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INTERVAL EXTENSION STRUCTURE BASIC TYPES OF SOLUTIONS OF BOUNDARY VALUE PROBLEMS FOR LINEAR PARTIAL DIFFERENTIAL EQUATIONS OF THE SECOND ORDER

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INTERVAL EXTENSION STRUCTURE BASIC TYPES OF SOLUTIONS OF BOUNDARY VALUE PROBLEMS FOR LINEAR PARTIAL DIFFERENTIAL EQUATIONS OF THE SECOND ORDER

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Abstract. In this article, the interval expansion of the structure of solving basic types of boundary value problems for partial differential equations of the second order of making the basic operations that compose interval arithmetic is developed. For the differential equation (1) of the type, when constructing the interval expansion of the structure of the formula, structural formulas were used to construct with the R-function method and 4 problems were studied — the Dirichlet problem, the Neumann problem, the third type problem, the mixed boundary conditions problem. For the Dirichlet problem, the solution is an interval expansion of the structure in the form (5), where $P = \{\underline{\omega\Phi}, \underline{\omega}\overline{\Phi}, \overline{\omega}\underline{\Phi}, \overline{\omega}\overline{\Phi}\}$ и $[\underline{\Phi}, \overline{\Phi}]$is an indefinite interval function. For the Neumann problem, a solution is solved in the interval extension of the structure (10), (11), $[\underline{\Phi_1}, \overline{\Phi_1}]$, $[\underline{\Phi_2}, \overline{\Phi_2}]$ is an indefinite interval function and D_1 is a differential operator of the form (9). For the problem of the third type, the solution is solved in the interval extension of the structure (16), (17), $[\underline{\Phi_1}, \overline{\Phi_1}]$, $[\underline{\Phi_2}, \overline{\Phi_2}]$ - indefinite, interval function, D_1- differential operator of the form (9). For the problem, mixed boundary conditions are treated. The solution In the interval extension of the structure (22), (23), (24), $[\underline{\Phi_1}, \overline{\Phi_1}]$, $[\underline{\Phi_2}, \overline{\Phi_2}]$ is an indefinite interval function and $D_1$is a differential operator of the form (9).

Keywords: structure solutions, interval arithmetic, boundary value problems, R-function method.

Introduction

R-functions method allows us to construct coordinate sequences satisfying the boundary conditions exactly, without any approximations [1]. However, when solving systems of differential (integro-differential) equations of high order due to bad of a matrix (full matrix of large orders, compiled as a result of sampling in the spatial variables using the method of R-functions) lost accuracy of the approximate solutions . Furthermore, such a loss of accuracy arise in cases where the initial data of the problem are not accurate, approximate values of the integrals are computed and error methods for solving governing equations, etc. These disadvantages can be eliminated by using interval method [2, 5, 6]. Hence the need to develop an algorithm combining R-functions method and interval method for solving practical problems, which we call the interval-valued R-function.

Application algorithm of interval- valued R-functions for solving boundary value problems for differential equations of various orders consists of the following steps:

1. Construct interval extensions systems of R-functions.
2. Construct interval extensions for formulas differential tuples.
3. Construct interval extensions of structural formulas.
4. Construct interval extensions of cubature formulas.
5. Construct interval extensions solutions resolving equations (in the case of a static setting - the system of algebraic equations, and in the case of dynamics - a system of ordinary differential equations with initial conditions).

Interval extensions systems of R-functions and formulas for the interval extensions of multidimensional differential tuples are given in [7-10]. Here we consider only the interval extensions of structural formulas for basic boundary value problems for partial differential equations of second order.

Constructing interval extensions of structural formula

In constructing the interval extension structure formula used to build structural formula method of R-functions given in [1.4, 10].

Suppose we are given the differential equation

$$\sum_{i,j=1}^{m} A_{i,j}\frac{\partial^2 \boldsymbol{u}}{\partial x_i x_j} + F\left(\boldsymbol{x}, \boldsymbol{u}, \frac{\partial \boldsymbol{u}}{\partial x_1}, \dots, \frac{\partial \boldsymbol{u}}{\partial x_m}\right) = 0 \qquad (1)$$

Where, $\mathbf{A} = \{\mathbf{a}_{i,j}\}$ - symmetric matrix, **u** function. As is known, the characteristic of the matrix A are real [5]. Equation (1) is elliptic in Ω, if at all points of Ω eigenvalues of the matrix A have the same sign.

The main types of boundary conditions for the equation (1) are the following tasks:

Dirichlet problem. Boundary condition on Γ has the form

$$u\big|_{\Gamma} = \varphi_0 \qquad (2)$$

Problem Digde $\varphi_0 : \Gamma \to R$ (or R^n, if **u** vector - function). φ_0 function can be set only on the boundary Γ and have on different parts of $\Gamma_i \subset \Gamma$ different analytical expressions φ_i:

$$u = \begin{cases} \varphi_1, & x \in \Gamma_1 \\ \dots \dots & \dots \\ \varphi_m, & x \in \Gamma_m \end{cases} \qquad (3)$$

We assume that $\varphi_i \in \mathfrak{M}(H)$ - some H - functions implemented. In practice, most often $H = H_\varepsilon$, ie, $\mathfrak{M}(H)$ - a plurality of elementary functions.

These limit values can be extended into the domain and glue by using the EC. $EC\varphi_0 = \varphi$ - operator bonding limits. For this operator can be given various options and formulas here $\varphi_0 = \frac{\varphi_1\tau_1+\varphi_2\tau_2+\cdots+\varphi_m\tau_m}{\tau_1+\tau_2+\cdots+\tau_m}$, $\tau_i = \frac{1}{\omega_i}$. $EC\varphi_0 = \varphi$ if the structure of the solution of the problem (1), (2) the construction of a polynomial of the form R-functions method V.L. Rvachev has the form [1]:

$$u = \varphi + \omega\Phi \qquad (4)$$

where Φ - undefined function and ω - equation of the boundary region, the following species,

$$\omega = \begin{cases} > 0, & (x,y) \in \Omega \\ = 0, & (x,y) \in \Gamma \\ < 0 & (x,y) \notin \Gamma \cup \Omega \end{cases}$$

Applying interval arithmetic operations, construct interval extension solution structure (4):

$$[\underline{u}, \bar{u}] = [\underline{\varphi}, \bar{\varphi}] + [\underline{\omega}, \bar{\omega}][\underline{\Phi}, \bar{\Phi}] = [\underline{\varphi}, \bar{\varphi}] + [\min\{P\}, \max\{P\}] = [\underline{\varphi} + \min\{P\}, \bar{\varphi} + \max\{P\}].$$

And we get the interval extension structure in the form:

$$[\underline{u}, \bar{u}] = [\underline{\varphi} + \min\{P\}, \bar{\varphi} + \max\{P\}] \qquad (5)$$

where $P = \{\underline{\omega\Phi}, \underline{\omega}\bar{\Phi}, \bar{\omega}\underline{\Phi}, \bar{\omega}\bar{\Phi}\}$ and $[\underline{\Phi}, \bar{\Phi}]$ - undefined gap function.

II.2. Neumann problem**.** The boundary condition for this problem has the form

$$\frac{\partial u}{\partial n}\bigg|_{\Gamma} = \varphi_0 \qquad (6)$$

Here n - normal to Γ. If Γ is the point at which the direction φ_0 is not defined, requires special consideration. Suppose, as in the preceding paragraph, $EC\varphi_0 = \varphi$. This allows the condition (6) rewritten as

$$\frac{\partial u}{\partial n}\bigg|_{\Gamma} = \varphi \qquad (7)$$

Structure of the solution of the problem (1), (7) is to construct a polynomial method of R-functions VL Rvachev has the form [1]:

where Φ_1, Φ_2-undefined function and D_1- differential operator with the following form

$$D_1 u = \sum_{i=1}^{n} \frac{\partial\omega}{\partial x_i}\frac{\partial u}{\partial x_i} \qquad (9)$$

Applying interval arithmetic operations, construct interval extension solution structure (8):

$$[\underline{u}, \bar{u}] = [\underline{\Phi_1}, \overline{\Phi_1}] + [\underline{\omega}, \bar{\omega}]\left([\underline{\omega}, \bar{\omega}][\underline{\Phi_2}, \overline{\Phi_2}] - D_1[\underline{\Phi_1}, \overline{\Phi_1}] + [\underline{\varphi}, \bar{\varphi}]\right) =$$
$$= [\underline{\Phi_1}, \overline{\Phi_1}] + [\underline{\omega}, \bar{\omega}]\left([\min\{Q\}, \max\{Q\}] - [D_1\underline{\Phi_1}, D_1\overline{\Phi_1}] + [\underline{\varphi}, \bar{\varphi}]\right) =$$
$$= [\underline{\Phi_1}, \overline{\Phi_1}] + [\underline{\omega}, \bar{\omega}]\left([\min\{Q\}, \max\{Q\}] - [D_1\underline{\Phi_1} + \underline{\varphi}, D_1\overline{\Phi_1} + \bar{\varphi}]\right) =$$
$$= [\underline{\Phi_1}, \overline{\Phi_1}] + [\underline{\omega}, \bar{\omega}]\left([\min\{Q\} - D_1\overline{\Phi_1} + \bar{\varphi}, \max\{Q\} - D_1\underline{\Phi_1} + \underline{\varphi}]\right) =$$
$$= [\underline{\Phi_1} + \min(Q_1), \overline{\Phi_1} + \max(Q_1)]$$

where,

$Q = \{\underline{\omega\Phi_2}, \underline{\omega}\overline{\Phi_2}, \bar{\omega}\underline{\Phi_2}, \bar{\omega}\,\overline{\Phi_2}\}$, $Q_1 = \{\underline{\omega}(\min\{Q\} - D_1\overline{\Phi_1} + \overline{\varphi}), \underline{\omega}(\max\{Q\} - D_1\underline{\Phi_1} + \underline{\varphi}), \bar{\omega}(\min\{Q\} - -D_1\overline{\Phi_1} + \bar{\varphi}), \bar{\omega}(\max\{Q\} - D_1\underline{\Phi_1} + \underline{\varphi})\} = \{\underline{\omega}\min\{Q\} - \underline{\omega}D_1\overline{\Phi_1} + \underline{\omega}\overline{\varphi}), \underline{\omega}\max\{Q\} - \underline{\omega}D_1\underline{\Phi_1} + \underline{\omega\varphi}), \bar{\omega}\min\{Q\} - -\bar{\omega}D_1\overline{\Phi_1} + \overline{\bar{\omega}\varphi}), \bar{\omega}\max\{Q\} - \bar{\omega}D_1\underline{\Phi_1} + \bar{\omega}\underline{\varphi})\}$

и получаем интервальное расширение структуры в виде:

$$[\underline{u}, \bar{u}] = [\underline{\Phi_1} + \min(Q_1), \overline{\Phi_1} + \max(Q_1)] \qquad (10)$$

or you can build a structure of the solution interval extension (8) alternatively:

$$[\underline{u}, \bar{u}] = [\underline{\Phi_1}, \overline{\Phi_1}] - [\underline{\omega}, \bar{\omega}]D_1[\underline{\Phi_1}, \overline{\Phi_1}] + [\underline{\omega}, \bar{\omega}]^2[\underline{\Phi_2}, \overline{\Phi_2}] + +[\underline{\omega}, \bar{\omega}][\underline{\varphi}, \bar{\varphi}] =$$
$$= [\underline{\Phi_1}, \overline{\Phi_1}] - [\underline{\omega}, \bar{\omega}]D_1[\underline{\Phi_1}, \overline{\Phi_1}] + +[\min\{F\}, \max\{F\}] + [\min\{L\}, \max\{L\}] =$$
$$= [\underline{\Phi_1}, \overline{\Phi_1}] - [\min\{K\}, \max\{K\}] + [\min\{F\} + \min\{L\}, \max\{F\} + \max\{L\}] =$$
$$= [\underline{\Phi_1} - \max\{K\} + \min\{F\} + \min\{L\}, \overline{\Phi_1} - \min\{K\} + \max\{F\} + \max\{L\}]$$

and obtain

$$[\underline{u}, \bar{u}] = [\underline{\Phi_1} - \max\{K\} + \min\{F\} + \min\{L\}, \overline{\Phi_1} - \min\{K\} + + \max\{F\} + \max\{L\}] \qquad (11)$$

$F = \{\underline{\omega}^2\underline{\Phi_2}, \bar{\omega}^2\underline{\Phi_2}, \underline{\omega}^2\overline{\Phi_2}, \bar{\omega}^2\overline{\Phi_2}\}$, $L = \{\underline{\omega}\,\underline{\varphi}, \bar{\omega}\,\underline{\varphi}, \underline{\omega}\bar{\varphi}, \bar{\omega}\bar{\varphi}\}$,

$K = \{\underline{\omega}\,D_1\underline{\Phi_1}, \bar{\omega}\,D_1\underline{\Phi_1}, \underline{\omega}\,D_1\overline{\Phi_1}, \bar{\omega}\,D_1\overline{\Phi_1}\}$

In the interval extension structure (10), (11), $\left[\underline{\Phi_1},\overline{\Phi_1}\right]$, $\left[\underline{\Phi_2},\overline{\Phi_2}\right]$ - undefined interval function D_1 - differential operator of the form (9).

II.3. The task of the third type. The boundary condition of this problem has the form

$$\left(\frac{\partial u}{\partial n}+h_0(x)u\right)\Big|_{\Gamma}=\varphi_0 \qquad (12)$$

Here h_0 and φ_0 - function of $\mathfrak{M}$(H) defined, in general, just on the border. Let $\mathrm{EC}h_0=h_1$, $\mathrm{EC}\varphi_0=\varphi$. Then the condition (11) can be replaced

$$\left(\frac{\partial u}{\partial n}+h_1(x)u\right)\Big|_{\Gamma}=\varphi(x) \qquad (13)$$

Structure of the solution of the problem (1), (12) the construction of a polynomial of the form R-functions method has the form [1]:

$$u=\Phi_1-\omega D_1\Phi_1-h_1\Phi_1\omega+\varphi\omega+\omega^2\Phi_2 \qquad (14)$$

where, Φ_1,Φ_2-undefined function and D_1 - differential operator of the form (9). Applying interval arithmetic operations, construct interval extension solution structure (14):

$$\left[\underline{u},\bar{u}\right]=\left[\underline{\Phi_1},\overline{\Phi_1}\right]-\left[\underline{\omega},\bar{\omega}\right]D_1\left[\underline{\Phi_1},\overline{\Phi_1}\right]-\left[\underline{h_1},\overline{h_1}\right]\left[\underline{\Phi_1},\overline{\Phi_1}\right]\left[\underline{\omega},\bar{\omega}\right]+\left[\underline{\varphi},\bar{\varphi}\right]\left[\underline{\omega},\bar{\omega}\right]+\left[\underline{\omega},\bar{\omega}\right]^2\left[\underline{\Phi_2},\overline{\Phi_2}\right] \qquad (15)$$

Taking into account that

$$\left[\underline{\omega},\bar{\omega}\right]D_1\left[\underline{\Phi_1},\overline{\Phi_1}\right]=\left[\underline{\omega},\bar{\omega}\right]\left[D_1\underline{\Phi_1},D_1\overline{\Phi_1}\right]=\left[\underline{K_1},\overline{K_1}\right];$$

where $\underline{K_1}=min\{K_1\}$, $\overline{K_1}=max\{K_1\}$, $K_1=\left\{\underline{\omega}D_1\underline{\Phi_1},\underline{\omega}D_1\overline{\Phi_1},\bar{\omega}D_1\underline{\Phi_1},\bar{\omega}D_1\overline{\Phi_1}\right\}$,

$$\left[\underline{h_1},\overline{h_1}\right]\left[\underline{\Phi_1},\overline{\Phi_1}\right]\left[\underline{\omega},\bar{\omega}\right]=\left[\underline{h_1},\overline{h_1}\right]\left[\underline{K_2},\overline{K_2}\right]=\left[\underline{K_3},\overline{K_3}\right]$$

Where $\underline{K_2}=min\{K_2\}$, $\overline{K_2}=max\{K_2\}$, $K_2=\left\{\underline{\omega}\,\underline{\Phi_1},\underline{\omega}\,\overline{\Phi_1},\bar{\omega}\,\underline{\Phi_1},\bar{\omega}\overline{\Phi_1}\right\}$,

$\underline{K_3}=min\{K_3\}$, $\overline{K_3}=max\{K_3\}$, $K_3=\left\{\underline{h_1}\,\underline{K_2},\underline{h_1}\,\overline{K_2},\overline{h_1}\,\underline{K_2},\overline{h_1}\overline{K_2}\right\}$,

$$\left[\underline{\varphi},\bar{\varphi}\right]\left[\underline{\omega},\bar{\omega}\right]=\left[\underline{K_4},\overline{K_4}\right],$$

where $\underline{K_4}=min\{K_4\}$, $\overline{K_4}=max\{K_4\}$, $K_4=\left\{\underline{\omega}\,\underline{\varphi},\underline{\omega}\,\bar{\varphi},\bar{\omega}\,\underline{\varphi},\bar{\omega}\bar{\varphi}\right\}$,

$$\left[\underline{\omega},\bar{\omega}\right]^2\left[\underline{\Phi_2},\overline{\Phi_2}\right]=\left[\underline{K_5},\overline{K_5}\right],$$

where $\underline{K_5}=min\{K_5\}$, $\overline{K_5}=max\{K_5\}$, $K_5=\left\{\underline{\omega}^2\,\underline{\varphi},\underline{\omega}^2\,\bar{\varphi},\bar{\omega}^2\,\underline{\varphi},\bar{\omega}^2\bar{\varphi}\right\}$,

then the interval extension (15) has the form:

$$\left[\underline{u},\bar{u}\right]=\left[\underline{\Phi_1},\overline{\Phi_1}\right]-\left[\underline{K_1},\overline{K_1}\right]-\left[\underline{K_3},\overline{K_3}\right]+\left[\underline{K_4},\overline{K_4}\right]+\left[\underline{K_5},\overline{K_5}\right] \qquad (16)$$

or

$$\left[\underline{u},\bar{u}\right]=\left[\underline{\Phi_1}-\overline{K_1}-\overline{K_3}+\underline{K_4}+\underline{K_5},\overline{\Phi_1}-\underline{K_1}-\underline{K_3}+\overline{K_4}+\overline{K_5}\right] \qquad (17)$$

In the interval extension structure (16), (17), $\left[\underline{\Phi_1},\overline{\Phi_1}\right]$, $\left[\underline{\Phi_2},\overline{\Phi_2}\right]$ – undefined, gap function, D_1 - differential operator of the form (9).

II.4. Mixed boundary conditions. The boundary conditions of this problem are of the form

$$\boldsymbol{u}\Big|_{\Gamma_1}=\boldsymbol{\varphi_0},\qquad \left(\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{n}}+\boldsymbol{h_0}(\boldsymbol{x})\boldsymbol{u}\right)\Big|_{\Gamma_2}=\boldsymbol{\psi_0} \qquad (\mathbf{18})$$

Functions φ_0, $h_0(x)$ and ψ_0 assumed, as previously defined on Γ (possibly different formulas of $\mathfrak{M}$(H) at different parts of the border). Their continuation in Γ denotes $\mathrm{EC}h_0=h_1(x)$, $\mathrm{EC}\varphi_0=\varphi(x)$, $\mathrm{EC}\psi_0=\psi(x)$. Then instead of (23) we obtain

$$u\Big|_{\Gamma_1}=\varphi,\qquad \left(\frac{\partial u}{\partial n}+h_1(x)u\right)\Big|_{\Gamma_2}=\psi \qquad (19)$$

Structure of the solution of the problem (1), (19) the construction of a polynomial of the form R-functions method has the form [1]:

$$u=\omega_1\Phi_1+\frac{\omega_1\omega_2}{\omega_1+\omega_2}\left(\psi+\omega_2\Phi_2-D_1^{(2)}(\omega_1\Phi_1)-D_1^{(2)}\varphi-h_1\omega_1\Phi_1-h_1\varphi\right)+\varphi. \qquad (20)$$

where Φ_1,Φ_2 -undefined function and D_1-differential operator of the form (9), $D_1{}^{(2)}=(\nabla\omega_2,\nabla)$; ω_1,ω_2 – equation boundary. Applying interval arithmetic operations, construct interval extension solution structure (20):

$$\left[\underline{u},\bar{u}\right]=\left[\underline{\omega_1},\overline{\omega_1}\right]\left[\underline{\Phi_1},\overline{\Phi_1}\right]+\frac{\left[\underline{\omega_1},\overline{\omega_1}\right]\left[\underline{\omega_2},\overline{\omega_2}\right]}{\left[\underline{\omega_1},\overline{\omega_1}\right]+\left[\underline{\omega_2},\overline{\omega_2}\right]}\Big(\left[\underline{\psi},\bar{\psi}\right]+\left[\underline{\omega_2},\overline{\omega_2}\right]\times\left[\underline{\Phi_2},\overline{\Phi_2}\right]-D_1^{(2)}\left(\left[\underline{\omega_1},\overline{\omega_1}\right]\left[\underline{\Phi_1},\overline{\Phi_1}\right]\right)-D_1^{(2)}\left[\underline{\varphi},\bar{\varphi}\right]-\left[\underline{h_1},\overline{h_1}\right]\left[\underline{\omega_1},\overline{\omega_1}\right]\times\left[\underline{\Phi_1},\overline{\Phi_1}\right]-\left[\underline{h_1},\overline{h_1}\right]\left[\underline{\varphi},\bar{\varphi}\right]\Big)+\left[\underline{\varphi},\bar{\varphi}\right] \qquad (21)$$

Taking into account that

$$\left[\underline{\omega_1},\overline{\omega_1}\right]\left[\underline{\Phi_1},\overline{\Phi_1}\right]=\left[\underline{Q_1},\overline{Q_1}\right],$$

where $\underline{Q_1}=min\{Q_1\}$, $\overline{Q_1}=max\{Q_1\}$, $Q_1=\left\{\underline{\omega_1}\,\underline{\Phi_1},\underline{\omega_1}\,\overline{\Phi_1},\overline{\omega_1}\,\underline{\Phi_1},\overline{\omega_1}\overline{\Phi_1}\right\}$,

$$\left[\underline{\omega_1},\overline{\omega_1}\right]\left[\underline{\omega_2},\overline{\omega_2}\right]=\left[\underline{q_1},\overline{q_1}\right],$$

where $\underline{q_1}=min\{q_1\}$, $\overline{q_1}=max\{q_1\}$, $q_1=\left\{\underline{\omega_1}\,\underline{\omega_2},\underline{\omega_1}\overline{\omega_2},\overline{\omega_1}\,\underline{\omega_2},\overline{\omega_1\omega_2}\right\}$,

$$\frac{\left[\underline{q_1},\overline{q_1}\right]}{\left[\underline{\omega_1}+\underline{\omega_2},\overline{\omega_1}+\overline{\omega_2}\right]}=\left[\underline{Q_2},\overline{Q_2}\right],$$

where $\underline{Q_2}=min\{Q_2\}$, $\overline{Q_2}=max\{Q_2\}$, $Q_2=\left\{\underline{q_1}\frac{1}{\underline{\omega_1}+\underline{\omega_2}},\underline{q_1}\frac{1}{\overline{\omega_1}+\overline{\omega_2}},\overline{q_1}\frac{1}{\underline{\omega_1}+\underline{\omega_2}},\overline{q_1}\frac{1}{\overline{\omega_1}+\overline{\omega_2}}\right\}$,

$$\left[\underline{\omega_2}, \overline{\omega_2}\right]\left[\underline{\Phi_2}, \overline{\Phi_2}\right] = \left[\underline{Q_3}, \overline{Q_3}\right],$$

Where

$\underline{Q_3} = min\{Q_3\}, \overline{Q_3} = max\{Q_3\}, Q_3 = \left\{\underline{\omega_2}\ \underline{\Phi_2}, \underline{\omega_2}\ \overline{\Phi_2}, \overline{\omega_2}\ \underline{\Phi_2}, \overline{\omega_2}\overline{\Phi_2}\right\},$

$$D_1^{(2)}\left(\left[\underline{\omega_1}, \overline{\omega_1}\right]\left[\underline{\Phi_1}, \overline{\Phi_1}\right]\right) = \left[\underline{\omega_1}, \overline{\omega_1}\right] D_1^{(2)}\left[\underline{\Phi_1}, \overline{\Phi_1}\right] + \left[\underline{\Phi_1}, \overline{\Phi_1}\right] D_1^{(2)}\left[\underline{\omega_1}, \overline{\omega_1}\right] = \left[\underline{Q_4}, \overline{Q_4}\right] + \left[\underline{Q_5}, \overline{Q_5}\right] = \left[\underline{Q_6}, \overline{Q_6}\right],$$

where $\left[\underline{Q_6}, \overline{Q_6}\right] = \left[\underline{Q_4}, \overline{Q_4}\right] + \left[\underline{Q_5}, \overline{Q_5}\right] = \left[\underline{Q_4} + \underline{Q_5}, \overline{Q_4} + \overline{Q_5}\right], \underline{Q_4} = min\{Q_4\},$ $\overline{Q_4} = max\{Q_4\}, Q_4 = \left\{\underline{\omega_1} D_1^{(2)}\ \underline{\Phi_1}, \underline{\omega_1}\ D_1^{(2)}\overline{\Phi_1}, \overline{\omega_1} D_1^{(2)}\ \underline{\Phi_1}, \overline{\omega_1} D_1^{(2)}\overline{\Phi_1}\right\},$ $\underline{Q_5} = min\{Q_5\}, \overline{Q_5} = max\{Q_5\}, Q_5 = \left\{\underline{\Phi_1} D_1^{(2)}\ \underline{\omega_1}, \underline{\Phi_1}\ D_1^{(2)}\overline{\omega_1}, \overline{\Phi_1} D_1^{(2)}\ \underline{\omega_1}, \overline{\Phi_1} D_1^{(2)}\overline{\omega_1}\right\},$

$$\left[\underline{h_1}, \overline{h_1}\right]\left[\underline{\omega_1}, \overline{\omega_1}\right]\left[\underline{\Phi_1}, \overline{\Phi_1}\right] = \left[\underline{Q_7}, \overline{Q_7}\right],$$

where $\underline{Q_7} = min\{Q_7\}, \overline{Q_7} = max\{Q_7\}, Q_7 = \left\{\underline{h_1}\ \underline{Q_1}, \underline{h_1}\ \overline{Q_1}, \overline{h_1}\ \underline{Q_1}, \overline{h_1}\overline{Q_1}\right\},$

$$\left[\underline{h_1}, \overline{h_1}\right]\left[\underline{\varphi}, \overline{\varphi}\right] = \left[\underline{Q_8}, \overline{Q_8}\right],$$

where $\underline{Q_8} = min\{Q_8\}, \overline{Q_8} = max\{Q_8\}, Q_8 = \left\{\underline{h_1}\ \underline{\varphi}, \underline{h_1}\ \overline{\varphi}, \overline{h_1}\ \underline{\varphi}, \overline{h_1}\overline{\varphi}\right\}.$

and obtain the interval extension structure (21) as

$$\left[\underline{u}, \bar{u}\right] = \left[\underline{Q_1}, \overline{Q_1}\right] + \left[\underline{Q_2}, \overline{Q_2}\right]\left(\left[\underline{\psi}, \bar{\psi}\right] + \left[\underline{Q_3}, \overline{Q_3}\right] - \left[\underline{Q_6}, \overline{Q_6}\right] - \left[D_1^{(2)}\ \underline{\varphi}, D_1^{(2)}\bar{\varphi}\right] - \left[\underline{Q_7}, \overline{Q_7}\right] - \left[\underline{Q_8}, \overline{Q_8}\right]\right) + \left[\underline{\varphi}, \bar{\varphi}\right] \quad (22)$$

or

$$\left[\underline{u}, \bar{u}\right] = \left[\underline{Q_1}, \overline{Q_1}\right] + \left(\left[\underline{Q_2}, \overline{Q_2}\right]\left[\underline{\psi}, \bar{\psi}\right] + \left[\underline{Q_2}, \overline{Q_2}\right]\left[\underline{Q_3}, \overline{Q_3}\right] - \left[\underline{Q_2}, \overline{Q_2}\right]\left[\underline{Q_6}, \overline{Q_6}\right] - \left[\underline{Q_2}, \overline{Q_2}\right]\left[D_1^{(2)}\ \underline{\varphi}, D_1^{(2)}\bar{\varphi}\right] - \left[\underline{Q_2}, \overline{Q_2}\right]\left[\underline{Q_7}, \overline{Q_7}\right] - \left[\underline{Q_2}, \overline{Q_2}\right]\left[\underline{Q_8}, \overline{Q_8}\right]\right) + \left[\underline{\varphi}, \bar{\varphi}\right] \quad (23)$$

Taking into account that

$$\left[\underline{Q_2}, \overline{Q_2}\right]\left[\underline{\psi}, \bar{\psi}\right] = \left[\underline{L_1}, \overline{L_1}\right],$$

where $\underline{L_1} = min\{L_1\}, \overline{L_1} = max\{L_1\}, L_1 = \left\{\underline{Q_2}\ \underline{\psi}, \underline{Q_2}\bar{\psi}, \overline{Q_2}\ \underline{\psi}, \overline{Q_2}\bar{\psi}\right\},$

$$\left[\underline{Q_2}, \overline{Q_2}\right]\left[\underline{Q_3}, \overline{Q_3}\right] = \left[\underline{L_2}, \overline{L_2}\right],$$

where $\underline{L_2} = min\{L_2\}, \overline{L_2} = max\{L_2\}, L_2 = \left\{\underline{Q_2}\ \underline{Q_3}, \underline{Q_2}\overline{Q_3}, \overline{Q_2}\ \underline{Q_3}, \overline{Q_2 Q_3}\right\},$

$$\left[\underline{Q_2}, \overline{Q_2}\right]\left[\underline{Q_6}, \overline{Q_6}\right] = \left[\underline{L_3}, \overline{L_3}\right],$$

where $\underline{L_3} = min\{L_3\}, \overline{L_3} = max\{L_3\}, L_3 = \left\{\underline{Q_2}\ \underline{Q_6}, \underline{Q_2}\overline{Q_6}, \overline{Q_2}\ \underline{Q_6}, \overline{Q_2 Q_6}\right\},$

$$\left[\underline{Q_2}, \overline{Q_2}\right]\left[D_1^{(2)}\ \underline{\varphi}, D_1^{(2)}\bar{\varphi}\right] = \left[\underline{L_4}, \overline{L_4}\right],$$

where $\underline{L_4} = min\{L_4\}, \overline{L_4} = max\{L_4\},$ $L_4 = \left\{\underline{Q_2}\ D_1^{(2)}\ \underline{\varphi}, \underline{Q_2} D_1^{(2)}\bar{\varphi}, \overline{Q_2}\ D_1^{(2)}\ \underline{\varphi}, \overline{Q_2} D_1^{(2)}\bar{\varphi}\right\},$

$$\left[\underline{Q_2}, \overline{Q_2}\right]\left[\underline{Q_7}, \overline{Q_7}\right] = \left[\underline{L_5}, \overline{L_5}\right]$$

where $\underline{L_5} = min\{L_5\}, \overline{L_5} = max\{L_5\}, L_5 = \left\{\underline{Q_2}\ \underline{Q_7}, \underline{Q_2}\overline{Q_7}, \overline{Q_2}\ \underline{Q_7}, \overline{Q_2 Q_7}\right\},$

$$\left[\underline{Q_2}, \overline{Q_2}\right]\left[\underline{Q_8}, \overline{Q_8}\right] = \left[\underline{L_6}, \overline{L_6}\right]$$

where $\underline{L_6} = min\{L_6\}, \overline{L_6} = max\{L_6\}, L_6 = \left\{\underline{Q_2}\ \underline{Q_8}, \underline{Q_2}\overline{Q_8}, \overline{Q_2}\ \underline{Q_8}, \overline{Q_2 Q_8}\right\},$

and obtain the interval extension structure (22) as

$$\left[\underline{u}, \bar{u}\right] = \left[\underline{Q_1}, \overline{Q_1}\right] + \left[\underline{L_1}, \overline{L_1}\right] + \left[\underline{L_2}, \overline{L_2}\right] - \left[\underline{L_3}, \overline{L_3}\right] - \left[\underline{L_4}, \overline{L_4}\right] - \left[\underline{L_5}, \overline{L_5}\right] - \left[\underline{L_6}, \overline{L_6}\right] + \left[\underline{\varphi}, \bar{\varphi}\right] \quad (24)$$

or

$$\begin{aligned} \underline{u} &= \left(\underline{Q_1} + \underline{L_1} + \underline{L_2} + \overline{L_3} - \overline{L_4} - \overline{L_5} - \overline{L_6} + \underline{\varphi}\right) \\ \bar{u} &= \left(\overline{Q_1} + \overline{L_1} + \overline{L_2} - \underline{L_3} - \underline{L_4} - \underline{L_5} - \underline{L_6} + \bar{\varphi}\right). \end{aligned} \quad (25)$$

In the interval extension structure (22), (23), (24), $\left[\underline{\Phi_1}, \overline{\Phi_1}\right]$, $\left[\underline{\Phi_2}, \overline{\Phi_2}\right]$ - undefined interval function D_1 - differential operator of the form (9).

In constructing the interval extension of the structure of the formula, we used structural formulas with the R-function method and 4 problems studied - the Dirichlet problem, the Neumann problem, the third type problem, the mixed boundary conditions problem, and the Sol2Interval software library in C ++ was created. This billetka makes it possible to construct an interval 3D graph for interval expansion of structures (Fig-1).

There are three main areas for the successful use of interval analysis and interval methods on the basis of the Sol2Interval software library:

• Solving practical problems that have interval or, more generally, limited data uncertainty.

• Strict accounting of rounding errors in calculations with floating point numbers on digital computers.

• New approaches to solving traditional mathematical problems (such as, for example, the global optimization problem, global evidence-based solution of systems of nonlinear equations, etc.)

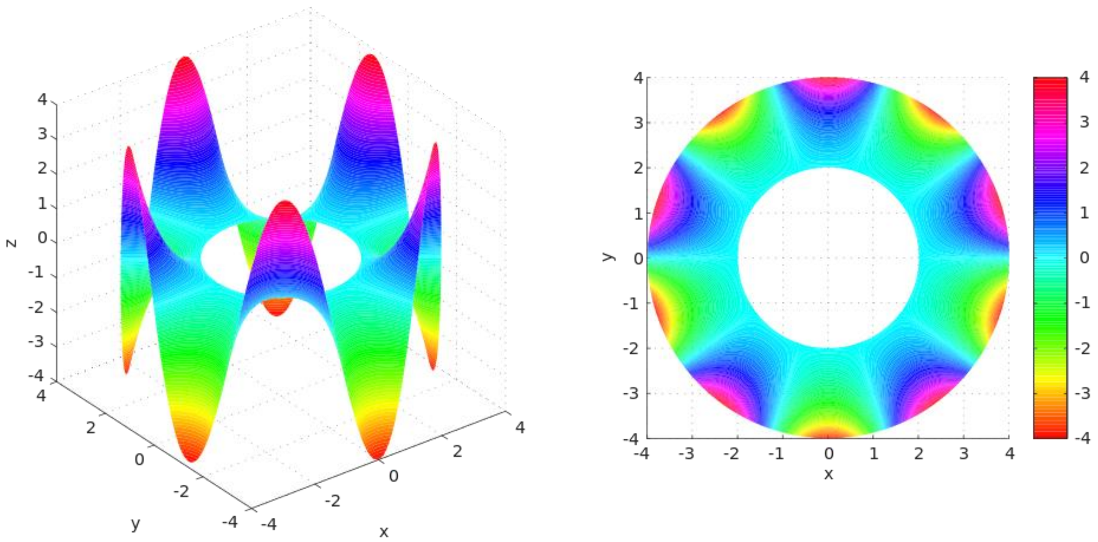

Figure 1. Graphic solutions for interval expansion of structures.

Conclusion

Built interval extension structure basic types of solutions of boundary value problems - Dirichlet problem, Neumann problem of the third type, mixed boundary conditions for partial differential equations of second order. The results can be used to construct interval - value structure solutions for other boundary value problems of differential equations of different order. The resulting interval - values the solution structure is easily implemented on computers.

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