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WAVELET METHODS FOR CARDIO SIGNALS PROCESSING

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Abstract. The article discusses the use of wavelet transforms and algorithms in the digital processing of cardiosignals in a number of medical applications due to their good adaptability to the analysis of non-stationary signals (that is, those whose statistical characteristics change with time). Since an electrocardiogram is a transient signal, wavelet methods can be used to recognize and detect key diagnostic features.

In real-time systems for digital signal processing, it is important that mathematical operations are performed quickly, and the time required to execute commands must be known precisely and in advance. For this, both the program and the hardware must be very effective. In digital signal processors, the most important mathematical operation and the core of all digital signal processing algorithms is multiplication, followed by summation. Fast execution of the multiplication operation followed by summation is very important for implementing real-time digital filters, signal processing, matrix multiplication, and graphic image manipulation. Therefore, all this requires the need to improve methods, algorithms and signal processing programs that determine the quality and performance of digital systems.

The technique is based on the transformation of the heart rate to simple harmonic oscillations (fast Fourier transform, autoregressive analysis) with different frequencies. In this case, the sequence of heartbeats is converted into a power spectrum of fluctuations of the duration of the RR intervals, which are a sequence of frequencies characterizing HRV. Most often, the area bounded by the spectral power curve corresponding to a certain defined frequency range, that is, the power within a limited frequency range, is estimated.

Keywords: spline, digital signal processor, functional dependencies, spline function, biomedical signals, signal processing methods and algorithms

Introduction

The state and prospects of the development of information technologies in the 21st century are characterized by the wide practical use of digital signal processing techniques - one of the most dynamic and fastest growing technologies in the world of telecommunications and computerization of society. Digital signal processing is, in fact, real-time computer science, designed to solve the problems of receiving, processing, reducing redundancy and transmitting information at a set speed.

The methods and techniques of digital signal processing are of great interest to scientists and specialists working in various fields, such as communications and control systems, radio engineering and electronics, acoustics and seismology, broadcasting and television, measurement technology and instrumentation. This also includes relatively new directions in the creation of hardware and software, such as processing audio and video signals, speech recognition, biometric systems, processing of dynamic images, multimedia learning systems.

The basic principles of the theory of digital signal processing were laid and tested on the theory of discrete systems and unitary transformations. Algorithms for fast Fourier transforms were developed, and a theory of binary-orthogonal transformations with local and integral bases was created.

A feature of the modern stage of scientific research is their wide automation based on computer technology, which is associated with large volumes of processed data. New methods and algorithms that ensure the timely and effective transformation of information are of great importance. For such areas as automation of bench tests, processing and restoration of geophysical data, analysis and processing of images, prediction of seismic events, the problems of combining the processes of collecting and processing analog measurement information and improving software are relevant.

An increase in productivity, interaction speed, and real-time computing is possible only through the development and implementation of new algorithms and software for processing and restoring signals.

Many existing methods and means of signal processing and reconstruction require solving a system of linear algebraic equations. This leads to a significant increase in computing costs, to complicate the implementation and implementation of the processing procedure in signal processing systems. All this poses new problems for the theory and practice of processing and reconstruction of signals in real time, which require the development and implementation of easily implemented and simple methods, algorithms, and signal processing tools.

In real-time systems, for digital signal processing, it is important that the mathematical operations are performed quickly, and the time required to execute the commands must be known accurately and in advance. For this, both the program and the hardware must be very effective. In digital signal processors, the most important mathematical operation and the core of all digital signal processing algorithms is multiplication followed by summation. The quick execution of the multiplication operation with the subsequent summation is very important for the implementation of real-time digital filters, signal processing, matrix multiplication, and manipulation of graphic images. Therefore, all this requires the need to improve methods, algorithms and signal processing programs that determine the quality and performance of digital systems.

Main part

The method of recording myocardial electrical activity - electrocardiography, proposed at the beginning of the century by Einthoven, remains one of the most common methods for diagnosing the state of the heart today. Modern technical capabilities, including computer technology, can significantly increase the information content of this research method due to new scientific approaches to the very essence of the cardiogram signal and new methods of special mathematical processing. At the first stage, tasks were set

• test existing methods for processing cardiac signals;

• draw conclusions about the appropriateness of using certain methods;

• create a fundamentally new mathematical apparatus for studying the dynamics of a cardiac signal based on methods of spectral analysis, wavelet transform, methods of the theory of dynamic chaos, a method of quantitative characteristics of the degree of irregularity of the electrical activity of the heart muscle.

One of the directions in this area is the study of heart rate variability (HRV) and the characteristic elements of cardiocycles. To carry out such an analysis, it is necessary to convert a digitized signal, which is a sequence of measurements of the potential difference with a certain sampling frequency into a vector of cardiac cycle parameters. In this case, each cardiocycle is represented by a set of parameters reflecting the size of the teeth and the intervals of this cardiocycle.

Analysis of heart rate variability (HRV) is a simple and convenient way to assess the nerve effects on the heart, which has made this method very popular in clinical practice. However, the potential of HRV analysis is still not fully realized, since the interpretation of the facts obtained is based on artificial or limited ideas about the formation of heart rhythm.

So, for example, the widely used methods of spectral analysis of heart rate (HR) are essentially abstract in nature, little projected on the specific mechanisms of regulation of the heart. And finally, all known analysis options ignore the fact that without determining the proper individual heart rate, all estimates of the current heart rate are superficial or erroneous.

Recently, there has been an increasing number of works devoted to the use of spectral analysis of heart rhythm (SARS) in clinical practice. However, many researchers used their algorithms to interpret the results. This was the occasion for the European Cardiology Association (JAC) and the North American Rhythmology Association (CAAP) to create a group of experts to develop standards for measuring heart rate variability, the results of which were published in 1996. However, not all problems were resolved, and a number of the points put forward is controversial and is refuted by the clinic. All this makes us look for new methodological approaches to the analysis of heart rate variability.

The technique is based on the conversion of the cardiac rhythm to simple harmonic oscillations (fast Fourier transform, autoregressive analysis) with different frequencies. In this case, the sequence of heart contractions is converted into a power spectrum of oscillations of the duration of the RR intervals, which are a sequence of frequencies characterizing HRV. Most often, the area bounded by the spectral power curve corresponding to a certain frequency range, that is, the power within a limited frequency range, is estimated. In the spectral analysis of HRV by the degree of change in the indicators of the rhythm spectrum, one can judge the level of adaptive capabilities of the body.

Based on the studies, the following problems are identified that arise during the spectral analysis of heart rhythm:

- the correct allocation of frequency ranges for calculating spectral power, as well as determining their optimal number for a full interpretation of the survey results;

- interpretation of the main and additional wave peaks of the spectrum at rest, when performing functional tests, under the influence of extreme factors and pathology;

- determination of the range of normal values of indicators of spectral analysis of heart rhythm;

- spectral analysis for extrasystole.

The problem of allocating frequency ranges for calculating spectral power is one of the main ones. The problem of interpretation of the wave peaks obtained in the analysis is directly related to it. Most healthy people have three main peaks. The first is near 0.25 Hz, is associated with the respiratory rate and reflects parasympathetic modulation of the heart rhythm. The second peak is about 0.1 Hz, correlates with the frequency of impulses from postganglionic sympathetic fibers and reflects sympathetic effects on the heart. The interpretation of the third peak in the near-zero frequency range is not completely clear. It is assumed that it reflects the effects on the heart rhythm of the higher centers of autonomic regulation.

It is necessary to increase the number of analyzed ranges, narrow their boundaries, evaluate the average frequencies and maximum amplitudes of wave peaks, as well as their shift to various types of functional loads and pathology. Determination of the average frequency of LF and HF should be carried out using simultaneous registration of BH.

The next problem, no less important in importance than the ones discussed above, is the determination of the range of normal values of the parameters of the spectral analysis of heart rhythm. It arises in connection with high heart rate variability in healthy people both in the same age group and between groups of different ages. So, for example, the values of the total spectrum power can be in the range of 643-16469 s2 / Hz. Naturally, people with severe pathology fall into the normal range. In this regard, it is necessary to individually normalize the values of the parameters of the spectral analysis of the heart rhythm, and then determine the normal boundaries of the indicators. The best is individual SD standardization (i.e., the spectral power values in a specific range should be divided by the SD value). In this case, individual differences in heart rate variability are taken into account.

An important problem is also the spectral analysis in the presence of extrasystoles in the ECG records. There are several approaches to solving this problem:

• do not conduct spectral analysis;

• analyze with the exception of extrasystoles, if their number does not exceed 5%;

• analyze using the following procedure - the extrasystole complex and countervailing pause count as two normal contractions.

The wavelet transform has become a powerful alternative to the Fourier transform in a number of medical applications due to its good adaptability to the analysis of unsteady signals (that is, those whose statistical characteristics change over time) [1,3]. Since the electrocardiogram is an unsteady signal, wavelet methods can be used to recognize and detect key diagnostic signs.

The Fourier transform represents the signal specified in the time domain in the form of an expansion in orthogonal basis functions (sines and cosines), thus highlighting the frequency components. The disadvantage of the Fourier transform is that the frequency components cannot be localized in time. This determines its applicability only to the analysis of stationary signals.

Most medical signals [2,4] have complex time-frequency characteristics. As a rule, such signals consist of close in time, short-lived high-frequency components and long-term, close in frequency low-frequency components (Levkovich-Maslyuk, 1998).

To analyze such signals, a method is needed that can provide good resolution in both frequency and time. The first is required to localize low-frequency components, the second - to resolve high-frequency components.

There are two approaches to the analysis of non-stationary signals of this type. The first is the local Fourier transform (short-time Fourier transform). Following this path, we work with a non-stationary signal, as with a stationary one, breaking it first into segments (frames) whose statistics do not change with time. The second approach is wavelet transform. In this case, an unsteady signal is analyzed by expanding into basic functions obtained from a certain prototype by means of compressions, stretches and shifts. The prototype function is called the analyzing, or maternal, wavelet (mother-wavelet), selected to study this signal.

There are reports of the development of a wavelet technique capable of detecting abnormal structures on cardiograms. The digitized ECG signal [5] is decomposed into wavelet functions at several resolution levels. At each level, the coefficients are the details that arise when moving from one scale to another. Regression analysis of the log-log graphs of the variation of wavelet coefficients depending on the scale indicates that the slope of the graphs of these signals is different in healthy people and in people with multiple coronary occlusions.

Another successful application of the wavelet technique relates to variations in heart rate. This technique, as described above, is based on the expansion of the ECG in a series of wavelets at different scales. It is known that the time series of intervals between heart contractions is unsteady and exhibit complex behavior. A typical feature of this kind of unsteady signals is the presence of torn structures that change over time. The appearance of these structures on the ECG changes in the presence of cardiac abnormalities. Anomalous contractions are usually located at large (low-frequency) scales, and normal structures - at smaller (high-frequency) scales.

The developed method for analyzing an ECG signal is based on a continuous wavelet transform (CWT). Continuous wavelet transform at different time scales characterizes the signal in different frequency ranges, while the discrete wavelet transform (DWT) is limited to scales that are powers of two.

Let s be a signal and Ψ a wavelet. With continuous conversion, the wavelet coefficients of the signal s corresponding to the scale factor a and position b are determined by the formula (1) [8]:

$$CWT(a,b)=\frac{1}{\sqrt{a}}\int_{-}^{\infty} s(t)\psi\cdot\left(\frac{t-b}{a}\right)dt \qquad (1)$$

To select the optimal wavelet used as the basis, several wavelet functions were tested. By the optimal wavelet, we mean the one that provides the correct location of the coordinates of the nine points of the cardiocycle: the beginning, peak, and offset of the T-wave, QRS complex, and P-wave. In [10, 11], biorthogonal wavelets with a compact support are used using scales that are multiples of powers of two. In [11, 12], Gaussian wavelets (pre - wavelets) are used. The properties of these wavelet families [8,9] are presented in table 1.

The best wavelet carrier that meets the above requirements, according to the results obtained (Table 2), is the biorthogonal wavelet "bior1.5".

As the scale used to determine the correct location of the nine coordinates of the points of the ECG signal, 15 scales were used to detect the QRS complex [11-15] and 41 scales to detect P and T teeth [11]. Scales 15 and 41 provide the greatest accuracy in detecting these teeth. The bior1.5 wavelet on scales 15 and 41 is shown in Figure 1. The process of analyzing a cardiosignal can conditionally be divided into two stages: the stage of preliminary processing and the selection of symptoms (Fig. 2). The pretreatment stage consists in the removal of noise (electromyographic muscle potentials, artifacts of the interaction of electrodes with the skin, electronic noise of amplifiers and background noise of the network) [6-9]. Noise is considered to be high-frequency components of the cardiac signal. Noise removal leads to compression and smoothing of the ECG signal. The stage of distinguishing features from a cardiac signal is the process of extracting the required information (teeth, complexes, etc.). Let's consider each stage in more detail.

Table 1

Properties of Gaussian and Biorthogonal Wavelets

Criterion	Gaussian wavelets (§ash)	Biorthogonal wavelets (bior)
The presence of the function f -	-	+
Availability function	+ (pronounced)	+
Orthogonal analysis	-	+
The presence of a compact medium	-	+ (ф, у, ф, у)
Potential	Recovery is not guaranteed	+
Symmetry	+	+
FIR- filter	-	+
Possible wavelet analysis	UWB without using fast algorithms	CWT and DWT using fast algorithms

Table 2.

Accuracy of detection of coordinates of points of an ECG signal.

Criteria	Gaus	Bior		
		«biorl.1»	«bior1.3»	«bior1.5»
The accuracy of detecting the coordinates of the points of the ECG signal,%	91-92	93-94	95-96	98-99

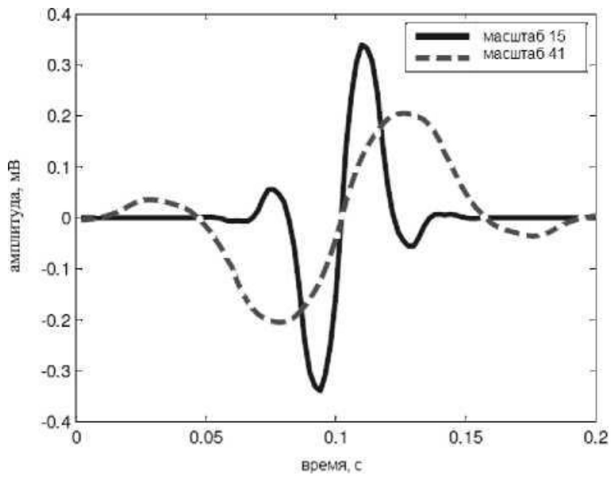

Figure 1. The bior 1.5 wavelet at scales 15 and 41

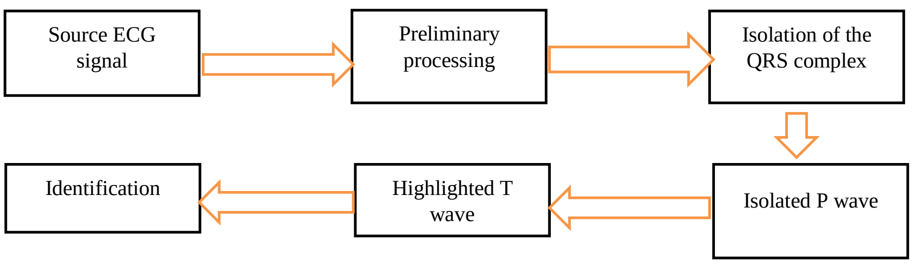

Figure 2: ECG signal processing structure

In the simplest model, it is assumed that the noisy signal has the form (2) [8,9]:

s (n) = f / (n) + Ó - e (n), (2)

where: f / (p) is the useful signal, Ó is the noise level and e (p) is the Gaussian white noise, i.e. stationary random sequence with zero mathematical expectation, absolutely uncorrelated and with a dispersion equal to unity.

In the wavelet analysis, the signal is decomposed into approximating coefficients, which represent a smoothed signal, and detailed coefficients describing the oscillations. Therefore, the noise component is better reflected in the detailed coefficients. Such components can be removed using the procedure of zeroing or recalculation of the coefficients of detail, the values of which are less than the threshold value. The threshold processing procedure, or "thresholding", is today a promising tool for "cleaning" the cardiac signal from noise (high-frequency components) [8]. . The following types of trash holding [8,9]:

Hard trashholding, in which all coefficients that exceed a certain threshold are considered to belong to the "original" signal, and the rest are related to noise and zero (3):

$$f(x) = \begin{cases} x, |x| \geq t, \\ 0, |x| < t. \end{cases} \quad (3)$$

where t is a certain threshold (thresholding coefficient).
Soft trashholding (4):

$$f(x) = \begin{cases} \boldsymbol{x - t, x \geq t,} \\ \boldsymbol{0}, |\boldsymbol{x}| < t, \\ \boldsymbol{x + t, x \leq t.} \end{cases} \quad (4)$$

The quality of the noise reduction of the signal and, consequently, the degree of increase in the signal-to-noise ratio depend not only on the type of the trashold function, but also on the method of its application. Depending on this, thrasholding is divided into global and local, and local in turn into general and multi-level [6,7].

To determine the threshold values, we will use the following methods:

SQR-LOG method (5) [16.17]:

$$t = \sqrt{2\left(\frac{(median\{c(i)\}\ \ i = 1 \ldots n)}{0{,}6745}\right)^2 \ln(n)} \quad (5)$$

where: 0.6745 is the estimate of the standard deviation of the white Gaussian noise, and s (G) are the wavelet coefficients.

1) Berg-Massard method (6) [17]:

$$t = |c(z)|,$$

$$Z = arg\ min[-\sum\{c^2(i), i < k\} + 2\sigma^2 k(a + \ln(\tfrac{n}{k}))];$$

k = 1......n (6)

where: o2 is the noise dispersion, and a is the rarefaction parameter a> 1.

The rarefaction parameter is the key in the Berg-Massard method, since it is precisely its value specified by the researcher that determines the degree of suppression of the noise present in the signal.

In the Berg – Massard method, three intervals of changes in the value of parameter a are determined, which specify the amount of the "penalty":

- "high", with 2.5 <a <10;
- "average", with 1.5 <a <2.5;
- "low", with 1 <a <1.5.

1) Stein Method (7):

$$T_w = \arg\min_{t\geq 0}\left[SURE(W)\right],$$

$$SURE(W) = \sigma^2 - \frac{1}{N}(2\sigma^2 \cdot \#\{n : |W(m,n)|\} -$$

$$-\sum_{k=1}^{L} \min\left(|W(m,n)|\right)^2\Big) \quad (7)$$

where: W (m, n) are wavelet coefficients at the decomposition level m; L is the length of the wavelet coefficient vector W (m, n) at the level m; G2 - noise variance; #S is the cardinality of S.

In the work, a wavelet from the Daubechies family was chosen as the base one. Let us introduce the following limitation: a basic wavelet can be applied to a discrete wavelet transform, the order of high-frequency and low-frequency filters designed to separate out the detailed and approximating components should not exceed 10 (due to the high resource consumption). The "db2" and "db4" wavelets satisfy these requirements[18.19].

It is believed that the upper boundary frequency of the cardiac signal, which significantly affects its shape, does not exceed 100 Hz [8]. Therefore, signal components of frequencies higher than 100 Hz can be removed without significantly changing the waveform. Based on this, the signal decomposition level for the "db2" and "db4" wavelets is calculated. The "db2" wavelet has a center frequency of Fr = 0.6667 Hz [8]. Since At = 1/1024, the central frequency of the first decomposition level is Fr1 = 0.6667 x 1024 = 682.70 Hz, then for the second level Fr2 = 341.35 Hz, for the third level Fr3 = 170.68, for fourth level Fr4 = 85.34 Hz. Similarly, for the db4 wavelet with the central frequency Fr = 0.7143 Hz: Fr1 = 734.30 Hz, Fr2 = 367.15 Hz, Fr3 = 183.57 Hz, Fr4 = 91.8 Hz. Thus, to remove components of a cardiosignal whose frequency is higher than 100 Hz, it is required to use the fourth decomposition level, and this results in a signal compression of 24 = 16 times[20].

Conclusion

Thus, as a result of applying digital processing methods in an electrocardiography system, the following conclusions can be drawn:

1. The choice of the type of wavelet transform and basic wavelet for the analysis of cardiac signals (continuous wavelet transform using the "bior1.5" basis) is justified.

2. The choice of a scale factor for continuous wavelet transform for detecting P, QRS, and T teeth is justified (15 scale for detecting a QRS complex and 41 scales for detecting P and T teeth).

3. The method for detecting P-QRS-T teeth has been improved: a threshold value and the use of signal approximation in the areas of the QRS complex have been proposed in order to increase the accuracy of detection of P and T teeth.

4. Existing techniques for cleaning signals from noise are analyzed.

5. The choice of the type and method of wavelet-thrasholding (local multi-level hard thrasholding using the Berg – Massard method and the "db4" wavelet at the fourth level of decomposition as a basis) is justified.

The developed method for the analysis of ECG signals based on wavelet transform in a high-resolution electrocardiography system allows you to "clean" the cardio signal from noise without loss of information. Using the method, you can detect nine important coordinates of the points of the cardiosignal: the beginning, peak, and offset of the P-wave, QRS complex, and T-wave with an accuracy of 98-99%. The proposed approach significantly increases the accuracy of detection of P and T teeth.

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