where $\varepsilon = 2M/R_s$ is the compactness parameter of the NS, $M = 4\pi\rho R_s^3/3$ and R_s are the total mass and radius of the NS, $\bar{r} = r/R_s$ is the dimensionless radial coordinate. Typically, the dimensionless parameter $\varepsilon = 0.3 \div 0.4$ for the relativistic neutron stars depending on the equation of state of the stellar matter.

For the spacetime metric (1) one can find determinants of the metric tensor $g_{\alpha\beta}$ and the spatial metric tensor γ_{ij} as

$$g = \det[g_{\alpha\beta}] = -e^{2(\Phi+\Lambda)}r^4 \sin^2 \theta, \quad \gamma = \det[\gamma_{ij}] = -g/g_{00} = -e^{2\Lambda}r^4 \sin^2 \theta. \quad (7)$$

Neutron stars have very strong magnetic field which can be estimated as 10^{12}G at their surface. Assume that the exterior magnetic field is dipolar one aligned with the axis of rotation of the neutron star and the interior magnetic field has constant magnetic density. Then the interior magnetic field of the rotating magnetized NS in a slowly rotating space-time is given as [6]

$$B^{\hat{r}} = B_0 f(1) \cos \theta, \quad B^{\hat{\theta}} = -B_0 f(1) e^{-\Lambda} \sin \theta, \quad r < R_s, \quad (8)$$

where $B_0 = 2\mu/R_s^3$ is the Newtonian value of the magnetic field at the pole of the NS, μ is the magnetic dipole moment of the NS, and

$$f(\bar{r}) = -3 \left(\frac{\bar{r}}{\varepsilon}\right)^3 \left[\ln \left(1 - \frac{\varepsilon}{\bar{r}}\right) + \frac{\varepsilon}{\bar{r}} \left(1 + \frac{\varepsilon}{2\bar{r}}\right) \right]. \quad (9)$$

Accordingly, the induced interior electric field is[6]

$$E^{\hat{r}} = -\frac{\bar{\omega}r}{c}e^{-(\Phi+\Lambda)}B_0f(1)\sin^2\theta, \quad (10)$$

$$E^{\hat{\theta}} = -\frac{\bar{\omega}r}{c}e^{-\Phi}B_0f(1)\sin\theta\cos\theta. \quad (11)$$

The internal space charge density corresponding to the electrical field (10)–(11) is[6]

$$\rho_e = \frac{B_0f(1)}{4\pi c} \left\{ \left[3e^{-\Phi}\bar{\omega} - \frac{e^{-\Lambda}}{r^2} \left(e^{-(\Phi+\Lambda)}\bar{\omega}r^3 \right)_{,r} \right] \sin^2\theta - 2e^{-\Phi}\bar{\omega} \right\}. \quad (12)$$

This charge density is similar to the Goldreich-Julian relativistic charge density $\rho_{GJ}(\bar{r})$ in plasma magnetosphere defined as [12, 13]

$$\rho_{GJ}(\bar{r}) = -\frac{\nabla}{4\pi} \left[e^{-\Phi} \left(1 - \frac{q}{\bar{r}^3} \right) \mathbf{w} \times \mathbf{B} \right], \quad r < R_s, \quad (13)$$

where $q = \varepsilon\beta$, $\beta = I/I_s$ is the moment of inertia of the NS in the units of $I_s = MR_s^2$ and $\mathbf{w} = \Omega\bar{r}R_s\sin\theta\mathbf{e}_{\hat{\phi}}$. One can either substitute the expression for the magnetic field (8) into (13) and define expression for the Goldreich-Julian relativistic charge density or estimate the radial part of the electric charge density (12) as

$$\rho_e(\bar{r}) = -\frac{\Omega B_0 f(1)}{2\pi} \frac{e^{-\Phi}}{e^{\Phi}} \left(1 - \frac{q}{\bar{r}^3} \right), \quad r < R_s. \quad (14)$$

In contrast, by the solutions of Maxwell equations together with the relativistic equations for superconducting currents, it has been shown[11] that the space charge density inside a superconducting medium of the rotating magnetized neutron star is equal to zero. According to the recent models, the neutron star observed as a pulsar is the relativistic compact object consisting of the conducting crust and superfluid core. In the inner crust of the neutron star the superfluid coexists with a crystal lattice, and the motion of the vortex lattice is constrained by pinning to the lattice. In the core of the neutron star, at the densities above $2 \times 10^{14}\text{gm/cm}^3$ there is a homogeneous mixture of superfluid neutrons and superconducting protons. According to[11] the above distinction of space charge distribution in different media is responsible for the existence of additional mechanism of electromagnetic radiation which can be produced inside the superfluid core of the magnetized rotating neutron star where the superconducting protons form a type II superconducting state, with the magnetic field penetrating in an array of vortices.

The time-dependent charge redistribution arising from the permanent changing of the medium state (in every point) from normal to superconducting and back causes electric charges to be oscillated (accelerated) in the radial direction and so is expected to result in electromagnetic radiation with the frequency ω at the radio range determined by the time of phase transition. The general relativistic effect of charge redistribution inside conductors leads to some possible mechanism of radio wave radiation within the neutron star. It should be noted that this effect gives a direct coupling between rotation, magnetic field and the physics of the neutron stars.

We consider the radio-wave radiation inside the rotating magnetized NS produced by the system of charges. The radio-wave radiation does not leave the interior of the rotating magnetized NS [11]. We choose the origin of coordinates O anywhere on the centre of the rotating magnetized NS. The vector from O to the point P , where we determine the field, we denoted by \mathbf{R}_s ($R_s \sim 10^6 \text{cm}$ is radius of the rotating magnetized NS and point P is situated near the surface of the rotating magnetized NS). Let the position vector of the charge element $de = \rho e^\Lambda dV$ be \mathbf{r} , and the vector from de to the point P be \mathbf{R} . Obviously $\mathbf{R} = \mathbf{R}_s - \mathbf{r}$.

The relativistic system of charges distribution inside of the rotating magnetized NS is defined as follows:

$$\rho(\bar{r}, t) = \rho_{GJ}(\bar{r}) e^{ik_\alpha x^\alpha}, \quad r < R_s, \quad (15)$$

where $k^\alpha = (k^0, k^r, k^\theta, k^\phi)$ is the 4-wave vector of the radio-wave radiation inside the NS and $x^\alpha = (t, r, \theta, \phi)$ is the 4-position vector.

We get for the components of the vector potential of the field the system of charges at the point P in the curved space-time [14]

$$A^i(t) = \int \frac{j^i(\bar{r}, t)}{R} \sqrt{-\gamma} dr d\theta d\phi = \int \frac{j^i(\bar{r}, t)}{R} e^\Lambda dV, \quad (16)$$

here we have used (7) and taken into account $dV = r^2 \sin \theta dr d\theta d\phi$ for the 3-volume element. Components of the current density at the moment t in the curved space-time are

$$j^i(\bar{r}, t) = \frac{\rho(\bar{r}, t) dx^i}{\sqrt{-g_{tt}} dt} = \rho(\bar{r}, t) e^{-\Phi} v^i, \quad (17)$$

where $x^i = (r, \theta, \phi)$ is the 3-position vector of the charge element de , where, for fixed values of r and θ , ϕ changes along time.

The components of the velocity perturbation are then

$$\delta u^\alpha = \Gamma \left(1, \delta v^i \right) = \Gamma \left(1, e^{-\Lambda} \delta v^{\hat{r}}, \frac{\delta v^{\hat{\theta}}}{r}, \frac{\delta v^{\hat{\phi}}}{r \sin \theta} \right), \quad (18)$$

and

$$\delta u_\alpha = \Gamma \left(-e^{2\Phi}, e^\Lambda \delta v^{\hat{r}}, r \delta v^{\hat{\theta}}, r \sin \theta \delta v^{\hat{\phi}} \right), \quad (19)$$

where $\delta v^i \equiv dx^i/dt$ is the oscillation 3-velocity of the conducting stellar medium and $\delta v^{\hat{i}} \equiv \tilde{\omega}_k^{\hat{i}} \delta v^k$ are the components of the oscillation 3-velocity in the orthonormal frame carried by the ZAMO observers in the stellar interior. As we are interested in small velocity perturbations for which $\delta v^i/c \ll 1$, we can neglect $\mathcal{O}(\delta v^2)$ terms and use the normalization for the four-velocity $w_\alpha w^\alpha = -1$ to obtain

$$\Gamma = \left[-g_{00} \left(1 + g_{ik} \frac{\delta v^i \delta v^k}{g_{00}} \right) \right]^{-1/2} \simeq e^{-\Phi}. \quad (20)$$

For simplicity we assume de being in rest relative to NS; in that case expression for the components of the 3-velocity of the charge element de is

$$v^i = \frac{dx^i}{dt} = (0, 0, \Omega), \quad (21)$$

where $\Omega = d\phi/dt = \text{constant}$ is the angular velocity of the charge element de (the NS). Stellar matter moves with a velocity field

$$\delta u^\alpha = \frac{1}{N} \left(1, 0, 0, \Omega \right), \quad (22)$$

corresponding components of the 3-velocity are

$$\delta v^{\hat{i}} \equiv \left(0, 0, \Omega r \sin \theta \right), \quad (23)$$

where $r \leq R$ and Ω is the angular velocity of the star as measured by a distant observer.

We can find components of the vector potential of the system of charges at the point P in the curved space-time by (15), (16), (17) and (21):

$$A^r = A^\theta = 0, \quad A^\phi(t) = \Omega \int \frac{\rho_{GJ}(\bar{r})}{R} e^{ik_\alpha x^\alpha} e^{\Lambda - \Phi} dV. \quad (24)$$

Using (24) we calculate the components of of the electric field near the NS surface:

$$E_r = E_\theta = 0, \quad E_\phi(t) = \frac{1}{\sqrt{-g_{tt}^s}} \frac{\partial A_\phi(t)}{\partial t} = g_{\phi\phi}^s e^{-\Phi_s} \frac{\partial A^\phi(t)}{\partial t}, \quad (25)$$

here $g_{tt}^s = -e^{2\Phi_s}$ and $g_{\phi\phi}^s = R_s^2 \sin^2 \theta$ are values of the g_{tt} and $g_{\phi\phi}$ at the surface NS, so we have

$$E_\phi(t) = \Omega R_s^2 e^{-\Phi_s} \sin^2 \theta \int \frac{\rho_{GJ}(\bar{r})}{R} e^{ik_\alpha x^\alpha} ik_\alpha \frac{\partial x^\alpha}{\partial t} e^{\Lambda - \Phi} dV. \quad (26)$$

Let us reshape

$$ik_\alpha \frac{\partial x^\alpha}{\partial t} = ik_\alpha \frac{dx^\alpha}{ids} \frac{i\partial s}{\partial t},$$

where $dx^\alpha / ids = u^\alpha$ is the 4-velocity of observer, which defines the frequency of radio-wave radiation $\omega = -k_\alpha u^\alpha$ [15]. Using eq. (1) we can write $i\partial s / \partial t = e^\Phi$. Substituting it into the previous expression we get the following:

$$ik_\alpha \frac{\partial x^\alpha}{\partial t} = -i\omega e^\Phi,$$

and then we can use it to calculate $E_\phi(t)$:

$$E_\phi(t) = -i\omega \Omega R_s^2 e^{-\Phi_s} \sin^2 \theta \int \frac{\rho_{GJ}(\bar{r})}{R} e^{ik_\alpha x^\alpha} e^\Lambda dV. \quad (27)$$

We must take real part of the eq. (27)

$$E_{0r} = E_{0\theta} = 0, E_{0\phi}(t) = \omega \Omega R_s^2 e^{-\Phi_s} \sin^2 \theta \int \frac{\rho_{GJ}(\bar{r})}{R} \sin(k_\alpha x^\alpha) e^\Lambda dV. \quad (28)$$

To estimate luminosity L of the radio-wave radiation inside the rotating magnetized NS we have to calculate

$$dL(t) = \frac{E_0^2(t)}{4\pi} R_s^2 dO,$$

where $dO = 2\pi \sin \theta d\theta$ is the solid angle, then we have

$$dL(t) = \frac{1}{4\pi} \frac{E_{0\phi}^2}{g_{\phi\phi}^s} R_s^2 (2\pi \sin \theta d\theta), \quad (29)$$

and rewriting the expression using eq. (28) we have the following:

$$dL(t) = \frac{\sin^3 \theta d\theta}{2} \omega^2 \Omega^2 R_s^4 e^{-2\Phi_s} \left[\int \frac{\rho_{GJ}(\bar{r})}{R} \sin(k_\alpha x^\alpha) e^\Lambda dV \right]^2. \quad (30)$$

After straightforward integration we have the following (14):

$$L(t) = \frac{1}{6\pi^2} \omega^2 \Omega^4 B_0^2 R_s^4 f^2(1) e^{-2\Phi_s} \left[\int \frac{\sin(k_\alpha x^\alpha)}{R} \left(1 - \frac{q}{\bar{r}^3}\right) e^{\Lambda - \Phi} dV \right]^2, \quad (31)$$

because $\int_0^\pi \sin^3 \theta d\theta = 4/3$.

To estimate (31) we assume $\varepsilon \sim 0.3$, $\beta_s = 1$, $q_s = \varepsilon$, $e^{-2\Phi_s} = e^{\Lambda_s - \Phi_s} \sim 1 + \varepsilon$, and get

$$\left(1 - \frac{q_s}{\bar{r}_s^3}\right) e^{\Lambda_s - \Phi_s} \sim 1,$$

then

$$\left[\int \frac{\sin(k_\alpha x^\alpha)}{R} \left(1 - \frac{q}{\bar{r}^3}\right) e^{\Lambda - \Phi} dV \right]^2 \sim 1.77 \pi^2 R_s^4,$$

here index 's' corresponds to the surface values of quantities and $f^2(1) = 36\varepsilon^{-4} \sim 3.6 \times 10^3$. Doing quick calculations we can estimate luminosity L of radio emission:

$$L \sim 10^3 \omega^2 \Omega^4 B_0^2 R_s^8. \quad (32)$$

Inserting dimensional factor c (speed of light in vacuum) and using $\omega = 2\pi\nu$, we turn (32) into

$$\Sigma \sim \frac{L}{T R_s} \sim 10^4 \frac{\nu^2 \Omega^4 B_0^2 R_s^7}{c^5 T}. \quad (33)$$

Substituting $R_s \sim 10^6$ cm, $c \sim 10^{10}$ cm/s one can calculate minimum and maximum values of Σ corresponding to the values of the $\nu \sim (10^4 \div 10^5)$ Hz)

$$\Sigma_{max} = 100 \Sigma_{min} \sim 10^6 \frac{\Omega^4 B_0^2}{T}. \quad (34)$$

For example, we can calculate Σ_{max} and Σ_{min} for the case $\Omega \sim 10$ s⁻¹, $B_0 \sim 10^{18}$ G and $T \sim 10^{11}$ K (for very young NS):

$$\Sigma_{max} = 100 \Sigma_{min} \sim 10^{35} \text{ ergs}^{-1} \text{ K}^{-1} \text{ cm}^{-1}. \quad (35)$$

2 Additional Mechanism in NS Cooling Model and Relaxation Time Modification

NSs are born very hot in supernova explosion, with the internal temperature $T \sim 10^{11}\text{K}$, but gradually cool down [5]. About 20s after birth, they become fully transparent for the neutrinos generated in numerous reactions in stellar interiors. We consider the cooling in the following neutrino-transparent stage. The cooling is realized via two channels, by neutrino emission from the entire stellar body and by heat conduction from the internal layers to the surface resulting in thermal emission of photons. We also take account of the possible reheating mechanism is the radio-wave radiation inside the rotating magnetized NS.

The internal structure of NSs can be regarded as temperature-independent [16]. The relativistic equations of thermal evolution include the energy and flux equations [7, 5]

$$\frac{e^{-(\Lambda+2\Phi)}}{4\pi r^2} \frac{\partial}{\partial r}(e^{2\Phi} L_r) = -Q_\nu - C_v e^{-\Phi} \frac{\partial T}{\partial t}, \quad (36)$$

$$\frac{L_r}{4\pi r^2} = -(k - \Sigma) e^{-(\Lambda+\Phi)} \frac{\partial}{\partial r}(T e^\Phi), \quad (37)$$

where Q_ν is the neutrino emissivity ($\text{erg cm}^{-3}\text{s}^{-1}$), C_v is the specific heat capacity ($\text{erg cm}^{-3}\text{K}^{-1}$), k is the thermal conductivity ($\text{erg s}^{-1}\text{K}^{-1}\text{cm}^{-1}$), Σ is the conductivity for energy of the radio-wave radiation ($\text{erg} \cdot \text{s}^{-1}\text{K}^{-1}\text{cm}^{-1}$) and L_r is the "local luminosity" ($\text{erg} \cdot \text{s}^{-1}$), defined as the non-neutrino heat flux transported (with the thermal conductivity $k - \Sigma$) through a sphere of radius r . The problem has been studied in [5] for $\Sigma = 0$ case.

The effective temperature determines the photon luminosity:

$$L_\gamma = L_r(R_s, t) = 4\pi\sigma R_s^2 T_s^4(t), \quad (38)$$

where σ is the Stefan-Boltzmann constant. Both L_γ and T_s refer to the locally flat reference frame on the surface. A distant observer would register the apparent luminosity as

$$L_\gamma^\infty = L_\gamma(1 - \varepsilon), \quad (39)$$

the apparent effective temperature as

$$T_s^\infty = T_s(1 - \varepsilon)^{1/2}, \quad (40)$$

and the apparent radius as

$$R_s^\infty = R_s(1 - \varepsilon)^{-1/2}. \quad (41)$$

The duration of the thermal relaxation epoch is potentially interesting from the observational point of view. This problem has been studied in a number of papers, with the most detailed and thorough work by Lattimer et al. (1994, and references therein). Those authors considered thermal relaxation for the fast cooling and defined the relaxation time t_w as the moment of the most negative slope of the cooling curve, $\ln T_s(\ln t)$, of a young NS. This is a typical time for the cooling wave to reach the

surface. We will focus mainly on the case of rapid cooling, where the relaxation effects are more pronounced.

According to Lattimer et al. (1994), the relaxation time of rapidly cooling NSs of various masses is determined mainly by the crust thickness $\Delta R_{crust} = R_s - R_{core}$ and is given by a simple scaling relation

$$t_w \approx \alpha t_1, \alpha \equiv (\Delta R_{crust}/1km)^2 (1 - \varepsilon)^{-3/2}. \quad (42)$$

Here, t_1 is the normalized relaxation time which depends solely on the microscopic properties of matter, such as the thermal conductivity and heat capacity. In superfluid NSs, t_1 is sensitive to the magnitude and density dependence of the critical temperature of neutron superfluidity in the crust. It is important that t_1 appears to be almost independent of the NS model, its total mass M and radius R_s .

The dependence of t_w on the thermal conductivity k and heat capacity C_v follows from a simple estimate of the thermal relaxation time in a uniform slab of width l [5]:

$$t_w \sim C_v l^2 / k. \quad (43)$$

The proper width of a thin crust ($\Delta R_{crust} \ll R$), taking into account the effects of general relativity, is

$$l = \Delta R_{crust} (1 - \varepsilon)^{-1/2}. \quad (44)$$

This gives

$$t_w \sim (1 - \varepsilon)^{-1}, \quad (45)$$

in eq. (42). An additional factor of $(1 - \varepsilon)^{-1/2}$ accounts for the gravitational dilation of time intervals.

We define by analogy ($k \rightarrow k - \Sigma$) the relaxation time of rapid cooling of the rotating magnetized NS (from (43))

$$\tau_w \sim C_v l^2 / (k - \Sigma). \quad (46)$$

Using (43) and (46)), we can compare τ_w with t_w :

$$\frac{\tau_w}{t_w} = k / (k - \Sigma) = (1 - \Sigma/k)^{-1} > 1, \quad (47)$$

here $\tau_w > t_w$: the relaxation time of rapid cooling of the rotating magnetized NS is greater than the relaxation time of rapid cooling of the non-rotating or non-magnetized NS. From (34), one can find:

$$\Sigma/k \sim \frac{\epsilon \Omega^4 B_0^2}{kT}, \quad (48)$$

where $\epsilon \sim (10^4 \div 10^6 \text{ s}^3 \text{ cm}^2)$.

Using the data from [17], we choose $k \sim 10^{18} \text{ erg} \cdot \text{s}^{-1} \text{ cm}^{-1} \text{ K}^{-1}$ for $\rho \sim 10^{10} \text{ g} \cdot \text{cm}^{-3}$ and $T \sim 10^6 \text{ K}$.

$$\frac{\Sigma}{k} = 1 \left(\frac{10^{18} \text{ erg} \cdot \text{s}^{-1} \text{ cm}^{-1} \text{ K}^{-1}}{k} \right) \left(\frac{10^6 \text{ K}}{T} \right) \left(\frac{10^{12} \text{ Gs}}{B_0} \right)^2 \left(\frac{\Omega}{0.1 \text{ s}} \right)^4 \quad (49)$$

This shows that for NS with slowest rotation $\lim_{\Sigma \rightarrow k} \tau_w = \infty$, which means that there will be no surface cooling, i.e., surface temperature will be quite stable. However, with magnetic field of the star fading, Σ/k ratio will decrease and tend to zero, which will trigger cooling process to start. As the surface temperature decreases, thermal conductivity will grow and that will further accelerate cooling process, creating positive feedback loop. That claim is somewhat supported by accelerated cooling phase reported by [18] (see Fig.4)

3 Numerical temperature time series

In order to check our claim we solved Eqs. (36) and (37) numerically. We only considered infinitesimally thin layer of crust near the surface, so the temperature is almost the same everywhere and we may assume no radial dependance. Also, we assumed interior Schwarzschild metric 6. Taking all of these into account we may derive the following equation from (36) and (37):

$$\frac{dT}{dt} - \frac{(6 - 5\varepsilon)\varepsilon}{4R^2\sqrt{1 - \varepsilon}} \frac{k - \Sigma}{C_v} T = -\sqrt{1 - \varepsilon} \frac{Q_\nu}{C_v} \quad (50)$$

According to [19], neutrino emissivity Q_ν is a sum of T^6 and T^8 terms. We also adopt $C_v = \alpha T$ [18], so equation takes the following form:

$$\frac{dT_{11}}{d\tau} - \mu T_{11} + q_5 T_{11}^5 + q_7 T_{11}^7 + \frac{\sigma}{T} = 0, \quad (51)$$

Where we moved to dimensionless quantities and $\mu, q_{1,2}, \sigma$ are constants which depend on parameters of NS, such as compactness, magnetic field, rotation frequency and thermal conductivity of the crust. σ is zero when there's no additional cooling by radiowaves. As we are only concerned with qualitative behavior, we won't be paying too much attention to exact numbers. Figure 1 shows the numerical time series for temperature with and without accounting for radiowave heating. It is clearly visible, that constant-temperature plateau reported by [18] will not be present without taking radiowave heating into account. Also, reheating mechanism proposed by us increases the lifetime of NS.

There are some comparisons of cooling theory to observational data (see [20]) however all of them have quite low accuracy. Though we have clear prediction of the stars having redshifted temperature higher by several percents, accuracy of observations is not enough to confirm or deny the mechanism proposed by us.

4 Conclusions

We have studied the thermal evolution of rotating magnetized neutron star (NS) which includes new heating mechanism. The influence of general relativistic effects on electric charge distribution inside a NS leads to the qualitative distinction of space charge distribution inside the conducting crust from that inside the superconducting

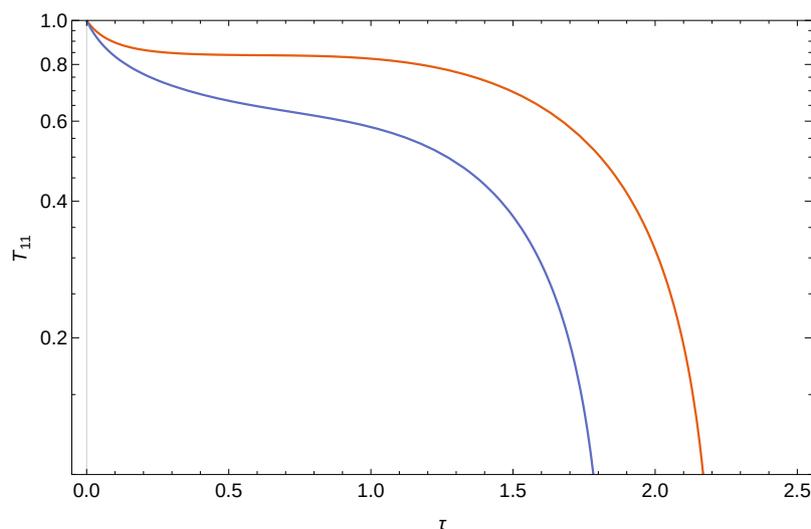


Figure 1: Numerical solution of eq. 51 for $\mu = 1$, $q_5 = 3$, $q_7 = 0.2$, $\sigma = 1$. Orange line depicts the temperature accounting for radiowave heating, blue line - without it.

core which may play a source of a possible mechanism of radio-wave radiation production in the intermediate medium inside a rotating neutron star. This radiation may interact the matter of the neutron star and energy of the electromagnetic radiation can be transformed into the heating energy. We have estimated the intensity of radio-wave radiation produced inside NS and its relation for the relaxation time of cooling of the rotating magnetized neutron star in the curved space-time. We have shown that the relaxation time of cooling of the rotating magnetized neutron star is increased by heating arisen from the interaction of the generated electromagnetic radiation with the conducting matter of the neutron star crust. New suggested physical mechanism for the heating of the crust by the radio-wave radiation is proposed.

We derived the expression (31) for the luminosity L of radio-wave radiation inside the rotating magnetized NS and the estimating relation (32) for L in the curved space-time.

We have considered a new physical quantity Σ which is an analogue of thermal conductivity for radio wave heating, and is estimated by (33) or, in more exact form, by (34) in the curved space-time.

We have calculated Σ_{max} and Σ_{min} for the case: $\Omega \sim 10\text{s}^{-1}$, $B_0 \sim 10^{18}\text{G}$ and $T \sim 10^{11}\text{K}$ (for the very young NS) $\Sigma_{max} = 100\Sigma_{min} \sim 10^{35}\text{ergs}^{-1}\text{K}^{-1}\text{cm}^{-1}$.

We have derived the estimating relation for the relaxation time τ_w of rapid cooling of the rotating magnetized NS (see (46)). We have shown that the relaxation time τ_w of rapid cooling of the rotating magnetized NS is always greater than the relaxation time t_w of rapid cooling of the non-rotating or non-magnetized NS (see (47)).

Using the known data it is shown that radio-wave heating becomes comparable to the cooling of NS crust at late stages of their evolution, albeit decay of magnetic field still breaks this balance. Numerical simulations show that proposed reheating mechanism allows to achieve qualitative agreement with observational data and extends

the lifetime of NS by several percents.

Numerical solution of equation for thermal evolution shows good qualitative agreement with previous results of [18].

Calculations in the paper are, surely, only a rough estimate, and future investigations using proper approach are needed to clarify the issue. It may be of benefit to conduct exact numerical simulations of thermal evolution taking into account all of the important factors.

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