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DESCRIPTION OF 2-LOCAL DERIVATIONS ON AN ALGEBRA OF MATRICES

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MATRITSALAR ALGEBRASIDA 2-LOKAL KO‘PAYTIRISHLARNING TAVSIFI

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Maqolada bir qator halqalarda aniqlangan har qanday 2-lokal chapdan ko‘paytirish chapdan ko‘paytirish bo‘lishi, ya’ni bir qator $\mathfrak{R}$ halqalarda shunday $a \in \mathfrak{R}$ element mavjud bo‘lib, ixtiyoriy $x \in \mathfrak{R}$ element uchun $\varphi(x) = ax$ tenglik o‘rinli bo‘lishi isbotlangan. Shu bilan birga, bir qator Yordan halqalarida aniqlangan har qanday 2-lokal Yordan ko‘paytirishi Yordan ko‘paytirishi bo‘lishi, ya’ni bir qator J Yordan halqalarida shunday $a \in J$ element mavjud bo‘lib, ixtiyoriy $x \in J$ element uchun $\varphi(x) = ax$ tenglik o‘rinli bo‘lishi isbotlangan.

Kalit so‘zlar: *assotsiativ halqa, chapdan ko‘paytirish, Yordan halqasi, Yordan ko‘paytirishi, 2-lokal akslantirish.*

В статье доказано, что 2-локальное левое умножение ряда ассоциативных колец является левым умножением, т.е. для всякого 2-локального левого умножения ряда колец $\mathfrak{R}$ существует такой элемент $a \in \mathfrak{R}$, что для всякого элемента $x \in \mathfrak{R}$ имеет место равенство $\varphi(x) = ax$. Кроме того, доказано, что всякое 2-локальное йорданово умножение ряда йордановых колец является йордановым умножением, т.е. для всякого 2-локального йорданова умножения ряда йордановых колец J существует такой элемент $a \in J$, что для всякого элемента $x \in J$ имеет место равенство $\varphi(x) = ax$.

Ключевые слова: *ассоциативное кольцо, левое умножение, йорданово кальцо, йорданово умножение, 2-локальное отображение.*

Kirish

Maqola assotsiativ va Jordan matritsalar halqasi ustida 2-lokal differensiallashlarning o‘xshashlarini o‘rganishga bag‘ishlangan. Ma’lumki, 2-lokal differensiallash quyidagicha kiritiladi: $\mathcal{R}$ halqa olamiz, $\Delta: \mathcal{R} \to \mathcal{R}$ akslantirish (umuman olganda additiv bo‘lmagan) 2-lokal differensiallash deb ataladi, agar ixtiyoriy $x, y \in \mathcal{R}$ uchun ularga bog‘liq bo‘lgan shunday $D_{x,y}: \mathcal{R} \to \mathcal{R}$ differensiallash mavjud bo‘lsaki, $\Delta(x) = D_{x,y}(x)$ va $\Delta(y) = D_{x,y}(y)$ tengliklar bajarilsa.

1997 yilda P.Semrl [1] 2-lokal differensiallash tushunchasini kiritgan va cheksiz o‘lchovli H separabl Gilbert fazosi ustida aniqlangan barcha chegaralangan chiziqli operatorlarning $B(H)$ algebrasida har qanday 2-lokal differensiallash differensiallash bo‘lishini ko‘rsatgan. Keyinroq [2] maqolada chekli o‘lchovli H Gilbert fazosida aniqlangan $B(H)$ algebra uchun ham shunday natija olingan [3]. Maqolada esa chekli o‘lchovli butunlik halqalari ustida aniqlangan matritsalar halqasida har qanday 2-lokal differensiallashning differensiallash bo‘lishi ko‘rsatilgan [4]. Maqolada mualliflar yangi isbotlash usulini ishlab chiqib, Gilbert fazolari uchun yuqorida aytib o‘tilgan [1] va [2] maqolalarning natijalarini umumlashtirishgan. Ya’ni ular ixtiyoriy olingan (separabellik talab etilmaydi) H Gilbert fazosi ustida aniqlangan barcha chiziqli operatorlar $B(H)$ algebrasida 2-lokal differensiallashlarni o‘rganishgan va $B(H)$ ustidagi har qanday 2-lokal differensiallash differensiallash bo‘lishini isbotlashgan [5, 6]. Maqolalarda mualliflar oldingi natijalarni kengaytirishgan va ixtiyoriy fon Neyman algebralari uchun teoremaning isbotini berishgan.

Ushbu maqolada ixtiyoriy halqani o‘zini-o‘ziga akslantiruvchi 2-lokal chapdan ko‘paytirish tushunchasi kiritilgan va o‘rganilgan. Bu tushuncha quyidagicha kiritiladi: aytaylik $\mathfrak{R}$ – ixtiyoriy halqa bo‘lsin. U holda $\mathfrak{R}$ halqani o‘zini-o‘ziga akslantiruvchi φ akslantirish uchun har qanday $x, y \in \mathfrak{R}$ elementlar uchun shunday $a \in \mathfrak{R}$ element mavjud bo‘lib, $\varphi(x) = ax$, $\varphi(y) = ay$ shartlar bajarilsa, φ 2-lokal chapdan ko‘paytirish deb ataladi.

Maqolada bir qator $\mathfrak{R}$ halqalarda aniqlangan har qanday φ 2-lokal chapdan ko‘paytirish chapdan ko‘paytirish bo‘lishi, ya’ni shunday $a \in \mathfrak{R}$ element mavjud bo‘lib, ixtiyoriy $x \in \mathfrak{R}$ element uchun $\varphi(x) = ax$ tenglik o‘rinli bo‘lishi isbotlangan. Bundan tashqari, ixtiyoriy Yordan halqasini o‘zini-o‘ziga akslantiruvchi 2-lokal Yordan ko‘paytirish tushunchasi kiritilgan va o‘rganilgan. Ushbu tushuncha quyidagicha kiritiladi: aytaylik J – ixtiyoriy Yordan halqasi bo‘lsin. U holda J Yordan halqasini o‘zini-o‘ziga akslantiruvchi φ akslantirish uchun har qanday $x, y \in J$ elementlar uchun shunday $a \in J$ element mavjud bo‘lib, $\varphi(x) = ax$, $\varphi(y) = ay$ shartlar bajarilsa, u holda φ 2-lokal Jordan ko‘paytirishi deb ataladi.

Maqolada bir qator $\mathfrak{R}$ Yordan halqalarida aniqlangan har qanday φ 2-lokal Yordan ko‘paytirishi Yordan ko‘paytirishi bo‘lishi, ya’ni shunday $a \in J$ element mavjud bo‘lib, ixtiyoriy $x \in J$ element uchun $\varphi(x) = ax$ tenglik o‘rinli bo‘lishi isbotlangan.

1. 2-lokal chapdan ko‘paytirishlar

Ta’rif. Aytaylik, $\mathcal{R}$ – assotsiativ halqa bo‘lsin. Akslantirish $\varphi: \mathcal{R} \to \mathcal{R}$ har qanday $x, y \in \mathcal{R}$ uchun shunday $a \in \mathcal{R}$ topilib $\varphi(x) = ax$ va $\varphi(y) = ay$ shartlarni qanoatlantirsa, u holda φ 2–lokal chapdan ko‘paytirish deyiladi.

Teorema 1.1. Aytaylik, $\mathcal{R}$–birlik elementli halqa va $\varphi: \mathcal{R} \to \mathcal{R}$– 2-lokal chapdan ko‘paytirish bo‘lsin. U holda shunday $a \in \mathcal{R}$ topiladiki,

$$\varphi(x) = ax \ \forall x \in \mathcal{R}$$

tenglik o‘rinli bo‘ladi.

Isbot. Ixtiyoriy $x \in \mathcal{R}$ va birlik element $e \in \mathcal{R}$ uchun, ta’rifga ko‘ra, shunday $a \in \mathcal{R}$ mavjudki,

$$\varphi(x) = ax \ \varphi(e) = ae = a$$

O‘z navbatida, ixtiyoriy $y \in \mathcal{R}$ element va $e \in \mathcal{R}$ uchun shunday $b \in \mathcal{R}$ topiladiki,

$$\varphi(y) = by \ \varphi(e) = be = b$$

Bundan $a = b$ ekanligi kelib chiqadi. x va y elementlar ixtiyoriy olingani uchun shunday $d \in \mathcal{R}$ topiladiki,

$$\varphi(z) = dz \ \forall z \in \mathcal{R}.$$

Isbot yakunlandi.

Aytaylik, l_2 – haqiqiy yoki kompleks sonlar maydonida berilgan ketma-ketliklar Gilbert fazosi bo‘lsin. U holda l_2 ustida aniqlangan chegaralangan chiziqli operatorlarni cheksiz o‘lchovli matritsalar ko‘rinishida tasvirlash mumkin. Agar a^{ij}–A cheksiz o‘lchovli matritsaning i–qatori va j–ustunida turgan komponenti bo‘lsa, u holda A matritsani quyidagicha berish mumkin:

$$A = (a^{ij})_{i,j=1}^{\infty}$$

Aytaylik, e_{ij}-bu i-qatori va j-ustuni kesishmasidagi komponenti 1ga teng, qolgan komponentlari 0 ga teng bo‘lgan cheksiz o‘lchovli matritsa bo‘lsin. U holda A matritsani

$$A = (a^{ij})_{i,j=1}^{\infty} = \sum_{i,j=1}^{\infty} a^{ij} e_{ij}$$

ko‘rinishida yozish mumkin. Aytaylik, $A_1 = a^{11}e_{11}$, $A_2 = a^{12}e_{12} + a^{22}e_{22} + a^{21}e_{21}$,

$$A_3 = \sum_{j=1}^{3} a^{3j} e_{3j} + \sum_{i=1}^{2} a^{i3} e_{i3}, \ldots A_n = \sum_{j=1}^{n} a^{nj} e_{nj} + \sum_{i=1}^{n-1} a^{in} e_{in} \ldots$$

matritsalar ketma-ketligi A matritsa orqali kiritilgan bo‘lsin. Agar l_2 Gilbert fazosi ustida chegaralangan chiziqli operator bo‘ladigan barcha A cheksiz o‘lchovli matritsalarning $M_{\infty}(F), F = \mathbb{R}, \mathbb{C}$ to‘plamini olsak, ma’lumki, $M_{\infty}(F) - l_2$ ustidagi barcha chegaralangan chiziqli operatorlar fon Neyman algebrasi $B(l_2)$ bilan ustma-ust tushadi. Endi $K(l_2)$ to‘plamni quyidagicha kiritamiz: $K(l_2)$ – bu matritsalar ketma-ketligi $A_1, A_2, \ldots, A_n, \ldots$ fundamental bo‘lgan barcha cheksiz o‘lchovli $A \in M_{\infty}(F)$ matritsalar to‘plami bo‘lsin. U holda $K(l_2)$ C^*-algebra bo‘ladi [7] va l_2 ustidagi barcha kompakt operatorlar C^*-algebrasi bilan ustma-ust tushadi.

Agar $\mathcal{R}$ halqa sifatida haqiqiy yoki kompleks sonlar maydoni ustida aniqlangan l_2 Gilbert fazosidagi kompakt operatorlar $K(l_2)$ algebrasini olsak, 2-lokal chapdan ko‘paytirish akslantirishi ta’rifi quyidagicha ko‘rinishda bo‘ladi.

Ta’rif. $\varphi: K(l_2) \to K(l_2)$ akslantirish ixtiyoriy $X, Y \in K(l_2)$ uchun shunday $A \in K(l_2)$ topilib, $\varphi(X) = AX$ va $\varphi(Y) = AY$ shartlarni qanoatlantirsa, u holda φ 2-lokal chapdan ko‘paytirish deyiladi.

Teorema 1.2. $\varphi: K(l_2) \to K(l_2)$ akslantirish 2-lokal chapdan ko‘paytirish bo‘lsin. U holda shunday $M \in K(l_2)$ topiladiki, ixtiyoriy $X \in K(l_2)$ element uchun

$$\varphi(X) = MX$$

tenglik o‘rinli bo‘ladi.

Isbot. $E_{\frac{1}{n}} \in K(l_2)$ elementni $E_{\frac{1}{n}} = \begin{pmatrix} 1 & 0 & \cdots & 0 & \cdots \\ 0 & \frac{1}{2} & & 0 & \cdots \\ \vdots & & \ddots & \vdots & \\ 0 & 0 & \cdots & \frac{1}{n} & 0 \\ \vdots & 0 & & 0 & \ddots \end{pmatrix}$ ko‘rinishida tanlasak, ta’rifga ko‘ra ixtiyoriy X element uchun shunday $A \in K(l_2)$ mavjudki,

$$\varphi(X) = AX, \ \varphi\left(E_{\frac{1}{n}}\right) = AE_{\frac{1}{n}}.$$

O‘z navbatida, ixtiyoriy Y element va $E_{\frac{1}{n}}$ uchun shunday $B \in K(l_2)$ element topiladiki,

$$\varphi(X) = AX, \ \varphi\left(E_{\frac{1}{n}}\right) = A$$

Bundan $A = B$ ekanligi kelib chiqadi. X va Y elementlar ixtiyoriy olingani sababli shunday $M \in K(l_2)$ topiladiki,

$$\varphi(Z) = MZ \ \forall Z \in K(l_2).$$

Isbot yakunlandi.

Aytaylik, $M_n(\mathbb{R})$– $\mathbb{R}$ haqiqiy sonlar maydonida aniqlangan n-o'lchovli matritsalar algebrasi va $c_o(M_n(\mathbb{R}))$ – komponentlari $M_n(\mathbb{R})$ algebradan olingan nolga yaqinlashuvchi ketma-ketliklar Banax fazosi bo'lsin. Ikkita ketma-ketlikning mos komponentlarini ko'paytirish orqali $c_o(M_n(\mathbb{R}))$ Banax fazosida ko'paytirish amalini kiritamiz, ya'ni

$$(A_n) \in c_o(M_n(\mathbb{R})), (B_n) \in c_o(M_n(\mathbb{R})),$$
$$(A_n) \circ (B_n) = (A_n B_n) \in c_o(M_n(\mathbb{R})).$$

U holda $(A_n B_n)$ ko'paytma ketma-ketlik sifatida $c_o(M_n(\mathbb{R}))$ da yotadi va $c_o(M_n(\mathbb{R}))$ Banax fazosi ushbu ko'paytirish amaliga nisbatan C^*-algebra bo'ladi. Ushbu C^*-algebrada norma quyidagicha aniqlanadi $(A_n) \in c_o(M_n(\mathbb{R}))$ uchun

$$\|(A_n)\| = sup_n \|A_n\|, (1)$$

Algebra $c_o(M_n(\mathbb{R}))$ halqa bo'lgani uchun 2-lokal chapdan ko'paytirish tushunchasi $c_o(M_n(\mathbb{R}))$ uchun ham yuqoridagidek kiritiladi. Quyidagi teorema o'rinli.

Teorema 1.3. $c_o(M_n(\mathbb{R}))$ C^*-algebrada $\varphi: c_o(M_n(\mathbb{R})) \to c_o(M_n(\mathbb{R}))$ 2-lokal chapdan ko'paytirish akslantirish berilgan. U holda shunday $(A_n) \in c_o(M_n(\mathbb{R}))$ mavjudki, φ quyidagicha aniqlanadi:

$$\varphi\big((X_n)\big) = (A_n)(X_n), \ (X_n) \in c_o(M_n(\mathbb{R}))$$

Isbot. $M_n(\mathbb{R})$ algebraning E birlik matritsasini olamiz. Ravshanki, $\left(\frac{1}{n}E\right)$ ketma-ketlik $c_o(M_n(\mathbb{R}))$ algebrada yotadi. Ixtiyoriy $(X_n) \in c_o(M_n(\mathbb{R}))$ element va $\left(\frac{1}{n}E\right)$ element uchun ta'rifga ko'ra shunday $(A_n) \in c_o(M_n(\mathbb{R}))$ mavjudki,

$$\varphi\big((X_n)\big) = (A_n)(X_n), \varphi\left(\left(\frac{1}{n}E\right)\right) = (A_n)\left(\frac{1}{n}E\right)$$

tengliklar bajariladi. O'z navbatida, ixtiyoriy $(Y_n) \in c_o(M_n(\mathbb{R}))$ element va $\left(\frac{1}{n}E\right)$ element uchun ta'rifga ko'ra shunday $(B_n) \in c_o(M_n(\mathbb{R}))$ mavjudki,

$$\varphi\big((Y_n)\big) = (B_n)(Y_n), \varphi\left(\left(\frac{1}{n}E\right)\right) = (B_n)\left(\frac{1}{n}E\right)$$

tengliklar bajariladi. U holda

$$\varphi\left(\left(\frac{1}{n}E\right)\right) = (A_n)\left(\frac{1}{n}E\right) = (B_n)\left(\frac{1}{n}E\right)$$

va

$$(A_n)\left(\frac{1}{n}E\right) = \left(\frac{1}{n}A_n E\right) = \left(\frac{1}{n}A_n\right) = (B_n)\left(\frac{1}{n}E\right) = \left(\frac{1}{n}B_n\right).$$

Bundan har qanday n natural son uchun

$$A_n = B_n$$

Demak, $(A_n) = (B_n)$. (X_n) va (Y_n) elementlar ixtiyoriy olingani uchun shunday $(D_n) \in c_o(M_n(\mathbb{R}))$ topiladiki, har qanday $(Z_n) \in c_o(M_n(\mathbb{R}))$ element uchun

$$\varphi\big((Z_n)\big) = (D_n)(Z_n).$$

Teorema isbotlandi.

Yuqorida kiritilgan l_2 Gilbert fazosida aniqlangan barcha kompakt operatorlar $K(l_2)$ algebrasini olamiz. Aytaylik, $c_o(K(l_2))$ – bu komponentlari $K(l_2)$ algebradan olingan nolga yaqinlashuvchi barcha ketma-ketliklar Banax fazosi bo'lsin. $c_o(K(l_2))$ uchun ko'paytirish amali aynan $c_o(M_n(\mathbb{R}))$ C^*-algebrasidagidek kiritiladi va $c_o(K(l_2))$ Banax fazosi ushbu ko'paytirishga nisbatan C^*-algebra bo'ladi. Bu yerda norma (1) tenglikka o'xshash aniqlanadi.

Yuqoridagidek $c_o(K(l_2))$ halqa bo'lgani uchun ushbu algebra ustida 2-lokal chapdan ko'paytirishni olishimiz mumkin. Quyidagi teorema o'rinli.

Teorema 1.4. $c_o(K(l_2))$ C^*-algebrada $\varphi: c_o(K(l_2)) \to c_o(K(l_2))$ 2-lokal chapdan ko'paytirish berilgan. U holda shunday $(A_n) \in c_o(K(l_2))$ element topiladiki, φ quyidagicha aniqlanadi:

$$\varphi\big((X_n)\big) = (A_n)(X_n), \ (X_n) \in c_o(K(l_2))$$

Isbot. $K(l_2)$ algebraning yuqorida kiritilgan $E_{\frac{1}{n}}$ elementini olamiz. U holda $\left(\frac{1}{m}E_{\frac{1}{n}}\right)$ ketma-ketlik nolga yaqinlashuvchi ketma-ketlik bo'ladi

$$\lim_{m\to\infty}\left\|\frac{1}{m}E_{\frac{1}{n}} - \bar{0}\right\| = \lim_{m\to\infty}\frac{1}{m}\left\|E_{\frac{1}{n}}\right\| = 0,$$

ya'ni $\left(\frac{1}{m}E_{\frac{1}{n}}\right)$ element $c_o(K(l_2))$ algebrada yotadi. 1.3 Teorema isbotidagi $\left(\frac{1}{n}E\right)$ketma-ketlik o'rniga $\left(\frac{1}{m}E_{\frac{1}{n}}\right)$ ketma-ketlikni olib 1.3 teorema isbotini takrorlasak, berilgan teoremaning isbotini quramiz. Isbot yakunlandi.

2. 2-lokal Yordan ko'paytirishlari

Ta'rif. Aytaylik, J – Yordan halqasi bo'lsin. Akslantirish $\varphi: J \to J$ har qanday $x, y \in J$ uchun shunday $a \in J$ topilib $\varphi(x) = ax$ va $\varphi(y) = ay$ shartlarni qanoatlantirsa, u holda φ 2–lokal Yordan ko'paytirishi deyiladi.

Teorema 2.1. Aytaylik, J–birlik elementli Yordan halqasi va $\varphi: J \to J$– 2-lokal Yordan ko'paytirishi bo'sin. U holda shunday $a \in J$ topiladiki,

$$\varphi(x) = ax \ \forall x \in J$$

tenglik o'rinli bo'ladi.

Isbot. Ixtiyoriy $x \in J$ va birlik element $e \in J$ uchun, ta'rifga ko'ra, shunday $a \in J$ mavjudki,

$$\varphi(x) = ax \ \varphi(e) = ae = a$$

O'z navbatida, ixtiyoriy $y \in J$ element va $e \in J$ uchun shunday $b \in J$ topiladiki,

$$\varphi(y) = by \ \varphi(e) = be = b$$

Bundan $a = b$ ekanligi kelib chiqadi. x va y elementlar ixtiyoriy olingani uchun shunday $d \in J$ topiladiki,

$$\varphi(z) = dz \ \forall z \in J.$$

Isbot yakunlandi.

Aytaylik, l_2 – haqiqiy yoki kompleks sonlar maydonida berilgan ketma-ketliklar Gilbert fazosi bo'lsin. U holda l_2 ustida aniqlangan chegaralangan o'z-o'ziga qoshma chiziqli operatorni cheksiz o'lchovli o'z-o'ziga qoshma matritsa ko'rinishida tasvirlash mumkin. Agar a^{ij}–A cheksiz o'lchovli o'z-o'ziga qoshma matritsaning i–qatori va j–ustunida turgan komponentsi bo'lsa, u holda A matritsani quyidagicha berish mumkin:

$$A = (a^{ij})_{i,j=1}^{\infty}, (a^{ij})^* = a^{ji}, i, j = 1,2, \ldots, n, \ldots$$

U holda A matritsani

$$A = (a^{ij})_{i,j=1}^{\infty} = \sum_{i,j=1}^{\infty} a^{ij} e_{ij}$$

ko'rinishida yozish mumkin. Aytaylik, $A_1 = a^{11}e_{11}$, $A_2 = a^{12}e_{12} + a^{22}e_{22} + a^{21}e_{21}$,

$$A_3 = \sum_{j=1}^{3} a^{3j} e_{3j} + \sum_{i=1}^{2} a^{i3} e_{i3}, \ldots A_n = \sum_{j=1}^{n} a^{nj} e_{nj} + \sum_{i=1}^{n-1} a^{in} e_{in} \ldots$$

matritsalar ketma-ketligi A matritsa orqali kiritilgan bo'lsin. Agar l_2 Gilbert fazosi ustida chegaralangan o'z-o'ziga qoshma chiziqli operator bo'ladigan barcha A cheksiz o'lchovli o'z-o'ziga qoshma matritsalarning $M_\infty(F)_{sa}, F = \mathbb{R}, \mathbb{C}$ to'plamini olsak, ma'lumki, $M_\infty(F)_{sa} - l_2$ ustidagi barcha chegaralangan o'z-o'ziga qoshma chiziqli operatorlar JW-algebrasi $B(l_2)_{sa}$ bilan ustma-ust tushadi. Endi $K(l_2)_{sa}$ to'plamni quyidagicha kiritamiz: $K(l_2)_{sa}$– bu matritsalari ketma-ketligi $A_1, A_2, \ldots, A_n, \ldots$ fundamental bo'lgan barcha cheksiz o'lchovli $A \in M_\infty(F)_{sa}$ matritsalar to'plami bo'lsin. U holda $K(l_2)_{sa}$ C^*-algebra bo'ladi va l_2 ustidagi barcha kompakt o'z-o'ziga qoshma operatorlar Yordan algebrasi bilan ustma-ust tushadi.

Agar J Yordan halqasi sifatida haqiqiy yoki kompleks sonlar maydoni ustida aniqlangan l_2 Gilbert fazosidagi kompakt o'z-o'ziga qoshma operatorlar $K(l_2)_{sa}$ algebrasini olsak 2-lokal Yordan ko'paytirishi akslantirishi ta'rifi quyidagicha ko'rinishda bo'ladi.

Ta'rif. $\varphi: K(l_2)_{sa} \to K(l_2)_{sa}$ akslantirish ixtiyoriy $X, Y \in K(l_2)_{sa}$ uchun shunday $A \in K(l_2)_{sa}$ topilib, $\varphi(X) = \frac{1}{2}(AX + XA)$ va $\varphi(Y) = \frac{1}{2}(AY + YA)$ shartlarni qanoatlantirsa, u holda φ 2-lokal Yordan ko'paytirishi deyiladi.

Teorema 2.2. $\varphi: K(l_2)_{sa} \to K(l_2)_{sa}$ akslantirish 2-lokal Yordan ko'paytirishi bo'lsin. U holda shunday $A \in K(l_2)_{sa}$ topiladiki, ixtiyoriy $X \in K(l_2)_{sa}$ element uchun

$$\varphi(X) = \frac{1}{2}(AX + XA)$$

tenglik o'rinli bo'ladi.

Isbot. $E_{\frac{1}{n}} \in K(l_2)_{sa}$ elementni $E_{\frac{1}{n}} = \begin{pmatrix} 1 & 0 & \cdots & 0 & \ldots \\ 0 & \frac{1}{2} & & 0 & \ldots \\ \vdots & & \ddots & \vdots & \\ 0 & 0 & \cdots & \frac{1}{n} & 0 \\ \vdots & 0 & & 0 & \ddots \end{pmatrix}$ ko'rinishida tanlasak, ta'rifga ko'ra ixtiyoriy X element uchun shunday $A \in K(l_2)_{sa}$ mavjudki,

$$\varphi(X) = \frac{1}{2}(AX + XA), \ \varphi\left(E_{\frac{1}{n}}\right) = \frac{1}{2}\left(AE_{\frac{1}{n}} + E_{\frac{1}{n}}A\right).$$

O‘z navbatida, ixtiyoriy Y element va $E_{\frac{1}{n}}$ uchun shunday $B \in K(l_2)_{sa}$ element topiladiki,

$$\varphi(Y) = \frac{1}{2}(BY + YB),\ \varphi\left(E_{\frac{1}{n}}\right) = \frac{1}{2}(BE_{\frac{1}{n}} + E_{\frac{1}{n}}B).$$

Bundan

$$AE_{\frac{1}{n}} + E_{\frac{1}{n}}A = BE_{\frac{1}{n}} + E_{\frac{1}{n}}B$$

va $A = B$ ekanligi kelib chiqadi. X va Y elementlar ixtiyoriy olingani sababli shunday $D \in K(l_2)_{sa}$ topiladiki,

$$\varphi(Z) = \frac{1}{2}(DZ + ZD)\ \forall Z \in K(l_2)_{sa}.$$

Isbot yakunlandi.

Aytaylik, $M_n(\mathbb{R})_{sa}$ – $\mathbb{R}$ haqiqiy sonlar maydonida aniqlangan n- o‘lchovli o‘z-o‘ziga qoshma matritsalar Yordan algebrasi va $c_o(M_n(\mathbb{R})_{sa})$ – komponentlari $M_n(\mathbb{R})_{sa}$ Yordan algebrasidan olingan nolga yaqinlashuvchi ketma-ketliklar Banax fazosi bo‘lsin. Ikkita ketma-ketlikning mos komponentlarini ko‘paytirish orqali $c_o(M_n(\mathbb{R})_{sa})$ Banax fazosida ko‘paytirish amalini kiritamiz, ya’ni

$$(A_n) \in c_o(M_n(\mathbb{R})_{sa}),\ (B_n) \in c_o(M_n(\mathbb{R})_{sa}),$$
$$(A_n) \circ (B_n) = \frac{1}{2}[(A_nB_n) + (B_nA_n)] \in c_o(M_n(\mathbb{R})_{sa}).$$

U holda (A_nB_n) ko‘paytma ketma-ketlik sifatida $c_o(M_n(\mathbb{R})_{sa})$ da yotadi va $c_o(M_n(\mathbb{R})_{sa})$ Banax fazosi ushbu ko‘paytirish amaliga nisbatan Yordan algebrasi bo‘ladi. Ushbu Yordan algebrasida norma quyidagicha aniqlanadi $(A_n) \in c_o(M_n(\mathbb{R})_{sa})$ uchun

$$\|(A_n)\| = sup_n\|A_n\|,\ (2)$$

Yordan algebrasi $c_o(M_n(\mathbb{R})_{sa})$ Yordan halqasi bo‘lgani uchun 2-lokal Yordan ko‘paytirishi tushunchasi $c_o(M_n(\mathbb{R})_{sa})$ uchun ham yuqoridagidek kiritiladi. Quyidagi teorema o‘rinli.

Teorema 2.3. $c_o(M_n(\mathbb{R})_{sa})$ Yordan algebrasida $\varphi: c_o(M_n(\mathbb{R})_{sa}) \to c_o(M_n(\mathbb{R})_{sa})$ 2-lokal Yordan ko‘paytirishi berilgan. U holda shunday $(A_n) \in c_o(M_n(\mathbb{R})_{sa})$ mavjudki, φ quyidagicha aniqlanadi:

$$\varphi\big((X_n)\big) = \frac{1}{2}[(A_n)(X_n) + (X_n)(A_n)],\ (X_n) \in c_o(M_n(\mathbb{R})_{sa})$$

Isbot. $M_n(\mathbb{R})_{sa}$ algebraning E birlik matritsasini olamiz. Ravshanki, $\left(\frac{1}{n}E\right)$ ketma-ketlik $c_o(M_n(\mathbb{R})_{sa})$ algebrada yotadi. Ixtiyoriy $(X_n) \in c_o(M_n(\mathbb{R})_{sa})$ element va $\left(\frac{1}{n}E\right)$ element uchun ta’rifga ko‘ra shunday $(A_n) \in c_o(M_n(\mathbb{R})_{sa})$ mavjudki,

$$\varphi\big((X_n)\big) = \frac{1}{2}[(A_n)(X_n) + (X_n)(A_n)],$$
$$\varphi\left(\left(\frac{1}{n}E\right)\right) = \frac{1}{2}\left[(A_n)\left(\frac{1}{n}E\right) + \left(\frac{1}{n}E\right)(A_n)\right]$$

tengliklar bajariladi. O‘z navbatida, ixtiyoriy $(Y_n) \in c_o(M_n(\mathbb{R})_{sa})$ element va $\left(\frac{1}{n}E\right)$ element uchun ta’rifga ko‘ra shunday $(B_n) \in c_o(M_n(\mathbb{R})_{sa})$ mavjudki,

$$\varphi\big((Y_n)\big) = \frac{1}{2}[(B_n)(Y_n) + (Y_n)(B_n)],$$
$$\varphi\left(\left(\frac{1}{n}E\right)\right) = \frac{1}{2}\left[(B_n)\left(\frac{1}{n}E\right) + \left(\frac{1}{n}E\right)(B_n)\right]$$

tengliklar bajariladi. U holda

$$\varphi\left(\left(\frac{1}{n}E\right)\right) = \frac{1}{2}\left[(A_n)\left(\frac{1}{n}E\right) + \left(\frac{1}{n}E\right)(A_n)\right] =$$
$$\frac{1}{2}\left[(B_n)\left(\frac{1}{n}E\right) + \left(\frac{1}{n}E\right)(B_n)\right]$$

va

$$(A_n)\left(\frac{1}{n}E\right) + \left(\frac{1}{n}E\right)(A_n) = \left(\frac{1}{n}A_nE\right) + \left(\frac{1}{n}EA_n\right) =$$
$$\left(\frac{2}{n}A_n\right) = (B_n)\left(\frac{1}{n}E\right) + \left(\frac{1}{n}E\right)(B_n) = \left(\frac{2}{n}B_n\right).$$

Bundan har qanday n natural son uchun

$$A_n = B_n$$

Demak, $(A_n) = (B_n)$. (X_n) va (Y_n) elementlar ixtiyoriy olingani uchun shunday $(D_n) \in c_o(M_n(\mathbb{R})_{sa})$ topiladiki, har qanday $(Z_n) \in c_o(M_n(\mathbb{R})_{sa})$ element uchun

$$\varphi\big((Z_n)\big) = \frac{1}{2}[(D_n)(Z_n) + (Z_n)(D_n)].$$

Teorema isbotlandi.

Yuqorida kiritilgan l_2 Gilbert fazosida aniqlangan barcha kompakt o‘z-o‘ziga qo‘shma operatorlar $K(l_2)_{sa}$Yordan algebrasini olamiz. Aytaylik, $c_o(K(l_2)_{sa})$–bu komponentlari $K(l_2)_{sa}$ algebradan olingan nolga yaqinlashuvchi barcha ketma-ketliklar Banax fazosi bo‘lsin. $c_o(K(l_2)_{sa})$ uchun Yordan ko‘paytirishi amali aynan $c_o(M_n(\mathbb{R})_{sa})$ Yordan algebrasidagidek kiritiladi va $c_o(K(l_2)_{sa})$ Banax fazosi ushbu ko‘paytirishga nisbatan Yordan algebrasi bo‘ladi. Bu yerda norma (2) tenglikka o‘xshash aniqlanadi.

Yuqoridagidek $c_o(K(l_2)_{sa})$ Yordan halqasi bo‘lgani uchun ushbu Yordan algebrasi ustida 2-lokal Yordan ko‘paytirishini olishimiz mumkin. Quyidagi teorema o‘rinli.

Teorema 2.4. $c_o(K(l_2)_{sa})$ Yordan algebrasida $\varphi: c_o(K(l_2)_{sa}) \to c_o(K(l_2)_{sa})$ 2-lokal Yordan ko‘paytirishi berilgan. U holda shunday $(A_n) \in c_o(K(l_2)_{sa})$ element topiladiki, φ quyidagicha aniqlanadi:

$$\varphi\big((X_n)\big) = \frac{1}{2}[(A_n)(X_n) + (X_n)(A_n)], \ \ (X_n) \in c_o(K(l_2)_{sa})$$

Isbot. $K(l_2)_{sa}$ Yordan algebrasining yuqorida kiritilgan $E_{\frac{1}{n}}$ elementini olamiz. U holda $\left(\frac{1}{m}E_{\frac{1}{n}}\right)$ ketma-ketlik nolga yaqinlashuvchi ketma-ketlik bo‘ladi

$$\lim_{m\to\infty}\left\|\frac{1}{m}E_{\frac{1}{n}} - \bar{0}\right\| = \lim_{m\to\infty}\frac{1}{m}\left\|E_{\frac{1}{n}}\right\| = 0,$$

ya’ni $\left(\frac{1}{m}E_{\frac{1}{n}}\right)$ element $c_o(K(l_2)_{sa})$ algebrada yotadi. 2.3 Teorema isbotidagi $\left(\frac{1}{n}E\right)$ ketma-ketlik o‘rniga $\left(\frac{1}{m}E_{\frac{1}{n}}\right)$ ketma-ketlikni olib 2.3 teorema isbotini takrorlasak, berilgan teoremaning isbotini quramiz. Isbot yakunlandi.

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DESCRIPTION OF 2-LOCAL DERIVATIONS ON AN ALGEBRA OF MATRICES

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Key words: associative ring, left multiplication, Jordan ring, Jordan multiplication, 2-local mapping.

The present paper is devoted to 2-local derivation on associative and Jordan matrix rings. 2-local derivation is defined as follows: given a ring R, a map Δ: $R \to R$ (not additive in general) is called 2-local derivation if for every $x,y \in R$, there exists a derivation $D_{x,y} : R \to R$ such that $\Delta(x) = D_{x,y}(x)$ and $\Delta(y) = D_{x,y}(y)$.

In 1997, P. Semrl introduced the notion of 2-local derivations and described 2-local derivations in the algebra $B(H)$ of all bounded linear operators on the infinite-dimensional separable Hilbert space H. A similar description for the finite-dimensional case appeared later in 2004. In the paper Y. Lin and T. Wong 2-local derivations have been described on matrix algebras over finite dimensional division rings. In 2012 Sh. Ayupov, K. Kudaybergenov suggested a new technique and have generalized the above mentioned results for abritrary Hilbert spaces. Namely they considered 2-local derivations on the algebra *B*(*H*) of all linear bounded operators on an arbitrary (no separability is assumed) Hilbert space *H* and proved that every 2-local derivation on *B*(*H*) is a derivation. Later Sh. Ayupov, K. Kudaybergenov and F. Arzikulov extended the above results and gave the proof of the theorem for arbitrary von Neumann algebras.

A number of papers were devoted to 2-local maps on different types of rings, algebras, Banach algebras and Banach spaces. The on 2-local derivations on finite dimensional Lie algebras, on weak-2-local derivations, 2-local □-Lie isomorphisms and 2-local Lie isomorphisms were obtained as a result. It proved a number of theorems on 2-local triple derivations. Other classes of 2-local maps on different types of associative and Jordan algebras were also studied.

A concept of 2-local left multiplication on an arbitrary ring is introduced and studied. This notion is introduced as follows: let $\Re$ be a ring. Then a mapping φ of $\Re$ into itself is called 2-local left multiplication if for every elements $x,y \in \Re$ there exists an element $a \in \Re$ such that $\varphi(x) = ax,\ \varphi(x) = ay.$

In the present paper, it is proved that every 2-local left multiplication of some associative rings is a left multiplication. Namely, we prove that for any element $x \in \Re$ there exists an element $a \in \Re$ such that $\varphi(x) = ax$.

Besides, in the present paper, a concept of 2-local Jordan multuplication on an arbitrary Jordan ring it is introduced and studied. This notion is introduced as follows: let J be a Jordan ring. Then a mapping φ of J into itself is called 2-local Jordan multiplication if for every elements $x,y \in J$ there exists an element $a \in J$ such that $\varphi(x) = ax,\ \varphi(x) = ay.$

Finally, it is proven that every 2-local Jordan multiplication φ of some Jordan rings J is a Jordan multiplication. Namely, it is proven that for every element $x \in J$ there exists an element $a \in J$ such that $\varphi(x) = ax$.

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