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TOPOLOGICAL PROPERTIES OF HYPERSPACES

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Abstract

In the work, it is given the normal functor F acting in the category of compacts and their continuous mappings. This functor does not preserve the Souslin number (or hereditary cellularity), the hereditary density, the hereditary π-weight and the hereditary Shanin number, the hereditary caliber, the hereditary precaliber, the hereditary preshanin number, the hereditary weak density, the hereditary Lindelöf number, and the hereditary extent of a compact. The example of the normal functor and the compact of Aleksandorv's two arrows are given. We study the action of functors $\exp_n$, $\exp_\omega$, $\exp_c$ and exp on finally compact, hereditarily disconnected, strongly zero-dimensional, extremely disconnected, paracompact spaces.

Keywords: normal functors, hyperspace, caliber, precaliber, Souslin number, finally compact space, hereditarily disconnected space, strongly zero-dimensional space, extremally disconnected space, paracompact space.

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Introduction

In [1], V.V. Fedorchuk stated the following general problems in the theory of covariant functors that defined a new direction of investigations in the given field of the topology:

Let X has a property P. Does it imply that $F(X)$ has the same property? Or, conversely, let P be a geometrical property. What conditions must a functor satisfy in order that $F(X)$ having property P would imply that X having the same property?

In 1997, S. Todorcevic and V. Fedorchuk proved [2] that X is an infinite compactum and F is a normal functor then $c\left(F\left(X\right)\right) = \sup\left\{c\left(X^n\right) : n \in N\right\}$.

Since the Souslin number is inherited by every everywhere dense subset, the theorem holds for any infinite Tychonoff space X, since it is everywhere dense in its Stone-Cech compact extension.

In the work of R.B.Beshimov [3] and $F : Comp \to Comp$ is weakly normal, then $d(F^\beta(X)) \leq d(X)$ and $wd(F^\beta(X)) \leq wd(X)$ hold for any infinite Tychonoff space X, here F^β denotes the natural extension of the functor F to the $Tych$ category of Tychonoff spaces, $d,\ wd$ density and weak density of topological spaces .

In the works of E.V. Shchepin [11], a far advanced and very meaningful general theory of covariant functors was constructed. He singled out a number of natural and low-restrictive properties of functors and defined a normal functor.

T. Radul [12] began to study the functor of weakly additive order-preserving normalized functionals in the category of compact sets and their continuous mappings. He showed that this functor is not normal and, therefore, the study of this functor is much more complicated than the study of normal functors. The appearance of this functor is also due to the fact that, recently, the theory of nonlinear functionals is being developed.

Albeverio S., Ayupov Sh.A., Zaitov A.A. [13] continued the functor O to the functor $O \circ \beta$, and investigated the categorical properties of the functor $O \circ \beta$. And in the works of R. Beshimov [3], the functor O is extended to a functor acting in the category of Tychonoff spaces and their continuous mappings. Further, Davletov D.E., Djabbarov G.F. [14] studied weak additive, order-preserving, normalized, positive-homogeneous and semiadditive functionals defined on the space of all continuous real functions of compact X, and studied the algebraic dimension of the space of semiadditive functionals. And also, Davletov D.E., Djabbarov G.F. [14] found a necessary and sufficient condition for the semiadditivity of a functional defined on a compact set.

We recall some definitions and facts for the presentation of the work.

Let X be a set and $\leq$ a relation on X. We say that [7] $\leq$ directs X, or that X is directed by $\leq$, if $\leq$ has the following properties:

a) if $x \leq y$ and $y \leq z$, then $x \leq z$;

b) for every $x \in X$ we have $x \leq x$;

c) for any $x,\ y \in X$ there exists a $z \in X$ such that $x \leq z$ and $y \leq z$.

Suppose that to every $\alpha \in \Sigma$ in a set Σ directed by the relation $\leq$ corresponds a topological space X_α, and that for any $\alpha,\ \beta\ \in \Sigma$ satisfying $\beta\ \leq \alpha$ a continuous mapping $\pi_\beta^\alpha : X_\alpha \to X_\beta$ is defined: suppose further that $\pi_\gamma^\beta \circ \pi_\beta^\alpha = \pi_\gamma^\alpha$ for any $\alpha,\ \beta,\ \gamma\ \in \Sigma$ satisfying $\gamma\ \leq\ \beta\ \leq\ \alpha$ and that $\pi_\alpha^\alpha = \mathrm{id}_{X_\alpha}$ for every $\alpha \in \Sigma$. In this situation we say that the family $S = \left\{X_\alpha,\ \pi_\beta^\alpha,\ \sum\right\}$ is an inverse spectrum of the spaces X_α, the mappings π_β^α are called bonding mappings of the inverse spectrum S (see [6], [7]).

Let $S = \left\{X_\alpha,\ \pi_\beta^\alpha,\ \sum\right\}$ be an inverse spectrum. An element $\{x_\alpha\}$ of the Cartesian product $\prod_{\alpha\in\sum} X_\alpha$ is called a thread of S if $\pi_\beta^\alpha\left(x_\alpha\right) = x_\beta$ for any $\alpha,\ \beta\ \in \sum$ satisfying $\beta\ \leq\ \alpha$, and the subspace of $X \subseteq \prod_{\alpha\in\sum} X_\alpha$ consisting of all threads of S, is called the limit of the inverse spectrum $S = \left\{X_\alpha,\ \pi_\beta^\alpha,\ \sum\right\}$ and is denoted by $X = \varprojlim S$ or by $X = \varprojlim \left\{X_\alpha,\ \pi_\beta^\alpha,\ \sum\right\}$. The mappings $\pi_\alpha : \varprojlim S \to X_\alpha$, i.e. projections $p_\alpha : \Pi \to X_\alpha$, considered only at the limit X of the spectrum S, are called end-to-end projections, and mappings $\pi_\beta^\alpha : X_\alpha \to X_\beta$ for $\beta\ \leq\ \alpha$ are called projections. Since the projections $p_\alpha : \Pi \to X_\alpha$ are continuous, the through projections $\pi_\alpha : X \to X_\alpha$ are also continuous. Note that the limit X of the inverse spectrum $\left\{X_\alpha,\ \pi_\beta^\alpha,\ \sum\right\}$ of nonempty compact sets X_α is nonempty (see [6]).

Let an inverse spectrum $S = \left\{Y_\alpha,\ \pi_\beta^\alpha,\ \sum\right\}$ be given, and let a mapping $f_\alpha : X \to Y_\alpha$ be defined for each $\alpha \in \Sigma$, and $f_\alpha = \pi_\alpha^\beta \circ f_\beta$, if $\beta > \alpha$. Then for any point $x \in X$ the set $\{f_\alpha\left(x\right)\} \in \Pi = \prod_{\alpha\in\Sigma} Y_\alpha$ is a thread of the spectrum S, i.e. the map $f : X \to Y$of the set X to the limit Y of the spectrum S is defined. This map is called the limit of the maps f_α, $\alpha \in \Sigma$ and is denoted by $f = \lim\limits_{\leftarrow} f_\alpha$.

Let $\xi = \{\theta, \mathrm{M}\}$ be a family of elements of two sorts. Elements from θ are called objects, and elements from M are called morphisms. For each morphism f, a unique ordered pair of X, Y objects is defined, and f is called a morphism from X to Y. In this situation, X is sometimes denoted by $dom\, f$, and Y : $rng\, f$. The family of all morphisms from X and Y is denoted by $[X, Y]$.

A family $\xi = \{\theta, \mathrm{M}\}$ is called a category [6] if the following conditions are satisfied:

a) for each pair of morphisms f and g with $rng\, f = dom\, g$ a unique morphism h with $dom\, h = dom\, f$ and $rng\, h = rng\, g$ is defined, called the composition of morphisms f and g and denoted by $g \circ f$;

b) for every object $X \in \theta$ there is a unique morphism from X to X, denoted by id_X, such that $id_Y \circ f = f = f \circ id_X$ for every morphism $f : X \to Y$;

c) $(h \circ g) \circ f = h \circ (g \circ f)$ for any triple of morphisms with $rng\, f = dom\, g$ and $rngg = domh$.

Let $\xi = (\theta, \mathrm{M})$ and $\xi' = (\theta', \mathrm{M}')$ be two categories. A mapping $F : \xi \to \xi'$ that takes objects to objects, and morphisms to morphisms, is called a covariant functor [6] from the category ξ to the category ξ' if:

1) for every morphism $f : X \to Y$ from the category ξ, the morphism $F(f)$ acts from $F(X)$ to $F(Y)$.

2) $F(id_X) = id_{F(X)}$ for every $X \in \theta$.

3) $F(f \circ g) = F(f) \circ F(g)$.

Note that in the work by "functor" we mean a covariant functor.

Definition 1 ([6]). *A functor $F : Comp \to Comp$ is called continuous if, for any inverse spectrum $S = \{X_\alpha, p^\alpha_\beta, \Omega\}$, the inverse spectrum $F(S) = \{F(X_\alpha), F(p^\alpha_\beta), \Omega\}$ defined by the maps $F(p^\alpha_\beta) : F(\underleftarrow{\lim} S) \to F(X_\alpha)$ is defined, where p_α are the end-to-end projections from $\underleftarrow{\lim} S$ to X_α, and the map $\underleftarrow{\lim} F(p_\alpha)$ from space $F(\underleftarrow{\lim} S)$ to the space $\underleftarrow{\lim} F(S)$ is a homeomorphism.*

A functor $F : Comp \to Comp$ is called weight preserving if $\omega F(X) = \omega(X)$ for any infinite compact X.

A functor $F : Comp \to Comp$ is called monomorphic if for any embedding i of the compact space X into the compact space Y, the map $F(i) : F(X) \to F(Y)$ is also an embedding.

A functor $F : Comp \to Comp$ is called epimorphic if it preserves the surjectivity of compact maps.

A functor $F : Comp \to Comp$ is called intersection preserving if for any family $\{X_\alpha : \alpha \in \Omega\}$ of closed subsets of an arbitrary compact set we have $\cap\{F(X_\alpha) : \alpha \in \Omega\} = F(\cap\{X_\alpha : \alpha \in \Omega\})$.

A functor $F : Comp \to Comp$ is called preserving preimages if for any continuous mapping f of compact set Y and for any closed subset $B \subseteq Y$, we have $F(f^{-1}B) = F(f)^{-1}F(B)$.

A covariant functor $F : Comp \to Comp$ is called normal if it is continuous, preserves a weight, intersections and preimages, is monomophic and epimorhic, and transfers a one-point space into a one-point one, and an empty set into an empty one [8].

A covariant functor $F : Comp \to Comp$ is called weakly normal if it satisfies all the conditions of normality, except for preserving the inverse images [6].

For a covariant functor F, we denote by F_n a functor associating the space X with the set of all those elements $a \in F(X)$ whose supports consist of no more than n points.

A cardinal topological invariant is a cardinal-valued function defined on a class of topological spaces and associating the same cardinal number with topologically equivalent spaces.

A family λ of nonempty subsets of a topological space X is called a π-network if for any open subset U of the space X, there is an element of the family λ lying in the set U. A π-network of the topological space X consisting of open sets in X is called a π-base of the topological space X.

A set $A \subset X$ is called dense in X if $[A] = X$. The density of a space X is defined as the smallest cardinal number of the form $|A|$, where A is the dense subset of X. This cardinal number is denoted by $d(X)$. Moreover, if $d\,(X) = \tau,\ \ \tau \geq \aleph_0$, then the space X is called τ-dense. If $d\,(X) \leq \aleph_0$, then we say that the space X is separable.

A family $B(x)$ of neighborhoods of x is called a base for a topological space X at the point x if for any neighborhood V of x there exists a $U \in B(x)$ such that $x \in U \subset V$.

The character [7] of a point x in the topological space X is defined as the smallest cardinal number of the form $|B(x)|$, where $B(x)$ is a base for X at the point x; this cardinal number is denoted by $\chi(x, X)$. The character [7] of a topological space X is defined as the supremum of all numbers $\chi(x, X)$ for $x \in X$; this cardinal number is denoted by $\chi(X)$. If $\chi(X) \leq \aleph_0$, then we say that the space X satisfies the first axiom of countability or is first-countable.

A family γ is called a π-network [9] at $x \in X$ if for any neighborhood U of x, there exists a nonempty set $B \in \gamma$ such that $B \subset U$. A π-network at a point x consisting of open sets in X is called a π-base at a point x.

The π-character [7] of the point x in the topological space X is the smallest cardinal number of the form $|B(x)|$, where $B(x)$ π -base X at the point x; this cardinal number is denoted by $\chi(x, X)$. The π-character of a topological space X is defined as follows:

$$\pi\chi(X) = \sup\{\pi\chi(x, X) : x \in X\}.$$

The smallest cardinal numbers $\tau \geq \aleph_0$ such that every family or pairwise disjoint non-empty open subsets of X has cardinality $\leq \tau$, is called the Souslin number, or cellularity, of the space X and is denoted by $c(X)$. If $c(X) \leq \aleph_0$, we say that the space X has the Souslin property.

The weak density [7] of a topological space X is the smallest cardinal number $\tau \geq\ \aleph_0$ such that in X there exists a π-base decomposing into τ centered systems of open sets, that is, there is a π-base $\mathrm{B} = \bigcup\{B_\alpha :\ \alpha \in A\}$, where B_α is the centered system of open sets for each $\alpha \in A$ and $|A| = \tau$.

The weak density of a topological space X is denoted by $wd\,(X)$. If $wd\,(X) = \aleph_0$, then the topological space X is called weakly separable.

The pseudocharacter [9] of a T_1-space at a point x is defined as the smallest cardinal $|\lambda|$, where λ is a family of open sets in X such that $\cap\lambda = \{x\}$; this cardinal is denoted by $\psi(x, X)$. The pseudocharacter [9] of a T_1-space X is defined as the supremum of all cardinals $\psi(x, X)$, where $x \in X$; this cardinal is denoted by $\psi(X)$. It is clear that $\psi(x, X) \leq \chi(x, X)$ and $\psi(X) \leq \chi(X)$ for each T_1-space X and any $x \in X$. If X is compact, then $\psi(x, X) = \chi(x, X)$ for all $x \in X$ and $\psi(X) = \chi(X)$.

The tightness [9] of the point x in the topological space X is the smallest cardinal number $\tau \geq \aleph_0$ with the following property:

if $x \in [C]$, then there exists a $C_0 \subset C$ such that $|C_0| \leq \tau$ and $x \in [C_0]$. This cardinal number is denoted by $t(x, X)$.

The tightness of a topological space X is the supremum of all numbers $t(x, X)$ for $x \in X$; this cardinal number is denoted by $t\,(X)$ i.e. $t\,(X) = \sup\{t\,(x,\,X) :\ t\,(x,\,X) -\ \text{tightness at point}\ x\ \text{space}\ X\}$.

A cardinal $\tau > \aleph_0$ is called a caliber [10] of the space X if for any family $\mu = \{U_\alpha : \alpha \in A\}$ of nonempty open sets in X such that $|A| = \tau$, there exists $B \subset A$, for which $|B| = \tau$, and $\cap\{U_\alpha : \alpha \in B\} \neq \emptyset$. Set $k(X) = \{\tau : \tau\ \text{is a caliber of the space}\ X\}$. A cardinal $\tau > \aleph_0$ is called a precaliber [10] of the space X, if for a family $\mu = \{U_\alpha : \alpha \in A\}$ of non-empty open sets in X such that $|A| = \tau$, there is $B \subset A$, for which $|B| = \tau$, and $\{U_\alpha : \alpha \in B\}$ is centered. Set $pk(X) = \{\tau : \tau\ \text{is a precaliber of the space}\ X\}$. The Shanin number $sh(X)$ [10] of a topological space X is defined as follows:

$sh(X) = \min\{\tau : \tau^+\ \text{is a caliber of the space}\ X\}$, where τ^+ is the least cardinal number from all cardinals strictly greater than τ.

The preshanin number $psh(X)$ [10] of a topological space X is defined in a following way:

$$psh(X) = \min\{\tau : \tau^+\ \text{is a precaliber of the space}\ X\}.$$

The smallest cardinal number $\tau \geq \aleph_0$ such that every closed subset of X consisting only of isolated points has cardinality $\leq \tau$, is called the extent of the space X and is denoted by $e\,(X)$, i.e. $e\,(X) = \sup\,\{\,|\,Y\,|\ :\ Y\ \text{is a closed disrete subspace in}\ X\,\}$ [6].

1 Preliminary results

In [12], T. Radul considered the functor of weakly additive normed and order-preserving functional on the category of compacts $O : Comp \to Comp$. He proved that the covariant functor $O : Comp \to Comp$ satisfied to all conditions of normality except preimage preservation, i.e. $O : Comp \to Comp$ is weakly normal functor.

Let X be a compact. We denote by $C\,(X)$ the space of all continuous functions $f : X \to \mathrm{R}$ with ordinary (pointwise) operations and supnorm, i.e. with the norm $\|f\| = \sup\,\{|\mathrm{f\,(x)}| : \mathrm{x \in X}\}$. For each $c \in \mathrm{R}$, let c_X denote the constant function defined by the formula $c_X(x) = c,\ x \in X$. Let $\phi,\ \psi\ \in C(X)$. The inequality $\phi\ \leq\ \psi$ means that $\phi\ (x) \leq\ \psi\ (x)$ for all $x \in X$.

The functional $\nu\ : C(X) \to \mathrm{R}$ is called [15]:

1) weakly additive if for all $c \in \mathrm{R}$ and $\phi\ \in C(X)$ the equality $\nu\ (\varphi\ + c_X) = \nu\ (\varphi) + c \cdot \nu\ (1_X)$ holds;

2) order preserving if for functions $\phi,\ \psi\ \in C(X)$ from $\phi\ \leq\ \psi$ follows $\nu\ (\phi)\ \leq \nu\ (\psi)$;

3) normalized if $\nu\ (1_X) = 1$;

4) positively homogeneous if $\nu\ (\lambda\ \phi) = \lambda\ \nu\ (\phi)$ for all $\phi\ \in C(X),\ \lambda\ \in \mathrm{R}_+$, where $\mathrm{R}_+ = [0,\ +\infty)$;

5) semiadditive if $\nu(f + g) \leq \nu(f) + \nu(g)$ for all $f, g \in C(X)$.

We endow the set $M(X)$ with a weak topology, i.e. we consider $M(X)$ a subset of the product of number lines $\prod \{R_\varphi :\ \varphi \in C(X)\}$. The neighborhood base of an element $\mu \in M(X)$ is formed by the sets $O(\mu,\ \varphi_1, ..., \varphi_k,\ \varepsilon)$ where $\varphi_1, ..., \varphi_k \in C(X)$, $\varepsilon > 0$ and $O(\mu,\ \varphi_1, ..., \varphi_k,\ \varepsilon) = \{\mu' \in M(X) :\ |\mu'(\varphi_i) - \mu(\varphi_i)| < \varepsilon,\ i = 1, ..., k\}$. So $M(X)$ is a completely regular space. The subspace $M(X)$ consisting of all probability measures is denoted by $P(X)$. By $P_n(X)$ we denote the set of all measures $\mu \in P(X)$ whose supports consist of at most n points. The set $P_n(X)$ is closed in $P(X)$ [6].

For a compact X by $O(X)$, we denote the set of all weakly additive, order-preserving, normed functionals [10]. Elements of the set $O(X)$, for brevity, are called weakly additive functionals. By $OH(X)$ we denote the set of all positively homogeneous functionals from $O(X)$, and by $OS(X)$ we denote the set of all semiadditive functionals from $OH(X)$. These sets are equipped with the pointwise convergence topology. The base of neighborhoods of the functional $\mu \in O(X)$ (respectively, $\mu \in OH(X)$, $\mu \in OS(X)$) is formed by sets of the form

$$\langle \mu; \varphi_1, ..., \varphi_k; \varepsilon \rangle = \{\nu \in O(X) :\ |\mu(\varphi_i) - \nu(\varphi_i)| < \varepsilon,\ i = 1, ..., k\}$$

respectively,

$$\langle \mu; \varphi_1, ..., \varphi_k; \varepsilon \rangle = \{\nu \in OH(X) :\ |\mu(\varphi_i) - \nu(\varphi_i)| < \varepsilon,\ i = 1, ..., k\},$$

$$\langle \mu; \varphi_1, ..., \varphi_k; \varepsilon \rangle = \{\nu \in OS(X) :\ |\mu(\varphi_i) - \nu(\varphi_i)| < \varepsilon,\ i = 1, ..., k\}$$

where $\varphi_i \in C(X),\ i = 1, ..., k,\ k \in N,\ \varepsilon > 0$.

Obviously, for each compact X the space $P(X)$ of probability measures (i.e., the space of all linear, nonnegative, normed functionals) is a subspace of the $OS(X)$ space.

In paper [31] introduced the OS_τ functor of semi-additive τ-smooth functionals, acting in the category of Tychonoff spaces $Tych$, which can be considered as a continuation of the functor OS of semiadditive functionals from the $Comp$ category of compact sets. It is proved that the semiadditive τ-smooth functional $\nu_A \in OS(\beta X)$ is uniquely determined by the convex compact set $A \subset P_\tau(X)$ of the space of τ-smooth probability measures.

Let X be an infinite Tychonoff space and a $C_b(X)$ space of all continuous bounded functions $f : X \to \mathrm{R}$ with ordinary (pointwise) operations and sup-norm, i.e. with norm $\|f\| = \sup\ \{|\mathrm{f}\ (\mathrm{x})| : \mathrm{x} \in \mathrm{X}\}$.

A functional $\mu \in O(\beta X)$ is called τ-smooth if $\mu(\phi_\alpha) \to 0$ for any monotonically decreasing directivity $\{\phi_\alpha\} \subset C(\beta X)$ that converges pointwise to zero [13].

For a Tychonoff space X, by $OS_\tau(X)$ we denote the set of all semiadditive τ-smooth functionals from $O(\beta X)$. $OS_\tau(X)$ considers the topology induced from $OS(\beta X)$. It is clear that $P_\tau(X) \subset OS_\tau(X) \subset O_\tau(X)$ for any Tychonoff space X.

Includes

$$OS_\beta(X) \subset OS_\tau(X) \subset OS(\beta X)$$

for any Tychonoff space X and the equality

$$OS_\beta(X) = OS_\tau(X) = OS(\beta X)$$

for any compact space X.

It was proved in [31] that the functor OS_τ is a normal functor in the category of Tychonoff spaces and their continuous mappings.

Proposition 1. *For any topological T_1-space X the following inequalities*

$$c(X) \leq wd(X) \leq d(X)$$

hold.

Proof. a) Let's show the following inequality $wd(X) \leq d(X)$. Let $d(X) = \tau$, i.e. there exists in X its subset $M = \{\, a_\alpha : \alpha \in A,\ |A| = \tau \,\}$ such that $[M] = X$. Denote by σ_α a system of all open subsets of X, consisting of the point a_α, i.e. $\sigma_\alpha = \{\, U_s^\alpha : a_\alpha \in U_s^\alpha$ and U_s^α is open in X for each $\alpha\}$. Consider the system σ consisting of σ_α, i.e. $\sigma = \bigcup \{\sigma_\alpha : \alpha \in A\}$.

At first, show that the system σ is a π-base in X. In fact, let G be an arbitrary non-empty open set in X. Then, by virtue of everywhere density of M, there is a point $a_\alpha \in M$ such that $a_\alpha \in G$. By virtue of openness of G, the point a_α is an inner point of the set G. Consequently, there exists $U_s^\alpha \in \sigma_\alpha \in \sigma$ such that $U_s^\alpha \subset G$. It means that the system σ is a π-base of X.

Now show that σ_α is the centered system for each $\alpha \in A$. Take arbitrary elements $U_{s_1}^\alpha, ..., U_{s_k}^\alpha$ of the family σ_α. We have from definition of σ_α: $a_\alpha \in \bigcap \left\{\, U_{s_i}^\alpha : i = 1, ..., k \,\right\} \neq \emptyset$. That means that σ_α is centered for each $\alpha \in A$. Thus, we have proved that $wd(X) \leq \tau$.

b) Let's show that $c(X) \leq wd(X)$. Let $wd(X) = \tau \geq \aleph_0$, i.e. $B = \bigcup \{B_\alpha : \alpha \in A,\ |A| = \tau\}$ be a π-base in X, and any $B_\alpha = \left\{\, U_{s_k}^\alpha : s_k \in A_\alpha \,\right\}$ be a centered system of open sets for each $\alpha \in A$. Suppose the opposite, i.e. the Souslin number of the space X be equal to $c\,(X) = \tau' > \tau$. Then there exists in X a system of non-empty disjoint open sets $\gamma = \left\{\, G_\beta : \beta \in B,\ |B| = \tau' > \tau \,\right\}$, where $G_\beta \bigcap G_{\beta'} = \emptyset$ at $\beta \neq \beta'$. For any open set $G_\beta \in \gamma$, there is a set $U_s^\alpha \in B_\alpha$ such that $U_s^\alpha \subset G_\beta$, since the system B is a π-base in X. From the centering of B_α it follows that in different sets the sets G_α there can lie sets U_s^α from different B_α only, which contradicts the centering of the π-basis property of system B. Proposition 1 is proved. □

An example of a cardinal function φ such that $\varphi(X) \neq \varphi(\exp X)$ is the character of the space X. It is clear [4] that if $\exp X$ satisfies the first axiom of countability, then the space X is completely normal. Therefore, $\chi(X) < \chi(\exp X)$.

Let ϕ be a cardinal invariant. Denote by $h\phi$ the new cardinal invariant defined by the following formula $h\phi\,(X) = \sup\{\,\phi\,(Y) : Y \subset X\,\}$. Invariants $hc\,(X)$, $hd\,(X)$, $h\pi w\,(X)$, $hsh\,(X)$, $hpsh\,(X)$, $hk\,(X)$, $hpk\,(X)$, $hwd\,(X)$, $hl\,(X)$, $he\,(X)$ denote the hereditary Souslin number (or hereditary cellularity), the hereditary density, the hereditary π-weight, the hereditary Shanin number, the hereditary preshanin number, the hereditary caliber, the hereditary precaliber, the hereditary weak density, the hereditary Lindelöf number and the hereditary extent of the space X, respectively. The spread [7] $s\,(X)$ of the space X is the least infinite cardinal τ such that the cardinality of the discrete space X does not exceed τ, i.e. $s\,(X) = \sup\{\,\tau : \tau = |Y|\,,\, Y \subset X,\, Y \text{ is discrete}\}$. One can easily see that the hereditary Souslin number $hc\,(X)$ of the space X coincides with its spread $s\,(X)$.

"*One arrow*" *by P.S. Aleksandrov* [9]. Consider the half-interval $[\,0,\,1\,)$ of the number line. Introduce in $[\,0,\,1\,)$ the following topology: all half-intervals $[\alpha,\,\beta)\,,\ 0 \leq \alpha < 1,\ 0 < \beta \leq 1$, form, by definition, the base of this topology. Denote the obtained topological space by X^*.

"*Two arrows*" *by P.S. Aleksandrov* [9]. Consider two half-intervals $X = [\,0,\,1\,)\,,$ $X' = (\,0,\,1\,]$ situated one under another. Denote by X^{**} the set of all points of these two half-intervals. Define in X^{**} the topology as follows. All possible sets of the kind $U_1 = [\,\alpha,\beta) \cup \left(\alpha',\beta'\right)$, $U_2 = (\alpha,\beta) \cup \left(\alpha',\beta'\right]$ form the base of the topology, here $[\alpha,\beta)$ is a half-interval in X, and $\left(\alpha',\beta'\right)$ is the projection of (α,β) onto X'; $\left(\alpha',\beta'\right]$ is a half-interval in X', and (α,β) is the projection of $\left(\alpha',\beta'\right]$ into X. One can easy see that X^{**} is a compact.

Let X be a topological T_1-space. Denote by $\exp X$ the set of all nonempty closed subsets of the space X. The family B of all sets in the form of

$$O\langle U_1, ..., U_n\rangle = \left\{ F :\ F \in \exp X,\ F \subset \bigcup_{i=1}^{n} U_i,\ F \cap U_i \neq \emptyset,\ i = 1,\,2,\,...,\,n \right\},$$

where $U_1, ..., U_n$ is a sequence of open subsets of X, generates the topology on the set $\exp X$. This topology is called the Vietoris topology. The set $\exp X$ with the Vietoris topology is called the exponential space or the hyperspace of the space X [6].

Theorem 1 ([6])**.** *Let X be a separable space. Then every uncountable cardinal is a caliber of X.*

Theorem 2 ([17])**.** *Let X be a compact. Then $hd\,(X) = h\pi w\,(X) = hsh\,(X)$.*

Theorem 3. *There exists a normal functor $F : Comp \to Comp$ and a compact X such that*

1) $s\,(F\,(X)) \neq s\,(X)$;
2) $hd\,(F\,(X)) \neq hd\,(X)$;
3) $h\pi w\,(F\,(X)) \neq h\pi w\,(X)$;
4) $hsh\,(F\,(X)) \neq hsh\,(X)$;

5) $hc\,(F\,(X)) \neq hc\,(X)$*;*
6) $hk\,(F\,(X)) \neq hk\,(X)$*;*
7) $hpk\,(F\,(X)) \neq hpk\,(X)$*;*
8) $hpsh\,(F\,(X)) \neq hpsh\,(X)$*;*
9) $hwd\,(F\,(X)) \neq hwd\,(X)$*;*
10) $hl\,(F\,(X)) \neq hl\,(X)$*;*
11) $he\,(F\,(X)) \neq he\,(X)$*.*

Proof. 1) Consider the one arrow space $X^* = [\,0,\,1\,)$, the base of which is formed by the sets in the form of $[\alpha, \beta)$, where $0 \leq \alpha < 1, \quad \alpha < \beta \leq 1$. Consider in $\exp_2^0 X^*$ the following set:

$$Y = \left\{ F_t = \{\,t,\,1-t\,\} : 0 < t < \frac{1}{2} \right\}.$$

Let's show that Y is a discrete set of the cardinality continuum. Let $OF_t = \langle\, O_1^t,\; O_2^t\,\rangle$, where $O_1^t = \left[\,t,\,\frac{1}{2}\right)$, $O_2^t = [\,1-t,\,1)$. Show that $OF_t \cap Y = F_t$. In fact, let a set $F_t \in OF_t$. Since $t^{'} < \frac{1}{2}$, we have $t^{'} \in O_1^t$, hence, $t^{'} > t$. But $t^{'} \in O_1^t$ implies $1-t^{'} \in O_2^t$, that means $1-t^{'} > 1-t$, we get from here $-t^{'} > -t$ or $t^{'} < t$. The obtained contradiction proves that $OF_t \cap Y = F_t$, hence, Y is a discrete set of the cardinality continuum c. By definition of a spread, we have $s\,(\exp_2^0 X^*) = c$. It is known that the space X^* is topologically embedded into X^{**}, what implies $s\,(\exp_2^0 X^{**}) = c$, hence, $hd\,(\exp_2^0 X^{**}) = c$. We proved that a spread of the space $c = s\,(\exp\,(X^{**})) \neq s\,(X^{**}) = \aleph_0$. It means that the functor exp does not preserve the spread of the compact X^{**}. The inequality 1) is proved.

2) We proved in the first item that the space $\exp\,(X^{**})$ consists of a discrete space Y of the cardinality continuum c. It means that $\exp\,(X^{**})$ is not a hereditary separable space, i.e. $c = hd\,(\exp\,(X^{**})) \neq hd\,(X) = \aleph_0$. Thus, the functor exp does not preserve the hereditary density of a compact X^{**}. The ineqaulity 2) is proved.

3), 4), 5). Theorem 2 [17] implies that $hd\,(X) = h\pi w\,(X) = hsh\,(X)$ for any compact X, moreover, $hc\,(X) = s\,(X)$. So, we obtain:

$c = hd\,(\exp X^{**}) = h\pi w\,(\exp X^{**}) = hsh\,(\exp X^{**}) \neq hd\,(X^{**}) = h\pi w\,(X^{**}) = hsh\,(X^{**}) = \aleph_0$ and

$$c = s\,(\exp X^{**}) = hc\,(\exp X^{**}) \neq s\,(X^{**}) = hc\,(X^{**}) = \aleph_0.$$

We have proved inequalities 3), 4), 5).

6) It is clear that Aleksandrov's one arrow space X^* is hereditarily separable. Then, by virtue of Theorem 1 [6], any uncountable cardinal is a caliber of the space X^*. Thus, the hereditary caliber of the compact $hk\,(X^{**}) = c$ the cardinality of continuum. On the other hand, we have shown that the set Y is a discrete set of cardinality continuum. It means that the hereditary caliber of the space $hk\,(\exp\,(X^{**})) = c^+$ is the next cardinal c. Thus, the functor exp does not preserve the hereditary caliber of a compact X^{**}. The inequality 6) is proved.

7) It is known, the caliber of the compact X^{**} is equal to its precaliber, therefore $hk\,(X^{**}) = hpk\,(X^{**}) = c$ the cardinality of continuum, and $hk\,(\exp\,(X^{**})) = hpk\,(\exp\,(X^{**})) = c^+$ is the next cardinal c. We have shown that the functor exp

does not preserve the hereditary caliber and precaliber of the compact X^{**}, i.e. we have proved 7).

8) The inequality 8) follows from definitions of the Shanin, preshanin numbers of the space X and from 6), 7).

9) Aleksandrov's two arrows space is hereditarily separable, and then, by virtue of Proposition 1, the space X^{**} is hereditarily separable. We have shown that $\exp(X^{**})$ consists of a discrete subset Y of cardinality continuum. Hence, the space $\exp(X^{**})$ is not hereditarily weakly separable, i.e. $c = hwd(\exp(X^{**})) \neq hwd(X^{**}) = \aleph_0$. We have proved that the functor exp does not preserve the hereditary weak density of the compact X^{**}, i.e. 9) has been proved.

10) It is known that Aleksandrov's one arrow space X^{*} is hereditarily Lindelöf, i.e. any subset of it is finally compact. We have shown that the compact $\exp(X^{**})$ consists of a discrete subset Y of cardinality continuum. This set is not a finally compact subset of the compact $\exp(X^{**})$. It means that the space $\exp(X^{**})$ is not hereditarily Lindelöf. We have proved that the functor exp does not preserve the hereditary Lindelöf number of the compact X^{**}. Thereby we proved the inequality 10).

11) In 1), we showed that the spread Aleksandrov's two arrows space $s(X^{**}) = \aleph_0$ is countable. We obtain from $e(X) \leq s(X) \leq \min\{hl(X), hd(X)\}$ that the extent of Aleksandrov's two arrows space $e(X^{**}) = \aleph_0$ is also countable. It is clear, that the subset Y is discrete. Consider a subset $Y_1 \subset Y$ of cardinality continuum. It is known that any subset of a discrete subset is also discrete and closed. Hence, $c = he\exp(X^{**}) \neq he(X^{**}) = \aleph_0$. The inequality 11) is proved. Theorem 3 is proved. $\square$

In [12], T. Radul also showed that the functor of probabilistic measures P, the functor of super-extension λ, the functor exp are subfunctors of the functor $O : Comp \to Comp$. We obtain from Theorem 3 that the weakly normal functor $O : Comp \to Comp$ does not preserve in the category of all compacts and their continuous mappings the spread or the hereditary Souslin number (or the hereditary cellularity), the hereditary density, the hereditary weak density, the hereditary π-weight and the hereditary Shanin number, the hereditary caliber, the hereditary precaliber, the hereditary preshanin number, the hereditary Lindelöf number, the hereditary character, the hereditary π-character, the hereditary pseudocharacter, the hereditary tightness, and the hereditary extent of any compact.

We obtain from Theorem 3.

Corollary 1. *The functor $O : Comp \to Comp$ does not preserve ALeksandrov's two arrows space.*

It is known that for any compact X, its square X^2 is topologically embedded into the space of probabilistic measures $P(X)$.

In fact, the space X^2 can be embedded into $P(X)$ as follows: with a point $(x_1, x_2) \in X^2$ we associate the point $\frac{1}{3}\delta_{x_1} + \frac{2}{3}\delta_{x_2} \in P(X)$, where δ_{x_1}, δ_{x_2} are the Dirac functional in the points x_1, x_2, respectively. It is clear, this correspondence is a topological embedding of X^2 into the space of probabilistic measures $P(X)$.

We obtain from these arguments the following

Corollary 2. *The functor of probabilistic measures $P : Comp \to Comp$ does not preserve Aleksandrov's two arrows space.*

We need the following

Theorem 4 ([6]). *For a continuous functor F preserving a point and acting in the category of $Comp$, there exists the unique natural transformation $\eta : Id \to F$. In addition, the mapping $\eta_X : X \to F\,(X)$ is defined as the composition*

$$X \to F\,(1) \times X \overset{jF1X}{\to} F\,(1 \times X) \to F\,(X)\,.$$

A compact space X is called a dyadic space if X is a continuous image of the Cantor cube D^m for some $m \geq \aleph_0$ [7].

Theorem 5 ([7]). *Let X be a dyadic compact. Then*

$$nw\ (X) = w\ (X) = hd\ (X) = hl\ (X) = hc\ (X) = \chi\,(X) = \psi\,(X) = t\ \ (X)\,.$$

We obtain from Theorem 2 [17] the following

Corollary 3. *Let X be a dyadic compact. Then*

$$nw\ (X) = w\ (X) = hd\ (X) = hl\ (X) = hc\ (X) =$$

$$= \chi\,(X) = \psi\,(X) = t\ \ (X) = h\pi w\,(X) = hsh\,(X)\,.$$

Proposition 2. *For any dyadic compact X and arbitrary normal functor F : $Comp \to Comp$, the following equalities are valid:*
1) $\chi\,(X) = \chi\,(F\,(X))$;
2) $t\ (X) = t\ (F\ (X))$;
3) $hd\,(X) = hd\,(F\,(X))$;
4) $h\pi w\ (X) = h\pi w\,(F\,(X))$;
5) $hsh\ (X) = hsh\ (F\,(X))$;
6) $hc\,(X) = hc\,(F\,(X))$;
7) $s\,(X) = s\,(F\,(X))$.

Proof. 1) $\chi\,(X) \leq \chi\,(F\,(X)) \leq w\,(F\,(X)) = w\,(X) = t\ (X) \leq \chi\,(X)$. Hence, $\chi\,(X) = \chi\,(F\,(X))$.

2) $t\ (X) \leq t\ (F\,(X)) \leq w\,(F\,(X)) = w\,(X) = t\,(X)$. It follows from here that $t\ (X) = t\ (F\ (X))$.

3) $hd\,(X) \leq hd\,(F\,(X)) \leq w\,(F\,(X)) = w\,(X) = t\ (X) \leq hc\,(X) \leq hd\,(X)$. That implies $hd\,(X) = hd\,(F\,(X))$.

Relations 4), 5) follow immediately from the equality $hd\,(X) = h\pi w\,(X) = hsh\,(X)$, valid for any compact X.

6) $hc\,(X) \leq hc\,(F\,(X)) \leq w\,(F\,(X)) = w\,(X) = t\ (X) \leq hc\,(X)$. We have from here that $hc\,(X) = hc\,(F\,(X))$. Relation 7) follows from the equality $hc\,(X) = s\,(X)$. Proposition 2 is proved. □

We obtain from Corollary 3 and Proposition 2 the following

Corollary 4. *Let X be a dyadic compact, and $F : Comp \to Comp$ be a normal functor. Then $nw\,(X) = w\,(X) = hd\,(X) = h\pi w\,(X) = hsh\,(X) = hc\,(X) = \chi\,(X) = \psi\,(X) = t\,\,(X) = s\,(X) = nw\,(F\,(X)) = w\,(F\,(X)) = hd\,(F\,(X)) = h\pi w\,(F\,(X)) = hsh\,(F\,(X)) = hc\,(F\,(X)) = \chi\,(F\,(X)) = \psi\,(F\,(X)) = t\,\,(F\,(X)) = s\,(F\,(X))$.*

Proposition 3. *For any infinite compact X and arbitrary normal functor $F : Comp \to Comp$, the following inequalities hold:*
1) $hd\,(F\,(X)) \leq 2^{hd(X)}$;
2) $h\pi w\,(F\,(X)) \leq 2^{h\pi w(X)}$;
3) $hsh\,(F\,(X)) \leq 2^{hsh(X)}$.

Proof. B.E. Shapirowskii showed in [17] that if X is a regular space of a point-countable type, then $w\,(X) \leq 2^{\,c(X)\,t(X)}$. Since $t\,\,(X) \leq hd\,(X)$ and $c\,(X) \leq hd\,(X)$, we have $hd\,(F\,(X)) \leq w\,(F\,(X)) = w\,(X) \leq 2^{\,c(X)\,t(X)} \leq 2^{hd(X)}$. Relations 2), 3) follow from the equality $hd\,(X) = h\pi w\,(X) = hsh\,(X)$, valid for any compact X. Proposition 3 is proved. □

Proposition 4. *Let X be an infinite compact such that $C_p\,(X)$ is a Lindelöf $\sum$-space, and $F : Comp \to Comp$ is a normal functor. Then:*
1) $hc\,(F\,(X)) = hc\,(X)$;
2) $hd\,(F\,(X)) = hd\,(X)$;
3) $h\pi w\,(F\,(X)) = h\pi w\,(X)$;
4) $hsh\,(F\,(X)) = hsh\,(X)$;
5) $s\,(F\,(X)) = s\,(X)$.

Proof. According to the Argiros-Negrepontis Theorem [18], if X is a compact and $C_p\,(X)$ is a Lindelöf $\sum$-space, then $c\,(X) = w\,(X)$.

1) $hc\,(X) \leq hc\,(F\,(X))$ because X is a subspace of $F\,(X)$ by virtue of Theorem 4 [6], and, obviously, $hc\,(F\,(X)) \leq w\,(F\,(X)) = w\,(X) = c\,(X) \leq hc\,(F\,(X))$. Thus, we get $hc\,(X) = hc\,(F\,(X))$.

2) Obviously, the following inequalities hold: $hd\,(X) \leq w\,(X)$ and $c\,(X) \leq hd\,(X)$. It is known that $hd\,(X) \leq hd\,(F\,(X))$ because X is a subspace of $F\,(X)$ by virtue of Theorem 4 [6]. Let's show that the inverse inequality $hd\,(X) \geq hd\,(F\,(X))$ is also correct. Indeed, $hd\,(F\,(X)) \leq w\,(F\,(X)) = w\,(X) = c\,(X) \leq hd\,(X)$. Therefore $hd\,(F\,(X)) = hd\,(X)$.

Relations 3), 4) follow directly from the equality $hd\,(X) = h\pi w\,(X) = hsh\,(X)$, valid for any compact X. Relation 5) follows from the equality $hc\,(X) = s\,(X)$. Proposition 4 is proved. □

A compact E is called Eberlein if there is a compact X such that E is homeomorphic to the subspace $C_p\,(X)$ [18].

Since the class of Eberlein compacts is contained in the class of compacts for which $C_p\,(X)$ is a Lindelöf $\sum$-space, we obtain from Proposition 4 the following

Corollary 5. *For any Eberlein compact X and a normal functor $F : Comp \to Comp$, the following equalities take place:*
1) $hc\left(F\left(X\right)\right) = hc\left(X\right)$;
2) $hd\left(F\left(X\right)\right) = hd\left(X\right)$;
3) $h\pi w\left(F\left(X\right)\right) = h\pi w\left(X\right)$;
4) $hsh\left(F\left(X\right)\right) = hsh\left(X\right)$;
5) $s\left(F\left(X\right)\right) = s\left(X\right)$.

Let A_τ be a one-point compactification (in the sense by P.S. ALeksandrov) of a discrete space of the cardinality $\tau \geq \aleph_0$. Since for each τ, A_τ is an Eberlein compact, and $w\left(A_\tau\right) = \tau$ [6], we obtain

Corollary 6. *Let A_τ be a one-point compactification (in the sense by P.S. Aleksandrov) of a discrete space of the cardinality $\tau \geq \aleph_0$, and $F : Comp \to Comp$ be a normal functor. Then:*
1) $hc\left(A_\tau\right) = hc\left(F\left(A_\tau\right)\right)$;
2) $hd\left(A_\tau\right) = hd\left(F\left(A_\tau\right)\right)$;
3) $h\pi w\left(A_\tau\right) = h\pi w\left(F\left(A_\tau\right)\right)$;
4) $hsh\left(A_\tau\right) = hsh\left(F\left(A_\tau\right)\right)$;
5) $s\left(A_\tau\right) = s\left(F\left(A_\tau\right)\right)$.

Corollary 7. *Let X be arbitrary pseudocompact subset of a Banach space Y in a weak topology, and $F : Comp \to Comp$ be a normal functor. Then:*
1) $hc\left(F\left(X\right)\right) = hc\left(X\right)$;
2) $hd\left(F\left(X\right)\right) = hd\left(X\right)$;
3) $h\pi w\left(F\left(X\right)\right) = h\pi w\left(X\right)$;
4) $hsh\left(F\left(X\right)\right) = hsh\left(X\right)$;
5) $s\left(F\left(X\right)\right) = s\left(X\right)$.

Proof of Corollary 7 is based on the fact that a pseudocompact subspace of a Banach space in a weak topology is an Eberlein compact [18].

The Corson compacts [18] are the compact subsets of the $\sum$-product of separable metrizable spaces (or, in other words, compact subsets of the $\sum$-product of segments).

Proposition 5. *Let X be an infinite Corson compact such that $C_p\left(C_p\left(X\right)\right)$ is a Lindelöf $\sum$-space, and $F : Comp \to Comp$ be a normal functor. Then:*
1) $hc\left(X\right) = hc\left(F\left(X\right)\right)$;
2) $hd\left(F\left(X\right)\right) = hd\left(X\right)$;
3) $h\pi w\left(F\left(X\right)\right) = h\pi w\left(X\right)$;
4) $hsh\left(F\left(X\right)\right) = hsh\left(X\right)$;
5) $s\left(F\left(X\right)\right) = s\left(X\right)$.

Proof of this assertion is based on the fact that if X is a Corson compact, for which $C_p\left(C_p\left(X\right)\right)$ is a Lindelöf $\sum$-space, then $C_p\left(X\right)$ is a Lindelöf $\sum$-space, and, hence, $c\left(X\right) = w\left(X\right)$ [18].

Let τ be an infinite cardinal number, X be a topological space, and X' be its subspace.

We call the subspace $X^{'}$ τ-monolite [20] in X, if for any $A \subset X^{'}$with $|A| \leq \tau$, $[A]_X$ is a compact of the weight $\leq \tau$.

We say that X τ-suppresses $X^{'}$[20] if $\lambda \geq \tau$ and $A \subset X^{'}$, $|A| \leq 2^{\lambda}$, imply that there is $A^{'} \subset X$ such that $\left[A^{'}\right] \supset A$ and $\left|A^{'}\right| \leq \lambda$.

A topological space X is called Dante space [20] if for any infinite cardinal number τ, there exists a subspace $X^{'}$ every where dense in X which is:

1) τ-monolite in itself;

2) τ-suppressed by X.

Proposition 6. *For any infinite Dante space X and arbitrary normal functor F : $Comp \to Comp$, the following equalities hold:*

1) $\chi(X) = \chi(F(X))$;

2) $t\ (X) = t\ (F\ (X))$;

3) $hd(X) = hd(F(X))$;

4) $h\pi w\ (X) = h\pi w(F(X))$;

5) $hsh\ (X) = hsh\ (F(X))$;

6) $hc(X) = hc(F(X))$;

7) $s(F(X)) = s(X)$.

Proof. Let X be a Dante space. Then it is compact, and $w(X) = t(X)$ [17]. By virtue of compactness of X, we have $t\ (X) \leq hc(X)$, and $w(X) = w(F(X))$. So,

1) $\chi(X) \leq \chi(F(X)) \leq w(F(X)) = w(X) = t\ (X) \leq \chi(X)$. Hence $\chi(X) = \chi(F(X))$.

2) $t\ (X) \leq t\ (F\ (X)) \leq w(F(X)) = w(X) = t(X)$. We have $t\ (X) = t\ (F\ (X))$.

3) $hd(X) \leq hd(F(X)) \leq w(F(X)) = w(X) = t\ (X) \leq hc(X) \leq hd(X)$. It follows from here $hd(X) = hd(F(X))$.

Relations 4), 5) follow immediately from the equality $hd(X) = h\pi w(X) = hsh(X)$ for any compact X.

6) $hc(X) \leq hc(F(X)) \leq w(F(X)) = w(X) = t\ (X) \leq hc(X)$. It follows that $hc(X) = hc(F(X))$. Relation 7) follows from the equality $hc(X) = s(X)$. Proposition 6 is proved. □

2 Main results

In this section, we study some topological properties of hyperspaces. Let X be a topological T_1-space. Denote by $\exp_n X$ the set of all non-empty closed subsets of X of cardinality not greater than the cardinal number n, i.e. $\exp_n X = \{F \in \exp X : |F| \leq n\}$. Put $\exp_\omega X = \bigcup\{\exp_n X : n = 1, 2, ...\}$, $\exp_c X = \{F \in \exp X : F \text{ is compact in } X\}$. It is clear, that $\exp_n X \subset \exp_\omega X \subset \exp_c X \subset \exp X$ for any topological space X.

Theorem 6 ([19]). *If X is an infinite T_1 space and $U_1, U_2, ..., U_n$ are arbitrary non-empty open sets in X, then the following equality holds*

$$[O\langle U_1,\ U_2,\ \dots,U_n\rangle] = O\langle [U_1],\ [U_2],\ \dots, [U_n]\rangle$$

A topological space X is called finally compact if X is regular and every open cover of X has a countable subcover [7].

A topological space X is called a Lindelöf space, or a space with the Lindelöf property, if X can regularly choose a countable subcover from every open cover of this space [7].

It is known that every closed subspace of a Lindelöf space is a Lindelöf space.

Theorem 7. *The space X is finally compact if and only if the space $\exp_c X$ is finally compact.*

Proof. Necessity. Let X be a finally compact space and $\mu = \{O\langle U_1^\alpha, U_2^\alpha, ..., U_n^\alpha\rangle : \alpha \in A\}$ an arbitrary open cover of the space $\exp_c X$. Consider the trace cover of μ, that is, $\mu_1 = \{U_1^\alpha,\ U_2^\alpha,\ \dots,\ U_n^\alpha,\ \alpha \in A\}$ of X. We show that the system $\{U_i^\alpha :\ \alpha \in A,\ i = 1, 2, ..., n\}$ is a cover of the space X.

We prove by contradiction, i.e. Suppose that there exists an element $x_1 \in X \setminus \bigcup \{U_i^\alpha :\ \alpha \in A,\ i = 1, 2, ..., n\}$. Then $\{x_1\} \in \exp_c X \setminus \bigcup \{O\langle U_1^\alpha, U_2^\alpha, ..., U_n^\alpha\rangle : \alpha \in A\}$. It follows that $\{x_1\} \in \exp_c X$, but $\{x_1\} \notin \bigcup \{O\langle U_1^\alpha, U_2^\alpha, ..., U_n^\alpha\rangle : \alpha \in A\}$. We get a contradiction. Therefore, the system $\{U_i^\alpha :\ \alpha \in A,\ i = 1, 2, ..., n\}$ is a cover of the space X.

Since X is finally compact space. Then there exists a countable cover $\mu_2 = \{U_i^s :\ \ i = 1, 2, ..., n;\ \ s \in S \subset A\,,\ |S| \leq \aleph_0\}$, such that μ_2 covers the space X. Consider all possible finite open subsets of the family μ_2, where $O\langle U_1^s,\ U_2^s,\ \dots, U_k^s\rangle$, $s \in S$, $|S| \leq \aleph_0$. Let us show that the system $\{O\langle U_1^s,\ U_2^s,\ \dots, U_k^s\rangle :\ s \in S\}$ is a cover of the space $\exp_c X$.

Suppose that there exists a compact set Φ such that

$$\Phi \in \exp_c X \setminus \bigcup \{O\langle U_1^s,\ U_2^s,\ \dots, U_k^s\rangle :\ s \in S,\ n \in N\}\,.$$

Since the system μ_2 is a cover of the space X, there are sets $U_1^s, U_2^s, \dots, U_k^s$ such that $\Phi \in O\langle U_1^s,\ U_2^s,\ \dots, U_k^s\rangle$ We get a contradiction. Therefore, $\mu_3 = \{O\langle U_1^s,\ U_2^s,\ \dots, U_n^s\rangle :\ s \in S,\ n \in N\}\,,\ |S| \leq \aleph_0$ is a cover of the space $\exp_c X$ and the space $\exp_c X$ is finally compact.

Sufficiency. Let $\exp_c X$ be finally compact. It is known that the final compactness is inherited in the transition to a closed subspace. Since the space X is closed in $\exp X$, the space X is closed in $\exp_c X$. This implies that the space X is also finally compact. Theorem 7 is proved. □

Corollary 8. *A topological T_1-space X is finally compact if and only if the $\exp_n X$ is finally compact.*

A topological space$X$ is called hereditarily disconnected if X does not contain any connected subsets of cardinality larger than one. Hence, the space X is hereditarily disconnected if and only if the component of any point $x \in X$ consists of the point x alone. It is known that the hereditary incoherence is inherited by every subset, any finite or countable product, and any sum [7]. Hence we get the following theorem.

Theorem 8. *If X is an infinite T_1-space. Then the space X is hereditarily disconnected if and only if the space $\exp_n X$ is hereditarily disconnected.*

Corollary 9. *If X is an infinite T_1-space, then the space X is hereditarily disconnected if and only if the $\exp_\omega X$ space is hereditarily disconnected.*

A subset A of a topological space X is called functionally closed if $A = f^{-1}(0)$ for some continuous function $f : X \to I$. Obviously, every functionally closed subset of X is closed in X. Let $f,\, g : X \to I$; since for $h_1,\, h_2 : X \to I$ defined by the formulas

$$h_1(x) = f(x) \cdot g(x) \quad \text{and} \quad h_2(x) = \frac{1}{2}(f(x) + g(x)).$$

We have [7]

$$h^{-1}(0) = f^{-1}(0) \cup g^{-1}(0) \quad \text{and} \quad h^{-1}(0) = f^{-1}(0) \cap g^{-1}(0).$$

It follows that the union and intersection of two functionally closed sets are functionally closed. A countable intersection of functionally closed sets is a functionally closed set as well.

The complement of a functionally closed subset of X is called functionally open. Obviously, every functionally open subset of X is open in X. Countable unions and finite intersections of functionally open sets are functionally open [7].

A topological space X is called strongly zero-dimensional if X is a nonempty Tychonoff space and every finite functionally open cover $\{U_i :\ i = 1, 2, ..., n\}$ of the space X has a finite open refinement $\{V_i :\ i = 1, 2, ..., k\}$ such that $V_i \cap V_j = \emptyset$ whenever $i \neq j$ [7].

Clearly, that refinement $\{V_i :\ i = 1, 2, ..., k\}$ consists of open-and-closed sets and thus is a functionally open cover of X.

Theorem 9 ([7])**.** *A nonempty normal space X is strongly zero-dimensional if and only if every finite open cover $\{U_i :\ i = 1, 2, ..., n\}$ of the space X has a finite open refinement $\{V_i :\ i = 1, 2, ..., k\}$ such that $V_i \cap V_j = \emptyset$ whenever $i \neq j$.*

Theorem 10. *Let X be a strongly zero-dimensional space such that $\exp X$ is a normal space. Then $\exp X$ is also a strongly zero-dimensional space.*

Proof. Let $\mu = \{O\langle U_1^i, U_2^i, ..., U_n^i\rangle :\ i = 1, 2, ..., k\}$ be an open cover of the space $\exp X$. Consider the trace of this cover in X, i.e. $\mu_1 = \{U_1^i, U_2^i, ..., U_n^i :\ i = 1, 2, ..., k\}$ and $\bigcup_{i=1}^{k} \left\{U_j^i :\ j = 1, 2, ..., n\right\}$ are covered by X. It is clear that μ_1 is a cover of X. Let $U_1^1, U_2^1, ..., U_{n_1}^1,\ U_1^2, U_2^2, ..., U_{n_2}^2, ...,\ U_1^k, U_2^k, ..., U_{n_s}^k$ be the trace of μ in X. Since X is strongly zero-dimensional, there is a disjoint open cover μ_2 that is inscribed in μ_1; those $V_{11}^1, V_{12}^1, ..., V_{1l_1}^1$ is inscribed in U_1^1; $V_{21}^1, V_{22}^1, ..., V_{2l_2}^1$ is inscribed in U_2^1; ...;$V_{n_11}^1, V_{n_12}^1, ..., V_{n_1l_m}^1$ is inscribed in $U_{n_1}^1$; ...;$V_{11}^k, V_{12}^k, ..., V_{1l_1}^k$ is inscribed in the U_1^k; $V_{21}^k, V_{22}^k, ..., V_{2l_2}^k$is inscribed in U_2^k; ...; $V_{n_s1}^k, V_{n_s2}^k, ..., V_{n_sl_m}^k$is inscribed in $U_{n_s}^k$.

Consider all possible combinations of the sets V_{jl}^i, where, $i = 1, 2, ..., k$, $j = 1, 2, ..., n_s$, $l = 1, 2, ..., m$ i.e. $\mu_3 =$

$\left\{O\left\langle V^i_{jl_1},\,V^i_{jl_2},\,...,\,V^i_{jl_m}\right\rangle:\ i=1,2,...,k,\ j=1,2,...,n_s,\ l=1,2,...,m\right\}$ is an open cover of the space $\exp X$ and is inscribed in μ. $\mu_2=\left\{V^j_1,\,V^j_2,\,...,\,V^j_n:\ j=1,2,...,s\right\}$ is a finite open cover such that it can be inscribed in μ_1 and $V^j_{l_1}\cap V^j_{l_2}=\emptyset$ for $l_1\neq l_2$. For an open set $O\langle U^i_1,\,U^i_2,\,...,\,U^i_n\rangle$, $i=1,2,...,k$ there exists $O\left\langle V^j_1,\,V^j_2,\,...,\,V^j_n\right\rangle$, $j=1,2,...,s$ that is inscribed in μ. We show that $O\left\langle V^{j_1}_1,\,V^{j_1}_2,\,...,\,V^{j_1}_{l_1}\right\rangle\cap O\left\langle V^{j_2}_1,\,V^{j_2}_2,\,...,\,V^{j_2}_{l_2}\right\rangle=\emptyset$ for $j_1\neq j_2$. Assume the opposite, i.e. let there exist an element $F\in O\left\langle V^{j_1}_1,\,V^{j_1}_2,\,...,\,V^{j_1}_{l_1}\right\rangle\cap O\left\langle V^{j_2}_1,\,V^{j_2}_2,\,...,\,V^{j_2}_{l_2}\right\rangle$. It follows that $F\in O\left\langle V^{j_1}_1,\,V^{j_1}_2,\,...,\,V^{j_1}_{l_1}\right\rangle$ and $F\in O\left\langle V^{j_2}_1,\,V^{j_2}_2,\,...,\,V^{j_2}_{l_2}\right\rangle$, then $F\subset\bigcup_{i=1}^{l_1}V^{j_1}_i$, $F\cap V^{j_1}_i\neq\emptyset$, $i=1,2,...,l_1$ and $F\subset\bigcup_{i=1}^{l_2}V^{j_2}_i$, $F\cap V^{j_2}_i\neq\emptyset$, $i=1,2,...,l_2$. Let x be an arbitrary point of the set F. Then from $F\subset\bigcup_{i=1}^{l_1}V^{j_1}_i$ and $F\subset\bigcup_{i=1}^{l_2}V^{j_2}_i$ we have that there exist sets $V^{j_1}_i$ and $V^{j_2}_i$ such that $x\in V^{j_1}_i$, $x\in V^{j_2}_i$. Hence $V^{j_1}_i\cap V^{j_2}_i\neq\emptyset$. Got a contradiction. Therefore, $\exp X$ is strongly zero-dimensional. Theorem 10 is proved. □

A topological space X is called extremally disconnected if X is a Hausdorff space and for every open set $U\subset X$ the closure $[U]$ is open in X [7].

Theorem 11 ([7])**.** *A Hausdorff space X is extremally disconnected if and only if for every pair of U, V disjoint open subsets in X we have $[U]\cap[V]=\emptyset$.*

Theorem 12. *Let X be extremally disconnected, if and only if the space $\exp X$ is extremally disconnected.*

Proof. Necessity. Let X be an extremally disconnected space, then for each open set $U\subset X$ the closure of $[U]$ is open in X. We show that $\exp X$ is an extremally disconnected space. Let $O\langle U_1,\ U_2,\ \dots,U_n\rangle$ be an arbitrary open subset in $\exp X$, show that the set $[O\langle U_1,\ U_2,\ \dots,U_n\rangle]=O\langle[U_1],[U_2],\dots,[U_n]\rangle$ is open in $\exp X$. Choose an arbitrary element $F\in O\langle[U_1],[U_2],\dots,[U_n]\rangle$. Show that F is an interior point of $O\langle[U_1],[U_2],\dots,[U_n]\rangle$. It follows from $F\in O\langle[U_1],[U_2],\dots,[U_n]\rangle$ that $F\subset\bigcup_{i=1}^{n}[U_i]$ and $F\cap[U_i]\neq\emptyset$ are for every $i=1,2,...,n$. Since X is extremely disconnected, then for every element $x\in F\cap[U_i]$ there is a neighborhood $Ox\subset[U_i]$ for every $i=1,2,...,n$. Denote by $G_i=\cup\{Ox:\ x\in F\cap[U_i]\ ,\ Ox\subset[U_i]\}$ for each $i=1,2,...,n$. We have that $F\subset\bigcup_{i=1}^{n}G_i$ and $F\cap G_i\neq\emptyset$,$i=1,2,...,n$. Hence, $F\in O\langle G_1,\,G_2,\,...,\,G_n\rangle$ and $O\langle G_1,\,G_2,\,...,\,G_n\rangle$ is a neighborhood of the point F.

We show that $O\langle G_1,\,G_2,\,...,\,G_n\rangle\subset O\langle[U_1],[U_2],\dots,[U_n]\rangle$. Choose an arbitrary element $E\in O\langle G_1,\,G_2,\,...,\,G_n\rangle$, since $E\subset\bigcup_{i=1}^{n}G_i$ and $E\cap G_i\neq\emptyset$, $i=1,2,...,n$, then from, $G_i\subset[U_i]$, $i=1,2,...,n$, we have $E\subset\bigcup_{i=1}^{n}[U_i]$ and $E\cap[U_i]\neq\emptyset$, $i=1,2,...,n$. Therefore, $E\in O\langle[U_1],[U_2],\dots,[U_n]\rangle$ and every point $F\in O\langle[U_1],[U_2],\dots,[U_n]\rangle$ is internal. Hence, $\exp X$ is an extremally disconnected space.

Sufficiency. Let $\exp X$ be an extremally disconnected space, and let U be any non-empty open subset of X. Let us show that $[U]$ is open in X. It is clear that $O\langle U\rangle$ is an open set in $\exp X$. Since $\exp X$ is extremely disconnected, $[O\langle U\rangle]=O\langle[U]\rangle$ is open in $\exp X$. We show that U is open in X. Let x be an arbitrary point in $[U]$. Then $\{x\}\in O\langle[U]\rangle$. From the openness of the set $O\langle[U]\rangle$ we have a neighborhood

$O\langle G\rangle$ containing a point x, that $O\langle G\rangle \subset O\langle [U]\rangle$, hence we have $G \subset [U]$. Therefore, $[U]$ is an open subset in X and X is an extremely disconnected space. Theorem 12 is proved. □

We say that a cover $B = \{B_t : t \in T\}$ is a refinement of another cover $A = \{A_s : s \in S\}$ of the same set X, if for every $t \in T$ there exists an $s(t) \in S$ such that $B_t \subset A_{s(t)}$. In this situation we say also that B refines A [7].

A family $\{A_s : s \in S\}$ of subsets of a topological space X is locally finite if for every point $x \in X$ there exists a neighborhood U such that the set $\{s \in S : U \cap A_s \neq \emptyset\}$ is finite [7].

A topological space X is called a paracompact space if X is a Hausdorff space and every open cover of X has a locally finite open refinement [7].

Theorem 13 ([7]). *Every paracompact space is normal.*

Lemma 1. *Let a family of open sets* $\mu = \{V^\alpha : \alpha \in A\}$ *is a refinement of an open set* G *in the topological* T_1 *space* X. *Then the family* $\mu_1 = \{O\langle V_1^\alpha, V_2^\alpha, ..., V_n^\alpha\rangle : \alpha \in A\}$ *is a refinement of an open set* $O\langle G\rangle$ *in the hyperspace* $\exp X$, *where* $\{V_i^\alpha \in \mu : \alpha \in A,\ i = 1, 2, ..., n\}$.

Proof. Let a family of open sets $\mu = \{V^\alpha : \alpha \in A\}$ a refinement of an open set G in the topological T_1-space X. We show that $\bigcup\{O\langle V_1^\alpha, V_2^\alpha, ..., V_n^\alpha\rangle : \alpha \in A\} = O\langle G\rangle$. Assume the contrary, that there exists a point $F \in O\langle G\rangle \setminus \bigcup\{O\langle V_1^\alpha, V_2^\alpha, ..., V_n^\alpha\rangle : \alpha \in A\}$, then the set $F \subset G$ and $F \not\subset \bigcup\{V_i^\alpha : \alpha \in A,\ i = 1, 2, ..., n\}$. There exists a point $x \in F$ such that $x \in F \setminus \bigcup\{V_i^\alpha : \alpha \in A,\ i = 1, 2, ..., n\}$, then $x \notin \bigcup\{V_i^\alpha : \alpha \in A,\ i = 1, 2, ..., n\}$, hence we have that the family μ is not a refinement of the open set G. Got a contradiction. Lemma 1 is proved. □

Lemma 2. *Let the family of open sets* $\mu = \{V_i^\alpha : \alpha \in A_i,\ i = 1, 2, ..., n\}$ *is a refinement of the family* $\{G_1, G_2, ..., G_n\}$ *in open subsets of a topological space* X. *Then the family* $\mu_1 = \{O\langle V_1^{\alpha_1}, V_2^{\alpha_2}, ..., V_n^{\alpha_n}\rangle : \alpha_i \in A_i,\ i = 1, 2, ..., n\}$ *is a refinement of the family* $\{O\langle G_1, G_2, ..., G_n\rangle\}$ *in the hyperspace* $\exp X$.

The proof consists of repeating the arguments in the proof of Lemma 1.

Theorem 14. *A locally compact space* X *is paracompact if and only if* $\exp_c X$ *is paracompact.*

Proof. Necessity. Let X be a locally compact paracompact space and consider its arbitrary open cover $\mu = \{O\langle U_1^\alpha, U_2^\alpha, ..., U_n^\alpha\rangle : \alpha \in A\}$ of $\exp_c X$. Consider the trace of the family μ in the space X, i.e. $\mu_1 = \{U_i^\alpha : \alpha \in A,\ i = 1, 2, ..., n\}$. It is clear that μ_1 is an open cover of X. Since X is paracompact, there exists a locally finite open cover of $\nu_1 = \{V^\beta : \beta \in B\}$, which is a refinement of μ_1, by Lemma 2.

Consider all possible final combinations of elements of the cover ν and put $\nu_1 = \left\{O\left\langle V_1^\beta, V_2^\beta, ..., V_k^\beta\right\rangle : \beta \in B\right\}$. It is clear that ν_1 is a cover of the space $\exp_c X$ and ν_1 is refinement of the cover μ_1 by virtue of Lemma 2. Let us show that the system

ν_1 is locally finite. Let $F \in \exp_c X$ be an arbitrary element, then F is compact and $F \subset X$. Since the cover of $\nu = \left\{V^\beta : \beta \in B\right\}$ is locally finite, every point $x \in F$ has a neighborhood $O(x)$ such that $\left\{\beta \in B : O(x) \cap V^\beta \neq \emptyset\right\}$ is finite. Let the point x run along the compact set F. Since F is compact, there exists $O(x_1), O(x_2), ..., O(x_k)$, that $F \subset \bigcup\limits_{i=1}^{k} O(x_i)$ and $\left\{\beta \in B : O(x_i) \cap V^\beta \neq \emptyset\right\}$ are finite for every $i = 1, 2, ..., k$. Then the set $O\langle O(x_1), O(x_2), ..., O(x_k)\rangle$ is a neighborhood of the compact set $F \in \exp_c X$ and $\left\{\beta \in B : O\langle O(x_1), O(x_2), ..., O(x_k)\rangle \cap O\left\langle V_1^\beta, V_2^\beta, ..., V_k^\beta\right\rangle \neq \emptyset\right\}$ is finite.

Sufficiency. Let $\exp_c X$ be paracompact. Let $\mu = \{U^\alpha : \alpha \in A\}$ be an arbitrary open cover of the space X. Consider all possible finite combinations of the cover μ and put $\mu_1 = \{O\langle U_1^\alpha, U_2^\alpha, ..., U_n^\alpha\rangle : \alpha \in A, U_i^\alpha \in \mu, i = 1, 2, ..., n\}$. It is clear that μ is an open cover of the space $\exp_c X$. Since $\exp_c X$ is paracompact, there exists a locally finite open cover $\nu = \left\{O\left\langle V_1^\beta, V_2^\beta, ..., V_s^\beta\right\rangle : \beta \in B\right\}$ which is refinement of the cover μ_1.

Consider the trace ν_1 of a family in the space X. Let us show that the trace $\nu_1 = \left\{ V_i^\beta : \beta \in B, i = 1, 2, ..., n \right\}$ is a locally finite open cover of X. Let x be an arbitrary point of X, then $\{x\} \in \exp_c X$. Since $\exp_c X$ is paracompact and ν is a locally finite open cover of X, then there exists a neighborhood $O\langle G\rangle$ of $\{x\}$, that $\left\{\beta \in B : O\langle G\rangle \cap O\left\langle V_1^\beta, V_2^\beta, ..., V_s^\beta\right\rangle \neq \emptyset\right\}$ is finite. This means that $x \in G$ and $G \cap V_i^\beta \neq \emptyset$, $i = 1, 2, ..., s$ and for finite $\beta \in B$. Hence we have that the system ν_1 is locally finite. Theorem 14 is proved. □

Corollary 10. *A locally compact space X is paracompact if and only if the space $\exp_n X$ is paracompact.*

Remark 1. *In Theorem 14, the condition of locally compactness is essential. Let X^* be Aleksandrov's one arrow space. It was proved in [7] that X^* is a hereditarily paracompact space. By virtue of Theorem 13 [7], every paracompact Hausdorff space is normal. If $X^* \times X^*$ is normal. We get a contradiction.*

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