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THE SOLUTION EXISTENCE AND UNIQUENESS OF THE NONLOKAL BOUNDARY VALUE PROBLEM WITH INTEGRAL CONDITION

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INTEGRAL SHARTLI NOLOKAL CHEGARAVIY MASALA YECHIMINING MAVJUDLIGI VA YAGONALIGI

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Maqolada aralash-parabolik tipdagi tenglama uchun integral shartli nolokal chegaraviy masala o‘rganilgan. Qo‘yilgan masala yechimining mavjudligi va yagonaligi isbotlangan. Integral shartli masala yechimining yagonaligini isbotlashda parabolik tipdagi tenglamalar uchun maksimum prinsipi qo‘llangan. Integral shartli masala yechimining mavjudligi Fredgolm 2-tur integral tenglama yechimining mavjudligiga ekvivalent ravishda bog‘lash bilan isbotlangan.

Kalit so‘zlar: *aralash tipdagi tenglama, buzilish chizig‘i, chegaraviy masala, nolokal chegaraviy masala, integral shartli masala, regulyar yechim, integral tenglama.*

В статье изучена нелокальная краевая задача с интегральным условием для уравнения смешанно-параболического типа. Доказано существование и единственность решения поставленной задачи. К доказательству единственности решения задачи с интегральным условием используется принцип максимума для уравнения параболического типа. А существование решения доказано эквивалентным сведением к существованию решения интегрального уравнения Фредгольма 2 – го рода.

Ключевые слова: *уравнения смешанного типа, линия вырождения, краевая задача, нелокальная краевая задача, задача с интегральным условием, регулярное решение, интегральное уравнение.*

Kirish. Aralash-parabolik tipdagi tenglamalar nazariyasi amaliy masalalarni hal etishdagi ahamiyati bilan zamonaviy xususiy hosilali differensial tenglamalar nazariyasining muhim bo‘limi hisobalanadi.

Aralash-parabolik tipdagi tenglamalar, ayniqsa, gidrodinamikada tatbiq etiladi, ya’ni ishorasi o‘zgaruvchan koeffitsiyentli suyuqlik harakatini o‘rganishda qo‘llanadi.

1960 - 1970 yillarda J.Canon [1], N.I.Ionkin [2]larning ishlarida

$$u_{xx} - u_t = 0$$

- issiqlik tarqalish tenglamasi uchun quyidagi

$$\int_0^1 u(x,t)dx = \mu(t)$$

integral shartli masala o‘rganilgan, bu yerda $\mu(t)$ - berilgan funksiya. Bunday shartlar, masalan, sterjenda (yoki sterjen qismida) issiqlikning umumiy miqdori ma’lum deb hisoblanganda, ingichka sterjenni isitishda issiqlik jarayonini o‘rganishda kelib chiqadi.

Mazkur yo‘nalish bo‘yicha masalalar umumiy chiziqli bir o‘lchovli parabolik tenglama kabi, sistema yoki ikki o‘lchamli hol uchun bir qancha olimlar tomonidan o‘rganilgan.

M.X.Akbarovaning ilmiy tadqiqot ishlarida [3-4]

$$u_{xx} - \operatorname{sgn} x u_t = f(x,t)$$

ko‘rinishdagi aralash-parabolik tipdagi model tenglama uchun integral shartli masalalar tadqiq etilgan.

Asosiy qism. R^2- tekislikda $x>0$da $t=0,\ \ x=1,\ \ t=T$ to‘g‘ri chiziqlar va $x<0$ da $t=0,\ x=-1,\ t=T$ to‘g‘ri chiziqlar bilan chegaralangan $D = D^+ \cup D^- \cup I$ - sohani qaraymiz, bu yerda

$$D^+ = \Big\{(x,t): 0<x<1, 0<t\le T\Big\},\ D^- = \Big\{(x,t): -1<x<0, 0<t\le T\Big\},$$

$$I = \Big\{(x,t): x=0, 0<t<T\Big\};$$

Belgilashlar kiritamiz:

$$I^+ = \Big\{(x,t): 0\le x\le 1, t=0\Big\},\ I^- = \Big\{(x,t): -1\le x\le 0, t=T\Big\},$$

$J^{+}=\{(x,t): x=1, 0\le t\le T\}$, $J^{-}=\{(x,t): x=-1, 0\le t\le T\}$.

Quyidagi aralash-parabolik tipdagi tenglamani qaraymiz:

$$\operatorname{sgn} x u_t - a^2 u_{xx} = 0. \quad (1)$$

Bu yerda $a^2 = const > 0$ bo‘lib, (1) tenglama D^{+}, D^{-} - sohalarda parabolik tipga tegishli [5. B. 181].

Ta’rif. (1) tenglamaning D sohadagi regulyar yechimi deb, D^{+}, D^{-} sohalarda bu tenglamani qanoatlantiruvchi, $C^{2,1}(D)$ sinfda yotuvchi $u(x,t)$ funksiyani ataymiz.

I masala. Quyidagi xossalarga ega $u(x,t)$ funksiyani aniqlang:

1) $u(x,t)\in C(\overline{D})\cap C^1(D\cup I^{+}\cup I^{-})$; 2) $u(x,t)$ - D / I sohada (1) tenglamaning regulyar yechimi; 3) $u(x,t)$ quyidagi shartlarni qanoatlantiradi:

$$u\Big|_{I^{+}} = \phi_1(x),\ x\in I^{+}, \quad (2)$$

$$u\Big|_{I^{-}} = \phi_2(x),\ x\in I^{-}, \quad (3)$$

$$\int_0^1 u(x,t)dx = \psi_1(t),\ \ t\in J^{+}, \quad (4)$$

$$u(-1,t) = \psi_2(t),\ \ t\in J^{-}, \quad (5)$$

bu yerda $\phi_i(x), \psi_i(t) (i=1,2)$ - berilgan funksiyalar bo‘lib,

$$\phi_i(x)\in C[0,1]\cap C^2(0,1), \quad (6)$$

$$\psi_1(t)\in C[0,T]\cap C^1(0,T], \quad (7)$$

$$\psi_2(t)\in C[0,T]\cap C^2(0,T], \quad (8)$$

$$\int_0^1 \phi_1(x)dx = \psi_1(0),\ \ \phi_2(-1) = \psi_2(T). \quad (9)$$

Teorema. (6) - (9) shartlar bajarilsin. U holda I masala yechimi mavjud va yagona.

Isbot. **I masala yechimining yagonaligi.** Ko‘rsatamizki, (1) tenglama uchun

$$u\Big|_{I^{+}} = 0,\ u\Big|_{I^{-}} = 0, \quad (10)$$

$$\int_0^1 u(x,t)dx = 0,\ \ u(-1,t) = 0, \quad (11)$$

shartli bir jinsli I masala noldan farqli yechimga ega emas.

Aralash parabolik tipdagi tenglamalar uchun ekstremum prinsipiga asosan [6. B. 69-77], $\overline{D}$ - sohaning $I^{+}\cup J^{+}\cup I^{-}\cup J^{-}$ chegarasida (1) tenglamaning $u(x,t)$ yechimi o‘zining maksimum yoki minimumiga erishishi mumkin.

Faraz qilamiz, biror $(x_0,t_0)\in J^{+}$ nuqtada $u(x,t)$ funksiya maksimumga erishsin:

$$u(x_0,t_0) = u(1,t_0) = \max_{D} u(x,t) = \max_{D^{+}} u(x,t).$$

Parabolik tenglamalar uchun urinma hosila ishorasi haqidagi teoremaga ko‘ra [7. B. 51], [8. B. 12]

$$u_x(1,t_0) > 0. \quad (12)$$

(1) tenglamani $x>0$ da qaraymiz: $u_t - a^2 u_{xx} = 0$. Bu yerdan quyidagini hosil qilamiz:

$$a^2\int_0^1 u_{xx}(x,t)dx-\int_0^1 u_t(x,t)dx=0,$$

yoki (11) shartni hisobga olgan holda,

$$u_x(1,t)=u_x(0,t)$$

tenglikni olamiz. (12) shartga ko‘ra,

$$u_x(0,t_0)>0,$$

bu esa parabolik tipdagi tenglamalar urinma hosila ishorasi haqidagi teoremaga zid, ya’ni bu nuqtada quyidagi tengsizlik o‘rinli

$$u_x(0,t_0)<0.$$

$u(x,t)$ funksiya minimumga erishgan holda, xuddi yuqoridagidek, zidlikka duch kelamiz.

Shunday qilib, (1), (10), (11) – masalaning $u(x,t)$ yechimi J^+ - da ekstremumga erishmaydi.

(10) va (11) shartlarga ko‘ra, $u(x,t)=0$. Bu yerdan D - sohada

$$u(x,t)\equiv 0$$

tenglikka ega bo‘lamiz. Shunday qilib, I masala yechimining yagonaligi isbotlandi.

Quyidagi ekstremum prinsipi o‘rinli. I masalaning $u(x,t)$ yechimi

$\phi_i(x)=0,\ \ \psi_i(t)=0,\ \ (i=1,2)$ bo‘lgan holda, $\overline{D}$ - sohada musbat maksimum va manfiy minimum qiymatlarga bu soha chegarasining $I^+\cup I^-\cup J^-$ qismida erishishi mumkin.

I masala yechimining mavjudligi.

Begilashlar kiritamiz:

$$u(0,t)=\tau(t), t\in\overline{I}, \text{ (13)}$$
$$u(1,t)=\mu(t),\ 0\le t\le T. \text{ (14)}$$

(1) tenglamaning D^+ sohadagi (2), (13), (14) shartlarni qanoatlantiruvchi yechimi quyidagicha[3. B. 6-9], [8. B. 66-67]:

$$u(x,t)=\int_0^t G_\xi^+(x,t;0,\eta)\tau(\eta)d\eta-\int_0^t G_\xi^+(x,t;1,\eta)\mu(\eta)d\eta+$$
$$+\int_0^1 G^+(x,t;\xi,0)\phi_1(\xi)d\xi,\ (x,t)\in D^+, \text{ (15)}$$

Bu yerda

$$G^+(x,t;\xi,\eta)=$$
$$=\frac{1}{2\sqrt{\pi a^2(t-\eta)}}\sum\left\{e^{-\frac{\left(x-\xi-2n\right)^2}{4a^2(t-\eta)}}-e^{-\frac{\left(x+\xi-2n\right)^2}{4a^2(t-\eta)}}\right\}$$

- Grin funksiyasi.

(1) tenglamaning D^- sohadagi (3), (5), (13) shartlarni qanoatlantiruvchi yechimi esa

$$u(x,t)=\int_t^T G_\xi^-(x,t;0,\eta)\tau(\eta)d\eta-\int_t^T G_\xi^-(x,t;-1,\eta)\psi_2(\eta)d\eta+$$
$$+\int_{-1}^0 G^-(x,t;\xi,T)\phi_2(\xi)d\xi,\ (x,t)\in D^- \text{ (16)}$$

ko‘rinishda ifodalanadi, bu yerda $G^{-}(x,t;\xi,\eta)=G^{+}(-x,1-t;-\xi,1-\eta)$ - Grin funksiyasi.

(15) ifoda asosida (4) shartdan foydalanib, quyidagi tenglikni olamiz:

$$\int_0^1 dx\int_0^t G_{\xi}^{+}(x,t;0,\eta)\tau(\eta)d\eta-\int_0^1 dx\int_0^t G_{\xi}^{+}(x,t;1,\eta)\mu(\eta)d\eta+$$

$$+\int_0^1 dx\int_0^1 G^{+}(x,t;\xi,0)\phi_1(\xi)d\xi=\psi_1(t),$$

bu yerdan

$$\int_0^t \frac{\tau(\eta)+\mu(\eta)}{\sqrt{\pi a^2(t-\eta)}}d\eta-\int_0^t K(t,\eta)[\tau(\eta)+\mu(\eta)]d\eta=$$

$$=\psi_1(t)-\int_0^1 G^{+}(x,t;\xi,0)\phi_1(\xi)d\xi,$$

yoki

$$\mu^{+}(t)-\int_0^t K^{+}(t,\eta)\mu^{+}(\eta)d\eta=f_1(t),\ (17)$$

bu yerda

$$\mu^{+}(t)=\tau(t)+\mu(t),\ (18)$$

$$K^{+}(t,\eta)=\frac{1}{\sqrt{\pi a^2}}\int_{\eta}^{t}\frac{K'(t,s)}{\sqrt{t-s}}ds,\ (!9)$$

$$K(t,\eta)=\frac{1}{\sqrt{\pi a^2(t-\eta)}}\sum_{n=1}^{+\infty}\left\{e^{-\frac{(2n-1)^2}{4a^2(t-\eta)}}-e^{-\frac{n^2}{a^2(t-\eta)}}\right\},\ (20)$$

$$f_1(t)=\frac{1}{\sqrt{\pi a^2}}\frac{d}{dt}\int_0^t\left\{\psi_1(t)-\int_0^1 G^{+}(x,t;\xi,0)\phi_1(\xi)d\xi\right\}(t-s)^{-1/2}ds.\ (21)$$

(17) – uzluksiz yadroli Volterra 2-tur integral tenglamasidir [9. B. 222-225].

Endi (15) dan x bo‘yicha hosila olamiz, u holda

$$\frac{\partial u}{\partial x}=\frac{\partial u_0}{\partial x}+\int_0^t \tau(\eta)\frac{\partial}{\partial x}g^{+}(x,t,\eta)d\eta-$$

$$-\int_0^t G_{\xi x}^{+}(x,t;1,\eta)\mu(\eta)d\eta+\int_0^1 G_{x}^{+}(x,t;\xi,0)\phi_1(\xi)d\xi,\ (x,t)\in D^{+}.$$

Bu tenglikdan $x\to+0$ da limitga o‘tsak, $\tau(t)$ va $\nu(t)=\lim\limits_{x\to 0}\frac{\partial u}{\partial x}$ orasidagi D^{+} sohadan olingan funksional munosabatni hosil qilamiz:

$$\nu(t)=-\frac{1}{a^2\sqrt{\pi a^2}}\int_0^t\frac{\tau'(\eta)}{\sqrt{(t-\eta)}}d\eta-\int_0^t K_0^{+}(t,\eta)\tau'(\eta)d\eta-$$

$$-\int_0^t G_{\xi x}^{+}(x,t;1,\eta)\mu(\eta)d\eta+\int_0^1 G_{x}^{+}(x,t;\xi,0)\phi_1(\xi)d\xi.\ (22)$$

bu yerda

$$K_0^+(t,\eta)=\frac{2}{a^2\sqrt{\pi a^2(t-\eta)}}\sum_{n=1}^{+\infty}e^{-\frac{n^2}{4a^2(t-\eta)}},\ 0<\eta<t. \quad (23)$$

Xuddi shunga o‘xshash, D^- sohadan ushbu munosabatni olish mumkin

$$\nu(t)=\frac{1}{a^2\sqrt{\pi a^2}}\int_t^T\frac{\tau'(\eta)}{\sqrt{\eta-t}}d\eta+\int_t^T K_0^-(\eta,t)\tau'(\eta)d\eta-$$

$$-\int_t^T G_{\xi x}^-(x,t;-1,\eta)\psi_2(\eta)d\eta+\int_{-1}^0 G_x^-(x,t;\xi,T)\phi_2(\xi)d\xi, \quad (24)$$

bu yerda

$$K_0^-(\eta,t)=\frac{2}{a^2\sqrt{\pi a^2(\eta-t)}}\sum_{n=1}^{+\infty}e^{-\frac{n^2}{4a^2(\eta-t)}},\ 0<t<\eta. \quad (25)$$

Endi (22), (24) dan $\nu(t)$ ni yo‘qotib, quyidagini olamiz

$$\frac{1}{a^2\sqrt{\pi a^2}}\int_0^t\frac{\tau'(\eta)}{\sqrt{t-\eta}}d\eta+\frac{1}{a^2\sqrt{\pi a^2}}\int_t^T\frac{\tau'(\eta)}{\sqrt{\eta-t}}d\eta+$$

$$+\int_0^t K_0^+(t,\eta)\tau'(\eta)d\eta+\int_t^T K_0^-(\eta,t)\tau'(\eta)d\eta+$$

$$+\int_0^t G_{\xi x}^+(x,t;1,\eta)\mu(\eta)d\eta=\int_t^T G_{\xi x}^-(x,t;-1,\eta)\psi_2(\eta)d\eta-$$

$$-\int_0^1 G_x^+(x,t;\xi,0)\phi_1(\xi)d\xi-\int_{-1}^0 G_x^-(x,t;\xi,T)\phi_2(\xi)d\xi. \quad (26)$$

Shunday qilib, $\tau(t),\mu(t)$- noma’lum funksiyalarni aniqlash uchun (17), (26) integral tenglamalar sistemasini hosil qildik.

(17) Volterra 2-tur integral tenglamasini yechib, (19) - (21) lar asosida, $\mu^+(t)\in C[0,T]\cap C^1(0,T)$ yechimini aniqlaymiz:

$$\mu^+(t)=f_1(t)+\int_0^t R(t,\eta)f_1(\eta)d\eta, \quad (27)$$

bu yerda $R(t,\eta)$- $K^+(t,\eta)$ yadroning rezolventasi.

(27) da $\mu^+(t)$ ning qiymatini (18) ga qo‘yib, quyidagini aniqlaymiz:

$$\mu(t)=\mu^+(t)-\tau(t).$$

Bu qiymatni (26) ifodaga qo‘ysak,

$$\frac{1}{a^2\sqrt{\pi a^2}}\int_0^t\frac{\tau'(\eta)}{\sqrt{(t-\eta)}}d\eta+\frac{1}{a^2\sqrt{\pi a^2}}\int_t^T\frac{\tau'(\eta)}{\sqrt{\eta-t}}d\eta+$$

$$+\int_0^t K_0^+(t,\eta)\tau'(\eta)d\eta - \int_t^T K_0^-(t,\eta)\tau'(\eta)d\eta +$$

$$+\int_0^t G_{\xi x}^+(x,t;1,\eta)\left[\mu^+(\eta)-\tau(\eta)\right]d\eta = \int_t^T G_{\xi x}^-(x,t;-1,\eta)\psi_2(\eta)d\eta -$$

$$-\int_0^1 G_x^+(x,t;\xi,0)\phi_1(\xi)d\xi - \int_{-1}^0 G_x^+(x,t;\xi,T)\phi_2(\xi)d\xi,$$

ya'ni

$$\frac{1}{a^2\sqrt{\pi a^2}}\int_0^t \frac{\tau'(\eta)}{\sqrt{t-\eta}}d\eta + \frac{1}{a^2\sqrt{\pi a^2}}\int_t^T \frac{\tau'(\eta)}{\sqrt{\eta-t}}d\eta +$$

$$+\int_0^T K_1(t,\eta)\tau'(\eta)d\eta + \int_0^t K_2(t,\eta)\tau(\eta)d\eta = f_2(t),$$

yoki

$$\frac{1}{a^2\sqrt{\pi a^2}}\int_0^t \frac{\tau'(\eta)}{\sqrt{t-\eta}}d\eta + \frac{1}{a^2\sqrt{\pi a^2}}\int_t^T \frac{\tau'(\eta)}{\sqrt{\eta-t}}d\eta +$$

$$+\int_0^T K_1(t,\eta)\tau'(\eta)d\eta + \int_0^T K_2^0(t,\eta)\tau(\eta)d\eta = f_2(t), \quad (28)$$

tenglama hosil bo'ladi, bu yerda

$$K_1(t,\eta) = \begin{cases} K_0^+(t,\eta), 0<\eta<t, \\ K_0^-(\eta,t), t<\eta<T, \end{cases}$$

$$K_2(t,\eta) = -G_{\xi x}^+\left(0,t;1,\eta\right), 0<\eta<t,$$

$$K_2^0(t,\eta) = \begin{cases} -G_{\xi x}^+\left(0,t;1,\eta\right), 0<\eta<t, \\ 0, t<\eta<T, \end{cases}$$

$$f_2(t) = \int_t^T G_{\xi x}^-(x,t;-1,\eta)\psi_2(\eta)d\eta -$$

$$-\int_0^1 G_x^+(x,t;\xi,0)\phi_1(\xi)d\xi - \int_{-1}^0 G_x^+(x,t;\xi,T)\phi_2(\xi)d\xi -$$

$$-\int_0^t G_{\xi x}^+(x,t;1,\eta)\mu^+(\eta)d\eta.$$

Endi quyidagi

$$\rho(t) = \frac{1}{\sqrt{T-t}} D_{0t}^{-1/2}\tau'(t)$$

belgilashni kiritib, va ushbu

$$\int_0^T K_1(t,\eta) D_{0\eta}^{1/2} \sqrt{T-\eta}\rho(\eta)d\eta = \int_0^T \sqrt{T-\eta}\rho(\eta) D_{0\eta}^{1/2} K_1(t,\eta)d\eta,$$

$$\int_0^T K_2^0(t,\eta)\tau(\eta)d\eta = \int_0^T \sqrt{T-\eta}\rho(\eta)\overline{K_2^0}(t,\eta)d\eta$$

tengliklarni hisobga olgan holda, (28) dan

$$\rho(t) + \frac{1}{\sqrt{\pi}} \int_0^T \frac{\rho(\eta)}{\eta - t} d\eta + \frac{1}{\sqrt{T-t}} \int_0^T \rho(\eta) K(t,\eta) d\eta =$$

$$= f(t), \quad (29)$$

integral tenglamani hosil qilamiz, bu yerda

$$K(t,\eta) = \sqrt{T-\eta}\left\{ D_{\eta 1}^{1/2} K_1(t,\eta) + \overline{K_2^0}(t,\eta) \right\},\ f(t) = \frac{1}{\sqrt{T-t}} f_2(t)$$

bo‘lib, $\left|\overline{K_2^0}(t,\eta)\right| \le const,\ f_2(t) \in C\left[0,T\right]$.

Bu tenglama $\rho(t)$ noma’lum funksiyaga nisbatan normal tipdagi, nol indeksli singulyar integral tenglamasidir. Singulyar integral tenglamalar nazariyasiga asosan, (29) tenglama Karleman – Vekua usuli yordamida Fredgolm 2-tur integral tenglamasiga keltiriladi [10. B. 200-214]. Olingan integral tenglamaning yechimga ega ekanligi I masala yechimining yagonaligidan kelib chiqadi.

I masalaning $\overline{D}$ - sohadagi regulyar yechimi, (1) tenglama uchun mos ravishda D^+, D^-- sohalardagi 1-chegaraviy masalalar yechimi sifatida aniqlanadi.

1-eslatma. I masalada (5) shartni quyidagi

$$\int_{-1}^0 u(x,t)dx = \psi_2(t),\ \ 0 \le t \le T$$

shart bilan almashtirish mumkin.

2-eslatma. (1) tenglama uchun $a^2 = 1$ bo‘lgan holda integral shartli masala [3] ishda o‘rganilgan.

Xulosa. Ushbu ishda aralash tipdagi tenglamalar nazariyasida fizik, biologik va boshqa jarayonlarni o‘rganishda katta ahamiyatga ega bo‘lgan aralash-parabolik tipdagi tenglamalar uchun chekli sohada integral shartli masala o‘rganildi. Masala yechimining mavjudligi va yagonaligi isbotlandi.

Bunda issiqlik tarqalish tenglamasi uchun 1-chegraviy masalalaning Grin funksiyasi va uning xossalaridan keng foydalanildi.

Parabolik tipdagi tenglamalar sterjenda issiqlik tarqalish jarayonini o‘rganishda muhim hisoblanadi. Shu sababli bunday tenglamalar uchun lokal chegaraviy masalalar bilan bir qatorda nolokal chegaraviy masalalarni o‘rganish ahamiyatlidir.

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Key words: equation of mixed type, lines of degenerasy, boundary value problem, nonlocal boundary value problem, problem with integral condition, regular solution, integral equation.

The theory of the equations of mixed parabolic type is one of the most important parts of the theory of differensial equations being important on solving practical problems.

The equations of a mixed parabolic type, in particular, are applied in hydrodynamics, when studying the motion of a fluid with an alternating coefficient

The nonlocal boundary value problem with integral condition for equation of mixed parabolic type is studied in this paper. Existence and uniqueness of the solution of the problem are proved.

In the limited with right quadrilateral domain $D = D^{+} \cup D^{-} \cup I$ of the plain $(x,t) \in R^{2}$, we consider the equation of mixed parabolic type:

$$\operatorname{sgn} x u_{t} - a^{2} u_{xx} = 0, \quad (1)$$

where D^{+} - parabolic part, D^{-} this is – inverse parabolic part of domain D;

$$I = \left\{(x,t) : x = 0, 0 < t < 1\right\};\ a^{2} = const > 0.$$

Use the following notation:

$$I^{+} = \left\{(x,t) : 0 \le x \le 1, t = 0\right\},\ I^{-} = \left\{(x,t) : -1 \le x \le 0, t = 1\right\},$$

Definition. A regular solution of the equation (1) at domain D / I is dedicated function $u(x,t)$, which satisfies this equation at the domains D^{+}, D^{-} and be in class $C^{2,1}(D / I)$.

Problem I. Find a function $u(x,t)$ with the following properties:

1) $u(x,t) \in C(\overline{D}) \cap C^{1}(D \cup I^{+} \cup I^{-});$ 2) $u(x,t)$ is a regular solution of the equation (1) at D / I; 3) $u(x,t)$ satisfies the conditions

$$u\Big|_{I^{+}} = \phi_{1}(x),\ x \in I^{+},\ (2)\quad u\Big|_{I^{-}} = \phi_{2}(x),\ x \in I^{-},\ (3)$$

$$\int_{0}^{1} u(x,t)dx = \psi_{1}(t),\ \ t \in J^{+}, (4)\quad u(-1,t) = \psi_{2}(t),\ \ t \in J^{-},\ (5)$$

where $\phi_{i}(x), \psi_{i}(t) (i = 1,2)$ - given sufficiently smooth functions,

$$\phi_i(x) \in C[0,1] \cap C^2(0,1), \text{ (6) } \psi_1(t) \in C[0,T] \cap C^1(0,T], \text{ (7) } \psi_2(t) \in C[0,T] \cap C^2(0,T], \text{ (8)}$$

$$\int_0^1 \phi_1(x)dx = \psi_1(0), \ \phi_2(-1) = \psi_2(T). \text{ (9)}$$

Th e o r e m. Let the conditions (6) - (9) be met. The solution of the problem I existence and uniqueness.

To prove the uniqueness of the solution of the problem with integral condition, use the principle of maximumes: the solution $u(x,t)$ of the problem I at $\phi_i(x) = 0, \ \psi_i(t) = 0, \ (i = 1,2)$ positive maximum and negative minimum in the domain $\overline{D}$ reaches only part of the boundary of this domain.

The existence of the solution is proved by equivalent reduction to an existence of a solution of Fredholm integral equation of the second kind, wich will be uniquely solvable due to the uniqueness of the solution of the problem.

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