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NODE-INDEPENDENT METHOD FOR GASTROENTEROLOGICAL SIGNAL PROCESSING BASED ON CUBIC SPLINES

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Abstract. This paper discusses a local cubic spline function built independently of node points using basic functions. the size of the calculations required to find the parameters to be determined during the construction of the spline function does not depend on the number of node points. Local-based splines are used to build such spline functions. Restoration of the gastroenterological signal was performed on the basis of the spline-function model discussed in the article. The result of a cubic spline-function error independent of the node points was compared with the result of the Lagrange classical polynomial error (Table 2).

Keywords. local basis functions, local cubic spline, Lagrange classical polynomial, gastroenterological signal, interpolation.

Introduction

Today, spline-function models are widely used by researchers in the field of digital signal processing [1, 2]. Because the accuracy of spline functions is higher than that of classical polynomials, and the algorithms based on them require less computation. Existing classical interpolation models, their application in signal recovery and digital processing algorithms are performed depending on the node points. In this case, a large number of node points are required for these models to have a high degree of convergence. In cases where the nodes are point-dependent, the order of the system of equations formed in the construction of the model under consideration increases. And this complicates the process of solving a system of equations and does not ensure a high level of accuracy of the model [3, 4, 14-20].

The process of digital processing of signals of the spline model discussed in this article was carried out independently of the node points. In this case, the increase in the degree of convergence of the model under consideration does not depend on the increase in the order of the system of equations. Methods for determining nonlinear spline functions, their derivatives, and estimating errors are performed in the same way as for simple interpolation splines [5, 9,10].

1. Local base functions

Let us be given the following function [12].

$$G(x,t)=(x-t)^3=\begin{cases}(x-t)^3, & x\ge t\\ 0, & x<t\end{cases} \qquad (1)$$

Assume that the $\{x_i\}$ node points on the OX axis are defined in steps h as follows.

$x_{i+1}=x_i+h,\ i=1,2,...,M$, $G(x,t)$ we enter the fourth-order divisor of the function separately for the $x_{i-2},x_{i-1},x_i,x_{i+1},x_{i+2}$ node points on the variable i:

$$\varphi_i(x)=G(x_{i-2},x_{i-1},x_i,x_{i+1},x_{i+2}),$$
$$i=3,4,...,M-2$$

The fourth-order difference of the $\varphi(x)$ function is determined by the following formula:

$$f(x_{i-2},x_{i-1},x_i,x_{i+1},x_{i+2})=$$
$$=\frac{f(x_{i-1},x_i,x_{i+1},x_{i+2})-f(x_{i-2},x_{i-1},x_i,x_{i+1})}{x_{i-2}-x_{i+2}}$$

.

In turn, $x_{i-1},x_i,x_{i+1},x_{i+2}$ and $x_{i-2},x_{i-1},x_i,x_{i+1}$ are also calculated sequentially as above.

After some simplification, the calculation formula for the fourth-order subtraction difference of the $G(x,t)$ function will look like this:

$$\varphi_i(x)=G(x_{i-2},x_{i-1},x_i,x_{i+1},x_{i+2})=$$
$$\frac{1}{4!h^4}[(x-x_{i+2})_+^3-4(x-x_{i+1})_+^3 \qquad (2)$$
$$+6(x-x_i)_+^3-4(x-x_{i-1})_+^3+(x-x_{i-2})_+^3]$$

Based on the features discussed above, the following forms the basis in the space of tertiary splines

$$S_i(x)=\frac{\varphi_i(x)}{\varphi_i(x_i)},\quad i=3,4,5,...,M-2 \qquad (3)$$

Let's look at the features.

This function forms the basis in the space of cubic splines and has the following features:

1) Smoothness

$$S_i(x)\in C^2[x_1,x_M] \qquad (3^1)$$

2) Locality

$$\begin{aligned}&S_i(x)>0,\ \ x\in(x_{i-2},x_{i+2})\\&S_i(x)\equiv 0,\quad x\notin(x_{i-2},x_{i+2}),\\&i=3,4,5...,\quad M-2\end{aligned} \qquad (3^2)$$

Here we see as proof

$$\varphi_i(x)=0 \qquad \text{if} \qquad x\notin(x_{i-2},x_{i+2})$$

Now let's calculate $\varphi_i(x_{i-2})$, $\varphi_i(x_{i-1})$, $\varphi_i(x_i)$, $\varphi_i(x_{i+1})$ *and* $\varphi_i(x_{i+2})$:

$$\varphi_i(x_{i-2})=\frac{1}{4!h_i^4}\Big[(x_{i-2}-x_{i+2})_+^3-$$
$$-4(x_{i-2}-x_{i+1})_+^3+6(x_{i-2}-x_i)_+^3$$
$$-4(x_{i-2}-x_{i-1})_+^3+(x_{i-2}-x_{i-2})_+^3\Big]=$$
$$=\left\{G(x,t)=(x-t)_-^3=\begin{cases}(x-t)^3, & x\ge t\\ 0, & x>t\end{cases}\right\}=0$$

$$\varphi_i(x_{i-2})=(4h)^3-4(3h)^3+6(2h)^3-4h^3=$$
$$=64h^3-108h^3+48h^3-4h^3=0$$

$$\varphi_i(x_{i-2})=0 \qquad (4)$$

The rest according to (3^2) are also found as follows

$$\varphi_i(x_{i-1})=\frac{1}{24h} \qquad (5)$$

$$\varphi_i(x_i)=\frac{1}{6h} \qquad (6)$$

$$\varphi_i(x_{i+1})=\frac{1}{24h} \qquad (7)$$

$$\varphi_i(x_{i+2})=0 \qquad (8)$$

As above, we find $\varphi_i(x_{i-2}),\varphi_i(x_{i-1}),\varphi_i(x_i),\varphi_i(x_{i+1}),\varphi_i(x_{i+2})$ based on the following nodes.

That is:

$$\varphi_{i-2}(x):\quad x_{i-4},\ i-3,\ i-2,\ i-1,\ i$$
$$\varphi_{i-1}(x):\quad i-3,\ i-2,\ i-1,\ i,\ i+1$$
$$\varphi_i(x):\quad i-2,\ i-1,\ i,\ i+1,\ i+2$$
$$\varphi_{i+1}(x):\quad i-1,\ i,\ i+1,\ i+2,\ i+3$$
$$\varphi_{i+2}(x):\ i,\ i+1,\ i+2,\ i+3,\ i+4$$

$$\varphi_{i-2}(x)=G(x,x_{i-4},x_{i-3},x_{i-2},x_{i-1},x_i)=$$
$$=\frac{1}{4!h^4}[(x-x_i)_+^3-4(x-x_{i-1})_+^3$$
$$+6(x-x_{i-2})_+^3-4(x-x_{i-3})_+^3+(x-x_{i-4})_+^3]=$$

$$=\frac{1}{4!h^4}[C_4^0(x-x_i)_+^3-C_4^1(x-x_{i-1})_+^3+$$
$$+C_4^2(x-x_{i-2})_+^3-C_4^3(x-x_{i-3})_+^3+$$
$$+C_4^4(x-x_{i-4})_+^3]=$$

$$=\frac{1}{4!h^4}\sum_{j=-4}^{0}(-1)^{4+j}C_4^{4+j}(x-x_{i+j})_+^3$$

We perform the $x_i<x<x_{i+1}$, $\quad t=\frac{x-x_i}{h}$ replacement as follows, which will be $x_{i+j}=h(t-j)$.

In that case

$$\varphi_{i-2}(x) = \frac{1}{4!h^4}\sum_{j=-4}^{0}(-1)^{4+j} *$$

$$* C_4^{4+j}\left(\frac{x - x_i + x_i - x_{i-4-j}}{h}\right)h^3 =$$

$$= \frac{1}{4!h}\sum_{j=-4}^{0}(-1)^{4+j}C_4^{4+j}[t + (j+4)]^3 =$$

$$= \frac{1}{24h}[(-1)^0 C_4^0[t+0]^3 + (-1)^1 C_4^1[t+1]^3 +$$

$$+ (-1)^2 C_4^2[t+2]^3 + (-1)^3 C_4^3[t+3]^3 +$$

$$(-1)^4 C_4^4[t+4]^3] = \frac{1}{24h}[t^3 - 4(t+1)^3 +$$

$$+ 6(t+2)^3 - 4(t+3)^3 + (t+4)^3] =$$

$$= \frac{1}{24h}[t^3 - 4t^3 - 12t^2 - 12t - 4 + 6t^3$$

$$+ 36t^2 + 72t + 48 - 4t^3 - 36t^2 - 108t -$$

$$- 108 + t^3 + 12t^2 + 48t + 64] =$$

$$= \frac{1}{24h}[(1 - 4 + 6 - 4 + 1)t^3 +$$

$$(-12 + 36 - 36 + 12)t^2 + (-12 + 72 - 108 + 48)t +$$

$$+ (-4 + 48 - 108 + 64)] = 0$$

$$\varphi_{i-2}(x) = 0$$

$$\varphi_{i+2}(x) = G(x, x_i, x_{i+1}, x_{i+2}, x_{i+3}, x_{i+4}) =$$

$$= \frac{1}{24h}[C_4^0[x - x_{i+4}]_+^3 + C_4^1[x - x_{i+3}]_+^3 +$$

$$C_4^2[x - x_{i+2}]_+^3 + C_4^3[x - x_{i+1}]_+^3 +$$

$$+ C_4^4[x - x_{i+1}]_+^3] = \frac{1}{24h}(x - x_i)^3$$

Now let's do a $t = \frac{x - x_i}{h}$ switch at $x \in [x_i, x_{i+1}]$, it will be $x - x_{i+j} = h(t - j)$.

In that case

$$\varphi_{i+2}(x) = \frac{1}{24h}\left(\frac{x - x_i}{h}h\right)^3 = \frac{1}{24h}h^3\left(\frac{x - x_i}{h}\right)^3$$

$$\varphi_{i+2}(x) = \frac{1}{24h}t^3$$

based on the above, we also substitute the $\varphi_{i-1}(x), \varphi_i(x), \varphi_{i+1}(x)$ for the variable t to form the following

$$\varphi_{i-2}(x) = 0 \tag{9}$$

$$\varphi_{i-1}(x) = \frac{1}{24h}(1-t)^3 \tag{10}$$

$$\varphi_i(x) = \frac{1}{24h}(3t^3 - 6t^2 + 4) \tag{11}$$

$$\varphi_{i+1}(x) = \frac{1}{24h}(1 + 3t + 3t^2 - 3t^3) \tag{12}$$

$$\varphi_{i+2}(x) = \frac{1}{24h}t^3 \tag{13}$$

We create based on the following functions (3) that form the basis in the field of tertiary splines.

We construct the basic functions based on (4) - (8) and (9) - (13).

$$S_{i-2}(x) = \frac{\varphi_{i-2}(x)}{\varphi_{i-2}(x_{i-2})} = 0 \tag{14}$$

$$S_{i-1}(x) = \frac{\varphi_{i-1}(x)}{\varphi_{i-1}(x_{i-1})} = \frac{\frac{1}{24h}(1-t)^3}{\frac{1}{6h}} = \frac{1}{4}(1-t)^3 \tag{15}$$

$$S_i(x) = \frac{\varphi_i(x)}{\varphi_i(x_i)} = \frac{\frac{1}{24h}(3t^3 - 6t^2 + 4)}{\frac{1}{6h}} = \tag{16}$$

$$= \frac{1}{4}(3t^3 - 6t^2 + 4)$$

$$S_{i+1}(x) = \frac{\varphi_{i+1}(x)}{\varphi_{i+1}(x_{i+1})} =$$

$$= \frac{\frac{1}{24h}(1 + 3t + 3t^2 - 3t^3)}{\frac{1}{6h}} = \tag{17}$$

$$= \frac{1}{4}(1 + 3t + 3t^2 - 3t^3)$$

$$S_{i+2}(x)=\frac{\varphi_{i+2}(x)}{\varphi_{i+2}(x_{i+2})}=\frac{\frac{1}{24h}t^3}{\frac{1}{6h}}=\frac{1}{4}t^3 \qquad (18)$$

The values of the $f(x)$ function at the $f_i=f(x_i),\ i=\overline{3,M-2}$ node points are given $x_4=a, x_{M-3}=b$.

Consider the following function.

$$S(x)=\sum_{i=3}^{M-2} f_i S_i(x), \quad x\in(x_4, x_{M-3})$$

From this function

$$S_i(x)>0, \quad x\in(x_{i-2}, x_{i+2})$$

$$S_i(x)\equiv 0, \quad x\notin(x_{i-2}, x_{i+2})$$

the local condition is required.

In that case, the values at the node point of the $S_i(x)$ function based on the localization condition are as follows.

$$S(x_i)=\sum_{j=i-1}^{i+1} f_j S_j(x_i), \quad i=\overline{4,M-3}$$

For simplicity

$$S(x_i)=\sum_{p=-1}^{1} f_{i+p} S_{i+p}(x_i)$$

That is

$$S(x_i)=f_{i-1}S_{i-1}(x_i)+f_iS_i(x_i)+$$
$$+f_{i+1}S_{i+1}(x_i), \qquad i=\overline{4,M-3}$$

The $S_i(x)$ function at values p= 0.1 is as follows

$$S_{i+p}(x_i)=S_{i-p}(x_i)=S_i(x_{i+p})=$$
$$=S_i(x_{i-p})=a_p$$

has properties

$$a_p=\begin{cases}1 & агар \quad p=0\\ 0{,}25 & агар \quad p=1\end{cases}$$

That is, at p = 0

$$S_i(x_i)=S_i(x_i)=S_i(x_i)=S_i(x_i)=1$$

really

$$S_i(x)=\frac{\varphi_i(x)}{\varphi_i(x_i)}\ ,$$

in this

$$S_i(x_i)=\frac{\varphi_i(x_i)}{\varphi_i(x_i)}=1,$$

The case p = 1 is similar.

As a result

$$S(x_i)=0{,}25f_{i-1}+1f_i+0{,}25f_{i+1}=$$
$$=f+0{,}25(f_{i-1}+f_{i+1}),\ i=\overline{4,M-3}$$

We now express the f_{i-1}, f_{i+1} through the f_i, for which we spread them to the Taylor series

$$f_{i-1}=f_i+hf_i'+\frac{h^2}{2}f_i''$$

$$f_{i+1}=f_i-hf_i'+\frac{h^2}{2}f_i''$$

By adding these expressions we get the following

$$f_{i-1}+f_{i+1}=2f_i+\frac{h^2}{2}(f_i''+f_i'')$$

$$f_{i-1}+f_{i+1}=2f_i+O(h^2)$$

$$0{,}25(f_{i-1}+f_{i+1})=0{,}5f_i+O(h^2)$$

In that case

$$S(x_i)=f_i+\frac{1}{4}(f_{i-1}+f_{i+1})=f_i+\frac{1}{2}f_i+O(h^2)\ ,$$

From this

$$S(x_i)=\frac{3}{2}f_i+O(h^2)$$

We will have $S(x_i)=\frac{3}{2}f_i+O(h^2)$.

The smoother the $f(x)$ function, the closer it is to f_i in $(f_{i-1}+f_{i+1})/2$ calculations.

And the $S_i(x)$ spline function is close to the $k\frac{3}{2}f(x)$ at the node points then we find the next approximate function

$$S^*(x_i)=\frac{2}{3}S(x_i)=\frac{2}{3}[\frac{3}{2}f_i+O(h^2)]$$

$$S^*(x_i)=f_i+O(h^2)$$

in which case it can be obtained

$$S^*(x)=\frac{2}{3}\sum_{i=3}^{M-2} f_iS_i(x)$$

$$S^*(x)=\frac{2}{3}S_{i-1}(x)f_{i-1}+\frac{2}{3}S_i(x)f_i+$$
$$+\frac{2}{3}S_{i+1}(x)f_{i+1}+\frac{2}{3}S_{i+2}(x)f_{i+2}$$

We enter the following notation, where $t=\frac{x-x_i}{h}, x=x_i+th$,

$$\psi_1(t)=\frac{2}{3}S_{i-1}(x);\quad \psi_2(t)=\frac{2}{3}S_i(x);$$
$$\psi_3(t)=\frac{2}{3}S_{i+1}(x);\quad \psi_4(t)=\frac{2}{3}S_{i+2}(x)$$

In that case

$$S^*(x)=\psi_1(t)f_{i-1}+\psi_2(t)f_i+$$
$$+\psi_3(t)f_{i+1}+\psi_4(t)f_{i+2}$$

It is not difficult to calculate $\psi_i(t)$.

We will see one of them count

$$\psi_1(t)=\frac{2}{3}S_{i-1}(x)=\frac{2}{3}\frac{1}{4}(1-t)^3=\frac{1}{6}(1-t)^3$$

the rest are calculated similarly.

$$\psi_2(t)=\frac{1}{6}(3t^3-6t^2+4)$$
$$\psi_3(t)=\frac{1}{6}(1+3t+3t^2+3t^3)$$
$$\psi_4(t)=\frac{1}{6}t^3$$

As a result, we write the general view of a tertiary spline function that is independent of node points as follows

$$S^*(x)=\sum_{j=1}^{4}\psi_j(t)f_{i-2+j}$$

Here

$$\begin{cases}\psi_1(t)=\frac{1}{6}(1-t)^3\\ \psi_2(t)=\frac{1}{6}\left(3t^3-6t^2+4\right)\\ \psi_3(t)=\frac{1}{6}\left(1+3t+3t^2+3t^3\right)\\ \psi_4(t)=\frac{1}{6}t^3\end{cases}$$

2. Digital processing of gastroenterological signals based on developed algorithms.

The construction of cubic spline models was performed on the basis of the gastroenterological signal given in Table 1 as the initial data [3, 6, 7]. Based on the above sequence, a cubic spline construction program in the MATLAB software environment of the gastroenterological signal given in Table 1 was developed and used in the processing of the gastroenterological signal [8, 11, 13]. The algorithm of this program is shown in Figure2.

Table 1. Values of gastroenterological signal.

№	X[i]	Y[i]
1.	1	6,370,065
2.	2	6,370,065
3.	3	5,835,327
4.	4	5,300,590
5.	5	4,765,852
6.	6	5,567,959
7.	7	6,637,434
8.	8	6,904,803
9.	9	6,904,803
10.	10	6,370,065
11.	11	6,370,065
12.	12	5,835,327
13.	13	5,300,590
14.	14	5,033,221
15.	15	5,033,221
16.	16	5,300,590
17.	17	6,102,696
18.	18	6,904,803
19.	19	7,172,172
20.	20	7,172,172
21	21	6,637,434
22	22	6,102,696
23	23	5,567,959
24	24	5,033,221
25	25	4,498,483
26	26	3,963,745
27	27	3,696,376
28	28	3,963,745
29	29	3,963,745
30	30	4,498,483
31	31	5,033,221
32	32	5,835,327
33	33	6,102,696
34	34	6,637,434
35	35	6,904,803
36	36	7,172,172
37	37	7,172,172
38	38	7,172,172
39	39	7,439,541
40	40	6,904,803
41	41	6,370,065
42	42	5,835,327

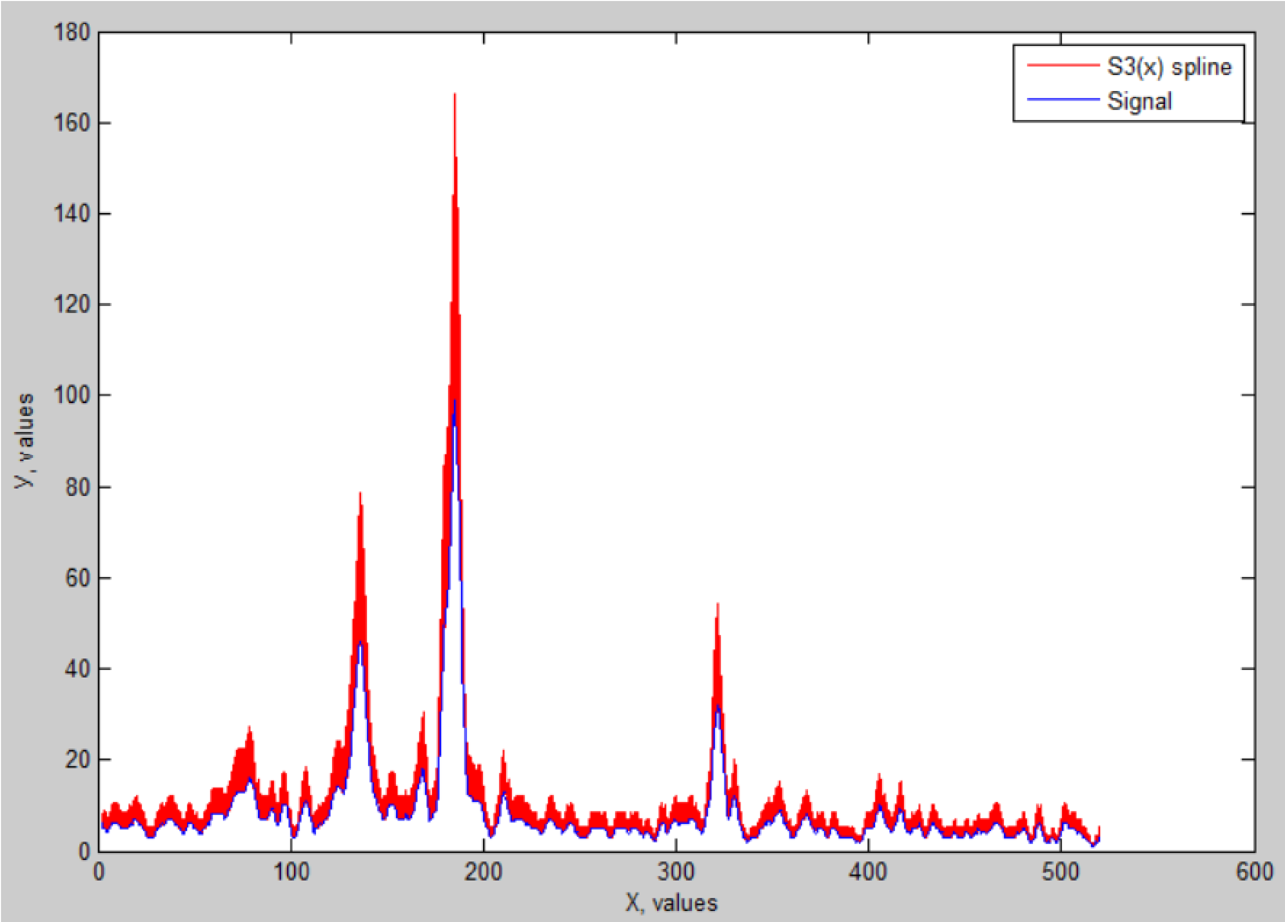

Fig. 1. Results of gastroenterological signal recovery.

Table 2. Error results in the process of digital processing of the gastroenterological signal.

№	Model types	Absolute error
1.	Lagrange classical interpolation model	1.0493
2.	A cubic spline model that does not depend on node points	0.1025

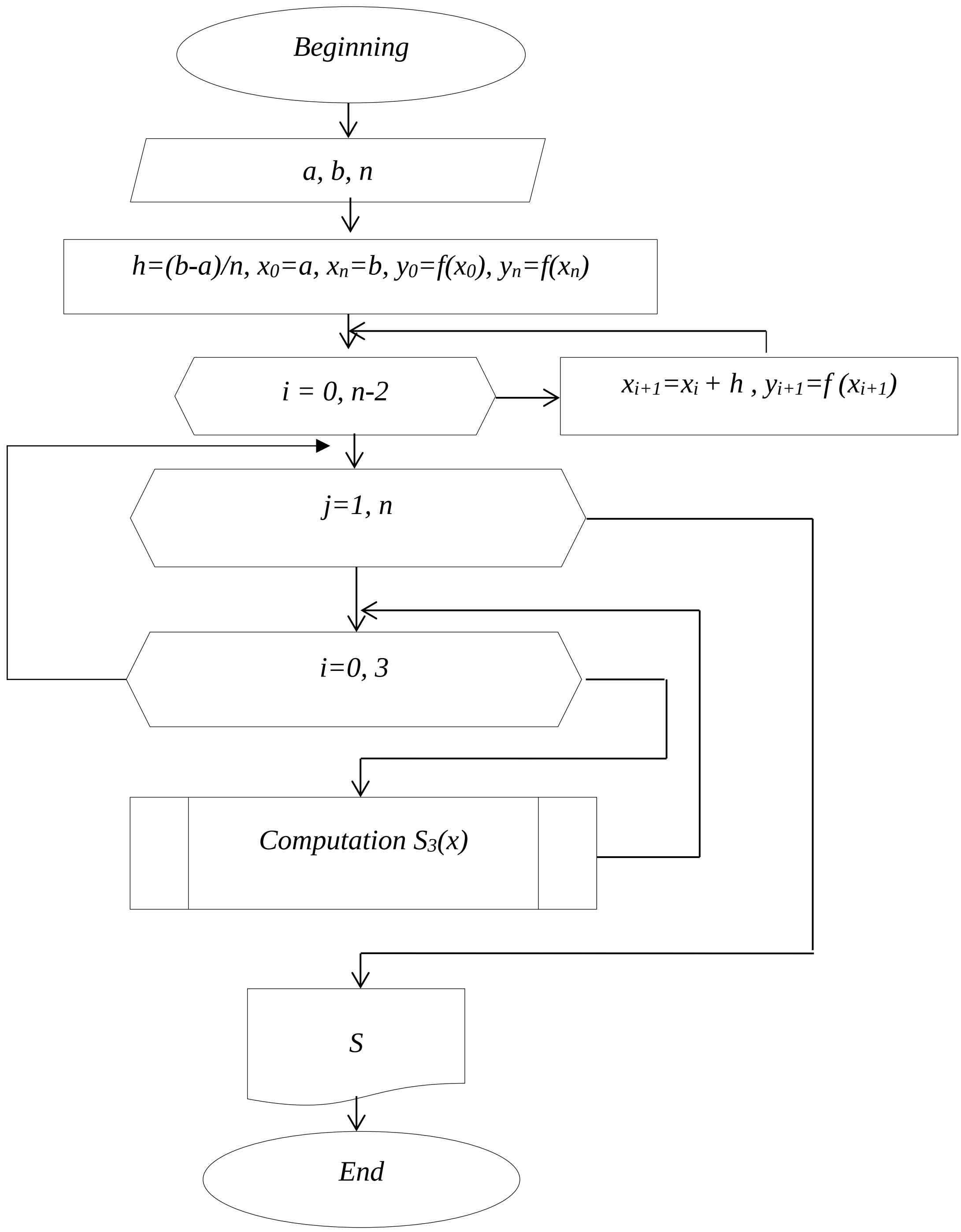

Fig. 2. Algorithm of cubic spline construction program.

3. Conclusion

This paper discusses a local cubic spline function built independently of node points using basic functions. And was used in the digital processing of the gastroenterological signal. The process of gastroenterological signal recovery was performed using the proposed method. The result showed that the method of tertiary spline functions independent of node points showed high accuracy in digital processing of signals (Figure 1).

The proposed cubic spline model has the following features compared to classical interpolation models:

-good proximity to the object when interpolating gastroentrological signals;

-the construction of the model is independent of node points and is much simpler than the classical polynomials;

-that the algorithm for determining the parameters of the spline is simple and convenient.

Hence, the use of cubic spline models built independently of the node points according to Table 2 gives good results in the digital processing of gastroenterological signals.

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