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LOCAL MULTIPLICATION OPERATORS ON ALGEBRAS OF MATRICES

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MATRITSALAR ALGEBRASIDA LOKAL KO‘PAYTIRISHLAR

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Maqolada ixtiyoriy maydonda aniqlangan matritsalar algebrasida har qanday additiv lokal chapdan ko‘paytirish chapdan ko‘paytirish bo‘lishi isbotlangan. Ratsional sonlar maydonida aniqlangan simmetrik matritsalar Jordan algebrasidagi ixtiyoriy additiv lokal Jordon ko‘paytirishi Jordon ko‘paytirishi bo‘lishi hamda haqiqiy yoki kompleks sonlar maydonida aniqlangan simmetrik matritsalar Jordan algebrasidagi ixtiyoriy uzluksiz additiv lokal Jordon ko‘paytirishi Jordon ko‘paytirishi bo‘lishi isbotlangan.

Kalit so‘zlar: *matritsalar algebrasi, matritsalar Jordan algebrasi, chapdan assosiativ ko‘paytirish, Jordon ko‘paytirish, lokal akslantirish.*

В данной научной работе доказано, что всякое аддитивное локальное левое умножение на алгебре матриц над произвольным полем является левым умножением. Доказано, что всякое аддитивное локальное Йорданово умножение Йордановой алгебры симметрических матриц над полем рациональных чисел является Йордановым умножением. А также, доказано, что всякое непрерывное аддитивное локальное Йорданово умножение Йордановой алгебры симметрических матриц над полем комплексных или вещественных чисел является Йордановым умножением.

Ключевые слова: *алгебра матриц, Йорданова алгебра матриц, левое ассоциативное умножение, Йорданово умножение, локальное отображение.*

Kirish. Glison-Koxon-Zelazko teoremasi [1], [2] Banax algebralari nazariyasining muhim natijalaridan biri bo‘lib, quyidagi ko‘rinishga ega: birlik elementli Akompleks Banax algebrasining har bir $a \in A$ elementi uchun $F(a)$ element a ning $\sigma(a)$ spektriga tegishli bo‘lishini qanoatlantiruvchi A Banax algebrasidagi har bir F birlik $(F(1) = 1)$ chiziqli funksional multiplikativdir. Zamonaviy terminologiyada bu tasdiq quyidagiga ekvivalent: har bir birlik elementli Akompleks Banax algebrasini $\mathbb{C}$kompleks sonlar maydoniga akslantiruvchi birlik chiziqli lokal gomomorfizm multiplikativdir. Bu yerda A Banax algebrasini B Banax algebrasiga akslantiruvchi T chiziqli akslantirish lokal gomomorfizm bo‘ladi deymiz, agarda har bir $a \in A$ element uchun $T(a) = \Phi_a(a)$ shartni qanoatlantiruvchi a elementga bog‘liq bo‘lgan $\Phi_a: A \to B$ gomomorfizm topilsa. Lokal differensiallashlar ham shunga o‘xshash aniqlanadi. Kodison [3] va Larson, Shurier [4] mos ravishda Banax algebralarida lokal differensiallashlar va lokal avtomorfizmlar nazariyasida dastlabki natijalarni olishgan.

Jonson [5]C^*-algebrani Banax A-bimodulga akslantiruvchi har bir lokal differensiallash diffrensiallashdan iborat bo‘lishini ko‘rsatib, lokal differesiallashlarni o‘rganishda kulminatsion natijani oldi. Lokal differensiallash tushunchasi R.Kodisonning [3] maqolasida birinchi marta kiritilgan va o‘rganilgan. [3] Maqolada Kodison fon Neyman algebrasini uning qo‘shma Banax bimoduliga akslantiruvchi har qanday uzluksiz lokal differensiallash differensiallash bo‘lishini isbotladi. Ushbu natijalar asosida bir qator mualliflar operator algebralaridagi lokal differensiallashlar bo‘yicha ilmiy ishlar olib bordilar. Masalan, [6] maqolaga va undagi adabiyotlar ro‘yxatiga qarang.

Mazkur maqolada ixtiyoriy maydonda aniqlangan matritsalar algebrasini o‘ziga akslantiruvchi additiv lokal ko‘paytirish tushunchasi kiritilgan va o‘rganilgan. Ushbu tushuncha quyidagicha kiritiladi: aytaylik $\Re$ – ixtiyoriy maydon, $M_n(\Re)$ – shu maydon ustida aniqlangan n o‘lchovli matritsalar algebrasi bo‘lsin. U holda $M_n(\Re)$ algebrani o‘ziga akslantiruvchi φ additiv akslantirish uchun har qanday $x \in M_n(\Re)$ element uchun shunday $a \in M_n(\Re)$ element mavjud bo‘lib, $\varphi(x) = ax$ shart bajarilsa, u holda φ additiv lokal ko‘paytirish deb ataladi.

Mazkur maqolada $M_n(\Re)$ algebrada aniqlangan har qanday φ additiv lokal ko‘paytirish chapdan ko‘paytirish bo‘lishi, ya’ni shunday $a \in M_n(\Re)$ element mavjud bo‘lib, ixtiyoriy $x \in M_n(\Re)$ element uchun $\varphi(x) = ax$ tenglik o‘rinli bo‘lishi isbotlangan.

Bundan tashqari, berilgan maqolada simmetrik matritsalar Yordan algebrasida additiv lokal Yordan ko‘paytirishi tushunchasi kiritilgan va o‘rganilgan. Ushbu tushuncha matritsalar assosiativ algebrasidagi additiv lokal chapdan (o‘ngdan) ko‘paytirishga o‘xshash kiritiladi. Berilgan maqolada ratsional sonlar maydonida aniqlangan simmetrik matritsalar Yordan algebrasidagi ixtiyoriy additiv lokal Yordan ko‘paytirishi Yordan ko‘paytirishi bo‘lishi hamda haqiqiy yoki kompleks sonlar maydonida aniqlangan simmetrik matritsalar Yordan algebrasida ixtiyoriy uzluksiz lokal Yordan ko‘paytirishi va ixtiyoriy chiziqli bo‘lgan additiv lokal Yordan ko‘paytirishi Yordan ko‘paytirishi bo‘lishi isbotlangan.

1. Matritsalar algebrasida additiv lokal ko‘paytirishlar.

Faraz qilaylik, $\Re$ – ixtiyoriy maydon, $M_n(\Re)$ – elementlari $\Re$ maydonda yotuvchi

$$a=\begin{bmatrix} a^{1,1}a^{1,2} & \dots & a^{1,n} \\ a^{2,1}a^{2,2} & \dots & a^{2,n} \\ \vdots \quad \vdots & \ddots & \vdots \\ a^{n,1}a^{n,2} & \dots & a^{n,n} \end{bmatrix}, \quad a^{i,j}\in\Re, \quad \mathrm{i,j}=1,2,\dots,\mathrm{n}$$

ko‘rinishidagi matritsalar algebrasi bo‘lsin.

1-ta’rif. Bizga $\varphi: M_n(\Re)\to M_n(\Re)$ akslantirish berilgan bo‘lsin. Agar shunday $a\in M_n(\Re)$ matritsa topilsaki, ixtiyoriy $x\in M_n(\Re)$ uchun $\varphi(x)=ax$ tenglik bajarilsa, u holda φ akslantirish – **chapdan ko‘paytirish** deb ataladi.

2-ta’rif. Bizga $\psi: M_n(\Re)\to M_n(\Re)$ additiv akslantirish berilgan bo‘lsin. Agar ixtiyoriy $x\in M_n(\Re)$ uchun shunday $a_x\in M_n(\Re)$ matritsa topilsaki, $\psi(x)=a_x x$ tenglik bajarilsa, u holda ψ akslantirish – **additive local chapdan ko‘paytirish** deb ataladi.

$[a]_i^j$orqali shunday matritsani belgilaymizki, uning j- ustuni a matritsaning i-ustuniga teng, qolgan ustunlari elementlari 0 dan iborat.

1-teorema. Agar ψ akslantirish additiv lokal chapdan ko‘paytirish bo‘lsa, u holda bu akslantirish chapdan ko‘paytirish bo‘ladi.

Isbot. $e_{i,j}\in M_n(\Re)$ orqali i-satr, j-ustun elementi 1, qolgan elementlari 0 dan iborat matritsani belgilaylik. Additiv lokal chapdan ko‘paytirish ta’rifiga ko‘ra $e_{1,i},\ i=1,\dots,n$ matritsalar uchun $\psi(e_{1,i})=a_{1,i}e_{1,i},\ i=1,\dots,n$ tengliklarni qanoatlantiruvchi $a_{1,i}\in M_n(\Re),\ i=1,\dots,n$ matritsalar topiladi. $a_{1,i}, i=1,\dots,n$ matritsalarning birinchi ustunlari o‘zaro tengligini ko‘rsatamiz. Bu matritsalardan ixtiyoriy ikkitasini olamiz: $a_{1,i}$ va $a_{1,j},\ i\neq j$. Ular

$$\psi(e_{1,i})=a_{1,i}e_{1,i},\ \psi(e_{1,j})=a_{1,j}e_{1,j} \qquad (1)$$

tengliklarni qanoatlantiradi. Additiv lokal chapdan ko‘paytirishning ta’rifiga ko‘ra, $e_{1,i}+e_{1,j}$ matritsa uchun shunday $a\in M_n(\Re)$ matritsa topiladiki, $\psi(e_{1,i}+e_{1,j})=a(e_{1,i}+e_{1,j})$ tenglik bajariladi. ψ akslantirishning additivlik hossasidan va (1) tengliklardan foydalanib

$$a_{1,i}e_{1,i}+a_{1,j}e_{1,j}=a(e_{1,i}+e_{1,j})$$

tenglikka ega bo‘lamiz. Bundan

$$[a_{1,i}]_1^i+[a_{1,j}]_1^j=[a]_1^i+[a]_1^j.$$

Oxirgi tenglik $a_{1,i}$ va $a_{1,j}$ matritsalarning birinchi ustunlari tengligini ko‘rsatadi. Bunda $i\neq j$ va i,j indekslar ixtiyoriy bo‘lgani uchun barcha $a_{1,i}, i=1,\dots,n$ matritsalarning birinchi ustunlari tengligini ko‘ramiz.

Yuqoridagidek mulohazalar yuritib, ixtiyoriy $k\in\{1,\dots,n\}$ olinganda ham $\psi(e_{k,i})=a_{k,i}e_{k,i},\ i=1,\dots,n$ tengliklarni qanoatlantiruvchi $a_{k,i},\ i=1,\dots,n$ matritsalar topilishini hamda bu matritsalarning k-ustunlari o‘zaro tengligini ko‘rsatish mumkin.

$d\in M_n(\Re)$ matritsani quyidagicha quramiz: uning k-ustuni $a_{k,i},\ i=1,\dots,n$ matritsalarning k-ustuniga teng.

Endi ixtiyoriy $\lambda\in\Re$element va ixtiyoriy$i,j=1,\dots,n$ indekslar juftligi uchun $\psi(\lambda e_{i,j})=d\lambda e_{i,j}$ tenglik bajarilishini ko‘rsataylik. Additiv lokal chapdan ko‘paytirish xossasiga ko‘ra$\lambda e_{i,j}$va $\lambda e_{i,j}+e_{i,i}$matritsalar uchun shunday $b,c\in M_n(\Re)$ matritsalar topiladiki, ular quyidagi tengliklarni qanoatlantiradi:

$$\psi(\lambda e_{i,j})=b\lambda e_{i,j},\ \ \psi(\lambda e_{i,j}+e_{i,i})=c(\lambda e_{i,j}+e_{i,i})(2)$$

ψ akslantirishning additivlik xususiyatidan va (2) tengliklardan foydalanib quyidagi tenglikka ega bo‘lamiz:

$$c(\lambda e_{i,j}+e_{i,i})=b\lambda e_{i,j}+a_{i,i}e_{i,i}.$$

Bu tenglikni chapdan $e_{i,k}$ matritsaga ko‘paytiramiz, bunda k indeks $1,2,\dots,n$ sonlardan ixtiyoriy biri. Natijada

$$c^{k,i}\lambda e_{i,k}+c^{k,i}e_{i,i}=b^{k,i}\lambda e_{i,k}+a_{i,i}^{k,i}e_{i,i}.$$

Bundan ixtiyoriy $k\in\{1,2,\dots,n\}$ uchun $b^{k,i}=c^{k,i}=a_{i,i}^{k,i}=d^{k,i}$ tengliklarga ega bo‘lamiz. Demak, b va d matritsalarning i-ustunlari teng. U holda (2) tengliklarning birinchisidan

$$\psi(\lambda e_{i,j})=b\lambda e_{i,j}=\lambda[b]_i^j=\lambda[d]_i^j=\lambda d e_{i,j}.$$

Shunday qilib, ixtiyoriy $\lambda\in\Re$element va ixtiyoriy $i,j=1,\dots,n$ indekslar juftligi uchun $\psi(\lambda e_{i,j})=d\lambda e_{i,j}$ tenglik bajarilishi ko‘rsatildi.

Endi ixtiyoriy $x\in M_n(\Re)$ matritsa uchun $\psi(x)=dx$ tenglik bajarilishini ko‘rsatamiz.

$$\psi(x)=\psi\left(\sum_{i,j=1}^{n}x^{i,j}e_{i,j}\right)=\sum_{i,j=1}^{n}\psi(x^{i,j}e_{i,j})=$$

$$= \sum_{i,j=1}^{n} dx^{i,j} e_{i,j} = d \sum_{i,j=1}^{n} x^{i,j} e_{i,j} = dx \,.$$

Demak, ixtiyoriy $x \in M_n(\mathfrak{R})$ matritsa uchun $\psi(x) = dx$ tenglik bajariladi. 1-ta'rifga ko'ra ψ akslantirish chapdan ko'paytirishdan iborat. Teorema to'liq isbotlandi.

Agar $\varphi: M_n(\mathfrak{R}) \to M_n(\mathfrak{R})$ akslantirish chapdan ko'paytirish bo'lsa, u holda shunday $a \in M_n(\mathfrak{R})$ matritsa topiladiki, ixtiyoriy $x \in M_n(\mathfrak{R})$ uchun $\varphi(x) = ax$ tenglik bajariladi. Ta'kidlash joizki, bunday a matritsa yagonadir.

Haqiqatan ham, $b \in M_n(\mathfrak{R})$ matrisa va ixtiyoriy $x \in M_n(\mathfrak{R})$ uchun $\varphi(x) = bx$ tenglik bajariladi deb olaylik. U holda $e \in M_n(\mathfrak{R})$ birlik matritsani olsak, bir tomondan $\varphi(e) = ae = a$, ikkinchi tomondan $\varphi(e) = be = b$. Bundan $b = a$ kelib chiqadi.

Endi o'ngdan ko'paytirish va additiv lokal o'ngdan ko'paytirish tushunchalarini kiritamiz.

3-ta'rif. Bizga $\varphi: M_n(\mathfrak{R}) \to M_n(\mathfrak{R})$ akslantirish berilgan bo'lsin. Agar shunday $a \in M_n(\mathfrak{R})$ matritsa topilsaki, ixtiyoriy $x \in M_n(\mathfrak{R})$ uchun $\varphi(x) = xa$ tenglik bajarilsa, u holda φ akslantirish – **o'ngdan ko'paytirish** deb ataladi.

4-ta'rif. Bizga $\psi: M_n(\mathfrak{R}) \to M_n(\mathfrak{R})$ additiv akslantirish berilgan bo'lsin. Agar ixtiyoriy $x \in M_n(\mathfrak{R})$ uchun shunday $a_x \in M_n(\mathfrak{R})$ matritsa topilsaki, $\psi(x) = xa_x$ tenglik bajarilsa, u holda ψ akslantirish – **additiv lokal o'ngdan ko'paytirish** deb ataladi.

$\{a\}_i^j$orqali shunday matritsani belgilaymizki, uning j-satri a matritsaning i-satriga teng, qolgan satrlari elementlari 0 dan iborat.

2-teorema. Agar ψ akslantirish additiv lokal o'ngdan ko'paytirish bo'lsa, u holda bu akslantirish o'ngdan ko'paytirish bo'ladi.

Isbot. Lokal o'ngdan ko'paytirish ta'rifiga ko'ra, $e_{i,1},\ i = 1, \dots, n$ matritsalar uchun $\psi(e_{i,1}) = e_{i,1} a_{i,1},\ i = 1, \dots, n$ tengliklarni qanoatlantiruvchi $a_{i,1} \in M_n(\mathfrak{R}),\ i = 1, \dots, n$ matritsalar topiladi. $a_{i,1}, i = 1, \dots, n$ matritsalarning birinchi satrlari tengligini ko'rsatamiz. Bu matritsalardan ixtiyoriy ikkitasini olamiz: $a_{i,1}$ va $a_{j,1}$, $i \neq j$. Ular

$$\psi(e_{i,1}) = e_{i,1} a_{i,1},\ \psi(e_{j,1}) = e_{j,1} a_{j,1} \quad (3)$$

tengliklarni qanoatlantiradi. Additiv lokal o'ngdan ko'paytirishning ta'rifiga ko'ra $e_{i,1} + e_{j,1}$ matritsa uchun shunday $a \in M_n(\mathfrak{R})$ matritsa topiladiki, $\psi(e_{i,1} + e_{j,1}) = (e_{i,1} + e_{j,1})a$ tenglik bajariladi. ψ akslantirishning additivlik xossasidan va (3) tengliklardan foydalanib

$$e_{i,1} a_{i,1} + e_{j,1} a_{j,1} = (e_{i,1} + e_{j,1})a$$

tenglikka ega bo'lamiz. Bundan

$$\{a_{i,1}\}_1^i + \{a_{j,1}\}_1^j = \{a\}_1^i + \{a\}_1^j.$$

Oxirgi tenglik $a_{i,1}$ va $a_{j,1}$ matritsalarning birinchi satrlari tengligini ko'rsatadi. Bunda $i \neq j$va i, jindekslar ixtiyoriy bo'lgani uchun barcha $a_{i,1}, i = 1, \dots, n$ matritsalarning birinchi satrlari o'zaro tengligini ko'ramiz.

Yuqoridagidek mulohazalar yuritib, ixtiyoriy $k \in \{1, \dots, n\}$ olinganda ham $\psi(e_{i,k}) = e_{i,k} a_{i,k},\ i = 1, \dots, n$ tengliklarni qanoatlantiruvchi $a_{i,k},\ i = 1, \dots, n$ matritsalar topilishini hamda bu matritsalarning k-satrlari o'zaro tengligini ko'rsatish mumkin.

$d \in M_n(\mathfrak{R})$ matritsani quyidagicha quramiz: uning k-satri $a_{i,k},\ i = 1, \dots, n$ matritsalarning k-satriga teng. Bu yerdan boshlab 1-teorema isbotidagidek mulohazalar yuritib, ixtiyoriy $x \in M_n(\mathfrak{R})$ matritsa uchun $\psi(x) = xd$ tenglik bajarilishini ko'rsatish mumkin. Teorema isbotlandi.

Agar $\varphi: M_n(\mathfrak{R}) \to M_n(\mathfrak{R})$ akslantirish chapdan ko'paytirish bo'lsa, u holda shunday $a \in M_n(\mathfrak{R})$ matritsa topiladiki, ixtiyoriy $x \in M_n(\mathfrak{R})$ uchun $\varphi(x) = xa$ tenglik bajariladi. Ta'kidlash joizki, bunday a matritsa yagonadir. Bu tasdiqni yuqoridagidek asoslash mumkin.

2. Matritsalar Yordan algebrasida additiv lokal ko'paytirishlar.

Bu paragrafda Yordan ko'paytirishi va additiv lokal Yordan ko'paytirishi tushunchalarini kiritamiz. Faraz qilaylik Q – ratsional sonlar maydoni, $H_n(Q)$ – elementlari Q maydonda yotuvchi

$$a = \begin{bmatrix} a^{1,1} a^{1,2} & \dots & a^{1,n} \\ a^{2,1} a^{2,2} & \dots & a^{2,n} \\ \vdots \quad \vdots & \ddots & \vdots \\ a^{n,1} a^{n,2} & \dots & a^{n,n} \end{bmatrix},\quad a^{i,j} = a^{j,i} \in \mathfrak{R},\ \ i, j = 1, 2, \dots, n$$

ko'rinishidagi matritsalar Yordan algebrasi bo'lsin. Boshqacha aytganda $H_n(\mathfrak{R})$ – simmetrik matrisalar Yordan algebrasi.

5-ta'rif. Bizga $\varphi: H_n(Q) \to H_n(Q)$ akslantirish berilgan bo'lsin. Agar shunday $a \in H_n(Q)$ matritsa topilsaki, ixtiyoriy $x \in H_n(Q)$ uchun $\varphi(x) = \frac{1}{2}(ax + xa)$ tenglik bajarilsa, u holda φ akslantirish – **Yordan ko'paytirishi** deb ataladi.

6-ta'rif. Bizga $\psi: H_n(Q) \to H_n(Q)$ additiv akslantirish berilgan bo'lsin. Agar ixtiyoriy $x \in H_n(Q)$ uchun shunday $a_x \in H_n(\mathfrak{R})$ matritsa topilsaki, $\psi(x) = \frac{1}{2}(a_x x + x a_x)$ tenglik bajarilsa, u holda ψ akslantirish – **additiv lokal Yordan ko'paytirishi** deb ataladi.

3-teorema. Agar ψ akslantirish additiv lokal Yordan ko'paytirishi bo'lsa, u holda bu akslantirish Yordan ko'paytirishi bo'ladi.

Isbot. ψ akslantirish additiv lokal Yordan ko'paytirishi bo'lsin. U holda $e_{i,i},\ i = 1, \dots, n$ simmetrik matritslar uchun $\psi(e_{i,i}) = \frac{1}{2}(a_{i,i}e_{i,i} + e_{i,i}a_{i,i})$ tengliklarni qanoatlantiruvchi $a_{i,i},\ i = 1, \dots, n$ simmetrik matritsalar topiladi. Agar

$$\psi(e_{i,i}) = \frac{1}{2}(a_{i,i}e_{i,i} + e_{i,i}a_{i,i})$$

tenglik bajarilsa, u holda ixtiyoriy $m \in Z$ butun son uchun

$$\psi(me_{i,i}) = \frac{m}{2}(a_{i,i}e_{i,i} + e_{i,i}a_{i,i})(4)$$

tenglik bajarilishini ko'rsataylik. ψ akslantirishning additivlik xossasiga ko'ra

$$\psi(2e_{i,i}) = \psi(e_{i,i}) + \psi(e_{i,i}) = a_{i,i}e_{i,i} + e_{i,i}a_{i,i}\,.$$

Demak, $m = 2$ uchun (4) tenglik o'rinli. Shunga o'xshash, barcha m natural sonlar uchun (4) tenglik bajarilishini ko'rsatish mumkin. Yana ψ akslantirishning additivlik xossasidan foydalanaylik

$$\psi(e_{i,i}) = \psi(-e_{i,i} + 2e_{i,i}) = \psi(-e_{i,i}) + \psi(2e_{i,i}).$$

Bu tenglikdan

$$\frac{1}{2}(a_{i,i}e_{i,i} + e_{i,i}a_{i,i}) = \psi(-e_{i,i}) + a_{i,i}e_{i,i} + e_{i,i}a_{i,i}$$

yoki

$$\psi(-e_{i,i}) = -\frac{1}{2}(a_{i,i}e_{i,i} + e_{i,i}a_{i,i})\,.$$

$m = -1$ uchun (4) tenglik bajarilishini ko'ramiz. Shunga o'xshash, barcha m butun manfiy sonlar uchun (4) tenglik bajarilishini ko'rsatish mumkin. Demak, barcha m butun sonlar uchun (4) tenglik o'rinli.

d simmetrik matritsani quyidagicha tuzaylik: d matritsaning i-ustuni (i -satri) $a_{i,i}$ matritsaning i-ustunidan (i-satridan) iborat. Ravshanki, d simmetrik matritsani bunday qoida bo'yicha tuzish mumkin bo'lishi uchun har qanday $i, k\ (i \neq k)$ indekslar juftligi olinganida ham

$$a_{i,i}^{i,k} = a_{k,k}^{i,k}(5)$$

tenglik bajarilishi zarur va yetarli. (5) tenglikni ko'rsataylik. Quyidagi tengliklarga egamiz:

$$\psi(e_{k,k}) = \frac{1}{2}(a_{k,k}e_{k,k} + e_{k,k}a_{k,k}), \qquad \psi(-e_{i,i}) = -\frac{1}{2}(a_{i,i}e_{i,i} + e_{i,i}a_{i,i})\,.$$

Additiv lokal Yordan ko'paytirishi ta'rifiga ko'ra shunday a matritsa topiladiki,

$$\psi(e_{k,k} - e_{i,i}) = \frac{1}{2}\left(a[e_{k,k} - e_{i,i}] + [e_{k,k} - e_{i,i}]a\right)$$

tenglik bajariladi. ψ akslantirishning additivlik xossasiga ko'ra

$$(a_{k,k}e_{k,k} + e_{k,k}a_{k,k}) - (a_{i,i}e_{i,i} + e_{i,i}a_{i,i}) = a[e_{k,k} - e_{i,i}] + [e_{k,k} - e_{i,i}]a\,.$$

Bu tenglikni chap va o'ngdan mos ravishda $e_{i,i}$ va $e_{k,k}$ matritsalarga ko'paytiramiz. Natijada $a_{k,k}^{i,k} - a_{k,k}^{i,k} = a^{i,k} - a^{i,k} = 0$. Bundan (5) tenglik kelib chiqadi.

Shunday qilib, d simmetrik matritsani tuzib oldik. Bu usulda qurilgan matritsa va ixtiyoriy $i, k\ (i \neq k)$ indekslar juftligi uchun

$$\psi(e_{i,i}) = \frac{1}{2}(de_{i,i} + e_{i,i}d),\ \ \psi(e_{k,k}) = \frac{1}{2}(de_{k,k} + e_{k,k}d)$$

$$\psi(e_{i,i} + e_{k,k}) = \frac{1}{2}\left(d[e_{i,i} + e_{k,k}] + [e_{i,i} + e_{k,k}]d\right)$$

tenglik bajariladi. Chunki

$$\psi(e_{i,i}) = \frac{1}{2}(a_{i,i}e_{i,i} + e_{i,i}a_{i,i}) = \frac{1}{2}\left([a_{i,i}]_i^i + \{a_{i,i}\}_i^i\right) = \frac{1}{2}([d]_i^i + \{d\}_i^i) = \frac{1}{2}(de_{i,i} + e_{i,i}d),$$

$$\psi(e_{k,k}) = \frac{1}{2}(a_{k,k}e_{k,k} + e_{k,k}a_{k,k}) = \frac{1}{2}\left([a_{k,k}]_k^k + \{a_{k,k}\}_k^k\right) = \frac{1}{2}([d]_k^k + \{d\}_k^k) = \frac{1}{2}(de_{k,k} + e_{k,k}d),$$

$$\psi(e_{i,i}+e_{k,k})=\psi(e_{i,i})+\psi(e_{k,k})=\frac{1}{2}(de_{i,i}+e_{i,i}d)+\frac{1}{2}(de_{k,k}+e_{k,k}d)=\frac{1}{2}(d[e_{i,i}+e_{k,k}]+[e_{i,i}+e_{k,k}]d).$$

Endi ixtiyoriy olingan i,k $(i<k)$ indekslar juftligi uchun

$$\psi(e_{i,k}+e_{k,i})=\frac{1}{2}(d[e_{i,k}+e_{k,i}]+[e_{i,k}+e_{k,i}]d)(6)$$

tenglik o‘rinli bo‘lishini ko‘rsatamiz. Additiv lokal Yordan ko‘paytirishining ta’rifiga ko‘ra shunday a matritsa topiladiki

$$\psi(e_{i,k}+e_{k,i})=\frac{1}{2}(a[e_{i,k}+e_{k,i}]+[e_{i,k}+e_{k,i}]a)$$

tenglik bajariladi. Agar

$$a[e_{i,k}+e_{k,i}]+[e_{i,k}+e_{k,i}]a=d[e_{i,k}+e_{k,i}]+[e_{i,k}+e_{k,i}]d \qquad (7)$$

tenglikni ko‘rsatsak, (6) ni isbotlagan bo‘lamiz. (7) tenglikdagi matritsalarning i va j nomerli ustun va satrlaridan boshqa ustun va satrlaridagi elementlari 0 ga teng. (7) tenglikni chapdan $e_{1,j}$ matritsaga ko‘paytiraylik, bunda j nomer i va k ga teng emas. Natijada

$$a^{j,i}e_{1,k}+a^{j,k}e_{1,i}=d^{j,i}e_{1,k}+d^{j,k}e_{1,i}.$$

Demak, barcha $j\in\{1,\dots,n\}\{i,j\}$ nomerlar uchun

$$a^{j,i}=d^{j,i},\ \ a^{j,k}=d^{j,k}(8)$$

tengliklar bajarilishi kerak. (7) matritsalarning i-satr i-ustun elementlari orasidagi munosabatni aniqlash uchun tenglikni chapdan $e_{1,i}$ ga, o‘ngdan $e_{i,1}$ ga ko‘paytiramiz:

$$a^{i,k}e_{1,1}+a^{k,i}e_{1,1}=d^{i,k}e_{1,1}+d^{k,i}e_{1,1}\Rightarrow a^{i,k}=d^{i,k}(9)$$

(7) matritsalarning i-satr k-ustun elementlari orasidagi munosabatni aniqlash uchun tenglikni chapdan $e_{1,i}$ ga, o‘ngdan $e_{k,1}$ ga ko‘paytiramiz:

$$a^{i,i}e_{1,1}+a^{k,k}e_{1,1}=d^{i,i}e_{1,1}+d^{k,k}e_{1,1}\Rightarrow a^{i,i}+a^{kk}=d^{i,i}+d^{k,k}(10)$$

Qaralayotgan matritslar simmetrik bo‘lgani uchun k-satr i-ustun elementlari orasida ham yuqoridagi (10) bog‘liqlik bajarilishi zarur. (7) matritsalarning k-satr k-ustun elementlari orasidagi munosabatni aniqlash uchun tenglikni chapdan $e_{1,k}$ ga, o‘ngdan $e_{k,1}$ ga ko‘paytiramiz:

$$a^{k,i}e_{1,1}+a^{i,k}e_{1,1}=d^{k,i}e_{1,1}+d^{i,k}e_{1,1}.$$

Demak, (9) tenglik bajarilsa, (7) matritsalarning k-satr k-ustun elementlari o‘zaro teng bo‘ladi. Shunday qilib, (7) matritsalar tengligini ko‘rsatish uchun (8), (9) va (10) tengliklarni isbotlash zarur.

Additiv lokal Yordan ko‘paytirishining ta’rifiga ko‘ra shunday b va c matritsalar topiladiki,

$$\psi(e_{i,k}+e_{k,i}+e_{i,i}+e_{k,k})=\frac{1}{2}(b[e_{i,k}+e_{k,i}+e_{i,i}+e_{k,k}]+[e_{i,k}+e_{k,i}+e_{i,i}+e_{k,k}]b)$$
$$\psi(e_{i,k}+e_{k,i}-e_{i,i}-e_{k,k})=\frac{1}{2}(c[e_{i,k}+e_{k,i}-e_{i,i}-e_{k,k}]+[e_{i,k}+e_{k,i}-e_{i,i}-e_{k,k}]c)$$

tenglik bajariladi. Bu yerda ψ akslantirishning additivlik xossasidan foydalanib quyidagilarga ega bo‘lamiz:

$$\begin{aligned}&b[e_{i,k}+e_{k,i}+e_{i,i}+e_{k,k}]+[e_{i,k}+e_{k,i}+e_{i,i}+e_{k,k}]b\\&\qquad=a[e_{i,k}+e_{k,i}]+[e_{i,k}+e_{k,i}]a+d[e_{i,i}+e_{k,k}]+[e_{i,i}+e_{k,k}]d \quad (11)\\&c[e_{i,k}+e_{k,i}-e_{i,i}-e_{k,k}]+[e_{i,k}+e_{k,i}-e_{i,i}-e_{k,k}]c\\&\qquad=a[e_{i,k}+e_{k,i}]+[e_{i,k}+e_{k,i}]a-d[e_{i,i}+e_{k,k}]-[e_{i,i}+e_{k,k}]d \quad (12)\end{aligned}$$

(11), (12) tengliklarni chapdan $e_{1,j}$ matritsaga ko‘paytiraylik, bunda j nomer i va k ga teng emas. Natijada quyidagilarga ega bo‘lamiz

$$b^{j,i}e_{1,k}+b^{j,k}e_{1,i}+b^{j,i}e_{1,i}+b^{j,k}e_{1,k}=a^{j,i}e_{1,k}+a^{j,k}e_{1,i}+d^{j,i}e_{1,i}+d^{j,k}e_{1,k}$$
$$c^{j,i}e_{1,k}+c^{j,k}e_{1,i}-c^{j,i}e_{1,i}-c^{j,k}e_{1,k}=a^{j,i}e_{1,k}+a^{j,k}e_{1,i}-d^{j,i}e_{1,i}-d^{j,k}e_{1,k}$$

Bu tengliklardan

$$\left.\begin{aligned}b^{j,i}+b^{j,k}=a^{j,i}+d^{j,k}\\b^{j,i}+b^{j,k}=a^{j,k}+d^{j,i}\\c^{j,i}-c^{j,k}=a^{j,i}-d^{j,k}\\c^{j,k}-c^{j,i}=a^{j,k}-d^{j,i}\end{aligned}\right\}\Rightarrow\left.\begin{aligned}a^{j,i}+d^{j,k}=a^{j,k}+d^{j,i}\\a^{j,i}-d^{j,k}=d^{j,i}-a^{j,k}\end{aligned}\right\}$$

sistemalar ketma-ket hosil bo‘ladi. Oxirgi sistema barcha $j\in\{1,\dots,n\}\{i,j\}$ nomerlar uchun (8) tengliklar bajarilishini ko‘rsatadi. (11), (12) tengliklarni chapdan $e_{1,i}$ ga, o‘ngdan $e_{i,1}$ ga ko‘paytiramiz:

$$\left.\begin{aligned}b^{i,k}+b^{i,i}+b^{k,i}+b^{i,i}=a^{i,k}+a^{k,i}+d^{i,i}+d^{i,i}\\c^{i,k}-c^{i,i}+c^{k,i}-c^{i,i}=a^{i,k}+a^{k,i}-d^{i,i}-d^{i,i}\end{aligned}\right\}$$

Bundan

$$\left.\begin{aligned}b^{i,k}+b^{i,i}=a^{i,k}+d^{i,i}\\c^{i,k}-c^{i,i}=a^{i,k}-d^{i,i}\end{aligned}\right\}(13)$$

(11), (12) tengliklarni chapdan $e_{1,i}$ ga, o‘ngdan $e_{k,1}$ ga ko‘paytiramiz:

$$\left.\begin{array}{r} b^{i,i}+b^{i,k}+b^{k,k}+b^{i,k}=a^{i,i}+a^{k,k}+d^{i,k}+d^{i,k} \\ c^{i,i}-c^{i,k}+c^{k,k}-c^{i,k}=a^{i,i}+a^{k,k}-d^{i,k}-d^{i,k} \end{array}\right\} (14)$$

(11), (12) tengliklarni chapdan $e_{1,k}$ ga, o‘ngdan $e_{k,1}$ ga ko‘paytiramiz:

$$\left.\begin{array}{r} b^{k,i}+b^{k,k}+b^{i,k}+b^{k,k}=a^{k,i}+a^{i,k}+d^{k,k}+d^{k,k} \\ c^{k,i}-c^{k,k}+c^{i,k}-c^{k,k}=a^{k,i}+a^{i,k}-d^{k,k}-d^{k,k} \end{array}\right\}$$

Bundan

$$\left.\begin{array}{r} b^{k,i}+b^{k,k}=a^{i,k}+d^{k,k} \\ c^{k,i}-c^{k,k}=a^{i,k}-d^{k,k} \end{array}\right\} (15)$$

(13) va (15) sitemalarning birinchi tenglamalarini qo‘shib, (14)ning birinchi tenglamasini ayiramiz:

$$2a^{i,k}+d^{i,i}+d^{k,k}-a^{i,i}-a^{k,k}-2d^{i,k}=0$$

(13), (14) va (15) sitemalarning ikkinchi tenglamalarini qo‘shamiz:

$$2a^{i,k}-d^{i,i}+a^{i,i}+a^{k,k}-2d^{i,k}-d^{k,k}=0$$

Oxirgi ikki tenglikdan (9) va (10) tengliklar to‘griligi kelib chiqadi.

Shunday qilib, ixtiyoriy olingan i,k $(i<k)$ indekslar juftligi uchun (6) tenglik bajarilishi ko‘rsatildi.

Yuqorida (4) tenglikni ko‘rsatganimizdagi kabi mulohaza yuritib, ixtiyoriy olingan i,k indekslar juftligi ($i=k$ bo‘lishi ham mumkin) va ixtiyoriy m butun son uchun

$$\psi(m[e_{i,k}+e_{k,i}])=\frac{m}{2}(d[e_{i,k}+e_{k,i}]+[e_{i,k}+e_{k,i}]d)$$

tenglikni isbotlash mumkin.

Endi ixtiyoriy olingan i,k indekslar juftligi ($i=k$ bo‘lishi ham mumkin) va ixtiyoriy $\frac{m}{n}$ ratsional son (bunda m – butun son, n – natural son) uchun

$$\psi\left(\frac{m}{n}[e_{i,k}+e_{k,i}]\right)=\frac{m}{2n}(d[e_{i,k}+e_{k,i}]+[e_{i,k}+e_{k,i}]d)(16)$$

tenglik bajarilishini ko‘rsatamiz.

Ravshanki, tayin olingan i,k indekslar juftligi va tayin $\frac{m}{n}$ ratsional son uchun shunday $a\in H_n(\Re)$ matritsa topiladiki, ular quyidagi tenglikni qanoatlantiradi:

$$\psi\left(\frac{m}{n}[e_{i,k}+e_{k,i}]\right)=\frac{m}{2n}(a[e_{i,k}+e_{k,i}]+[e_{i,k}+e_{k,i}]a). \qquad (17)$$

(17) tengliklardan n tasini qo‘shib

$$\psi(m[e_{i,k}+e_{k,i}])=\frac{m}{2}(a[e_{i,k}+e_{k,i}]+[e_{i,k}+e_{k,i}]a)$$

tenglikni olamiz. Boshqa tomondan m – butun son bo‘lganligidan

$$\psi(m[e_{i,k}+e_{k,i}])=\frac{m}{2}(d[e_{i,k}+e_{k,i}]+[e_{i,k}+e_{k,i}]d)$$

tenglikka egamiz. Oxirgi ikki tenglikka ko‘ra

$$\frac{m}{2}(a[e_{i,k}+e_{k,i}]+[e_{i,k}+e_{k,i}]a)=\frac{m}{2}(d[e_{i,k}+e_{k,i}]+[e_{i,k}+e_{k,i}]d).$$

Buni (17) ga qo‘yib (16) tenglik bajarilishini ko‘ramiz. Demak, ixtiyoriy olingan i,k indekslar juftligi va ixtiyoriy r ratsional son uchun

$$\psi(r[e_{i,k}+e_{k,i}])=\frac{r}{2}(d[e_{i,k}+e_{k,i}]+[e_{i,k}+e_{k,i}]d)$$

tenglik bajariladi.

Endi ixtiyoriy $x\in H_n(Q)$ simmetrik matritsa uchun

$$\psi(x)=\frac{1}{2}(dx+xd) \qquad (18)$$

tenglik bajarilishini ko‘rsataylik. x simmetrik matritsani quyidagicha yozib olamiz:

$$x=\sum_{\substack{i,k=1\\ i<k}}^{n}x^{i,k}(e_{i,k}+e_{k,i})+\sum_{i=1}^{n}x^{i,i}e_{i,i},$$

bu yerda $x^{i,k}\in Q$. U holda

$$\psi(x)=\sum_{\substack{i,k=1\\ i<k}}^{n}\psi\left[x^{i,k}\left(e_{i,k}+e_{k,i}\right)\right]+\sum_{i=1}^{n}\psi\left(x^{i,i}e_{i,i}\right)$$
$$=\sum_{\substack{i,k=1\\ i<k}}^{n}\left[x^{i,k}\frac{1}{2}\left(d\left(e_{i,k}+e_{k,i}\right)+\left(e_{i,k}+e_{k,i}\right)d\right)\right]+\sum_{i=1}^{n}x^{i,i}\frac{1}{2}\left(de_{i,i}+e_{i,i}d\right)$$
$$=\frac{d}{2}\left(\sum_{\substack{i,k=1\\ i<k}}^{n}x^{i,k}\left(e_{i,k}+e_{k,i}\right)+\sum_{i=1}^{n}x^{i,i}e_{i,i}\right)+\left(\sum_{\substack{i,k=1\\ i<k}}^{n}x^{i,k}\left(e_{i,k}+e_{k,i}\right)+\sum_{i=1}^{n}x^{i,i}e_{i,i}\right)\frac{d}{2}$$
$$=\frac{1}{2}(dx+xd)\,.$$

Shunday qilib, yuqorida qurilgan d simmetrik matritsa va ixtiyoriy $x\in H_n(Q)$ matritsa uchun (18) tenglik bajariladi. 3-teorema isbotlandi.

Ta'kidlash joizki, 3-teoremada ψ akslantirish additivlik va birjinslilik shartini, ya'ni, bir so'z bilan aytganda, chiziqlilik shartini qanoatalntirishi talab qilinsa, u holda (4) tenglik va unga o'xshash tengliklarning bajarilishi avtomatik tarzda ko'rinar edi. Hattoki, m ixtiyoriy haqiqiy son bo'lishi ham mumkin. Bunday mulohazalar quyidagi teoremani isbotlaydi:

4-teorema. Agar $\Re$ – ixtiyoriy maydon, $\psi: H_n(\Re)\to H_n(\Re)$ chiziqli akslantirish additiv lokal Yordan ko'paytirishi bo'lsa, u holda bu akslantirish Yordan ko'paytirishi bo'ladi.

Agar 3-teoremada ψ additiv lokal Yordan ko'paytirishi uzluksizlik shartini qanoatlantirishi talab qilinsa, u holda quyidagi teoremani isbotlash mumkin bo'ladi:

5-teorema. Agar D – haqiqiy sonlar yoki kompleks sonlar maydoni, $\psi: H_n(D)\to H_n(D)$ uzluksiz akslantirish additiv lokal Yordan ko'paytirishi bo'lsa, u holda bu akslantirish Yordan ko'paytirishi bo'ladi.

Bu teoremaning isboti 3-teorema isbotidagi (16) tenglik hosil qilinishigacha birdek amalga oshiriladi. Keyin ixtiyoriy olingan i,k indekslar juftligi ($i=k$ bo'lishi ham mumkin) va ixtiyoriy α haqiqiy son uchun

$$\psi\left(\alpha\left[e_{i,k}+e_{k,i}\right]\right)=\frac{\alpha}{2}\left(d\left[e_{i,k}+e_{k,i}\right]+\left[e_{i,k}+e_{k,i}\right]d\right)(19)$$

tenglik bajarilishini ko'rsatamiz.

Agar α ratsional son bo'lsa, bu tasdiq bajarilishi 3-teoremada ko'rsatib o'tildi. α irratsional son bo'lsin. α songa yaqinlshuvchi biror $\{r_n\}$ ratsional sonlar ketma-ketligini olamiz:

$$\lim_{n\to\infty} r_n=\alpha\,.$$

Yuqorida ko'rsatilganlarga asosan, ixtiyoriy olingan i,k indekslar juftligi ($i=k$ bo'lishi ham mumkin) va ixtiyoriy r_n ratsional sonlar uchun

$$\psi\left(r_n\left[e_{i,k}+e_{k,i}\right]\right)=\frac{r_n}{2}\left(d\left[e_{i,k}+e_{k,i}\right]+\left[e_{i,k}+e_{k,i}\right]d\right)$$

tenglik o'rinli. Bu yerda ψ akslantirishning uzluksizligidan foydalanib, $n\to\infty$ da limitga o'tsak, (19) tenglik hosil bo'ladi.

Agar $\varphi: M_n(\Re)\to M_n(\Re)$ akslantirish Yordan ko'paytirishi bo'lsa, u holda shunday $a\in M_n(\Re)$ matritsa topiladiki, ixtiyoriy $x\in M_n(\Re)$ uchun $\varphi(x)=\frac{1}{2}(ax+xa)$ tenglik bajariladi. Ta'kidlash joizki, bunday a matritsa yagonadir. Bu tasdiqni yuqoridagidek asoslash mumkin.

LOCAL MULTIPLICATION OPERATORS ON ALGEBRAS OF MATRICES

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In the present paper a notion of additive local multiplication on the algebra of matrices over an arbitrary field is introduced and investigated. It is proved that every additive local left multiplication on this matrix algebra is a left multiplication. Also, in the present paper, a notion of additive local Jordon multiplication on a Jordon algebra of symmetric matrices is also introduced and investigated. It is proved that every additive local Jordon multiplication on the Jordon

algebra of symmetric matrices over the field of rational numbers is a Jordon multiplication. The fact that every continuous additive local Jordon multiplication and every linear additive local Jordon multiplication on the Jordon algebra of symmetric matrices over the field of complex or real numbers is a Jordon multiplication is also proved.

Recall that a local derivation as defined us follows: given an algebra A, a linear map $\nabla: A \to A$ is called a local derivation if for every $x \in A$ there exists a derivation $D: A \to A$ such that $\nabla(x) = D(x)$.

In 1990 R.Kadison introduced the concept of local derivation and proved that each continuous local derivation from a von Neumann algebra into its dual Banach bemodule is a derivation. B.Jonson extends the above result by proving that every local derivation from a C^*-algebra into its Banach bimodule is a derivation. In particular, Johnson gives an automatic continuity result by proving that local derivation of a C^*-algebra Ainto a Banach A-bimodule$X$ are continuous even if not assumed a priori to be so. Based on these results, many authors have studied local derivations on operator algebras.

The present paper develops a pure algebraic approach to the investigation of multiplication operators and local multiplications on associative and Jordon algebras. For this propose we introduce a notion of additive local left multiplication on an algebra as follows: given an algebra A, an additive map $\nabla: A \to A$ is called additive local left multiplication, if for every $x \in A$ there exists an element a in A such that $\nabla(x) = ax$. We prove that fo an arbitrary field F and $M_n(F)$ of $n \times n$ matrices over F every additive local left multiplication∇ on the algebra $M_n(F)$ is a left multiplication, ie. there exists $a \in M_n(F)$ such that $\nabla(x) = ax, x \in M_n(F)$. Similarly, every additive local right multiplication∇ on the algebra $M_n(F)$ is a right multiplication, ie. there exists $a \in M_n(F)$ such that $\nabla(x) = xa, x \in M_n(F)$.

Also we introduce a notion of additive local Jordon multiplication on a Jordon algebra, and, prove that for an arbitrary field F and the Jordon algebra $H_n(F)$ of $n \times n$symmetric matrices over F, every additive local Jordon multiplication ∇ on the Jordon algebra $H_n(F)$ is a Jordon multiplication, ie. there exists $a \in H_n(F)$ such that $\nabla(x) = ax + xa, x \in H_n(F)$.

The results above allow us to assert that for finite ring R genereted by its identity element and the addition (in particular the ring $\mathbb{Z}_n$). or the ring if integre numbers and the ring $M_n(\Re)$ of $n \times n$ matrices over $\Re$ every local left multiplication on the ring $M_n(\Re)$ is a left multiplication. Similarly, every local right multiplication on the ring $M_n(\Re)$ is a right multiplication. Similarly, for a finite ring $\Re$ generated by its identity element and the addition or the ring of integral numbers and the Jordon ring $H_n(\Re)$ of $n \times n$ symmetric matrices over $\Re$ every local Jordon multiplication on the Jordon ring $H_n(\Re)$ is a Jordon multiplication.

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