INTERVAL EXTENSION STRUCTURE BASIC TYPES OF SOLUTIONS OF BOUNDARY VALUE PROBLEMS FOR LINEAR PARTIAL DIFFERENTIAL EQUATIONS OF FORTH ORDER

Bohodir Muminov prof.
Tashkent university of information technologies named after Muhammad al-Khwarizmi, mbbahodir@gmail.com

Shodmankul Nazirov prof.

Follow this and additional works at: https://uzjournals.edu.uz/tuitmct

Recommended Citation
Available at: https://uzjournals.edu.uz/tuitmct/vol1/iss1/8

This Article is brought to you for free and open access by 2030 Uzbekistan Research Online. It has been accepted for inclusion in Bulletin of TUIT: Management and Communication Technologies by an authorized editor of 2030 Uzbekistan Research Online. For more information, please contact sh.erkinov@edu.uz.
UDC 681.03

INTERVAL EXTENSION STRUCTURE BASIC TYPES OF SOLUTIONS OF BOUNDARY VALUE PROBLEMS FOR LINEAR PARTIAL DIFFERENTIAL EQUATIONS OF THE FOURTH ORDER

Muminov B. B., Nazirov Sh. A.

Abstract. In this article, the interval expansion of the structure of solving basic types of boundary value problems for partial differential equations of the fourth order has been developed. Used Method of R-functions for constructed coordinate sequences. Constructing interval extensions of structural formulas, we consider problems (1) on the transverse bending of thin plates and 5 problems on a plate - rigidly clamped plate, loosely supported plate, elastically fixed plates, partially rigidly fixed and partially elastically fixed plates, plates, partially rigidly clamped and partially free. For the problem, the rigidly clamped plate Formula (7) is an interval structure for solving the boundary value problem (4). Here \( L = \{ -\omega - \psi, \omega - \psi, -\omega - \psi, \omega - \psi \} \), \( \Gamma_1 = \{ -\omega D_1 - \psi, D_1 - \psi, -\omega - \psi D_1 - \psi \} \), \( D_2, T_2, D_1 \) - differential operators of the form (15), (17), \((\Phi_1, \omega D_1 \Phi_1)\), \((\Phi_2, \omega D_1 \Phi_2)\) - indefinite interval functions; \( D_2, T_2 \) - differential operators of the form (11) and (12). For the problem of elastically fixed plates, a solution was obtained in the interval expansion of the structure in the form (15), (17), \((\Phi_1, \omega D_1 \Phi_1)\), \((\Phi_2, \omega D_1 \Phi_2)\) - indefinite interval functions, \( D_2, T_2 \) - differential operators of the form (11) and (12), (3). For the problem of partially rigidly clamped and partially elastically fixed plates, a solution was obtained in the interval expansion of the structure in the form of (28), (30), (32), \((\Phi_1, \omega D_1 \Phi_1)\), \((\Phi_2, \omega D_1 \Phi_2)\) - indefinite interval functions, \( D_2, T_2 \) - differential operators of the form (11) and (12), (6). For the plate problem, partially rigidly pinched and partially free, a solution is obtained in the interval expansion of the structure (40), (41), (42), \((\Phi_1, \omega D_1 \Phi_1)\), \((\Phi_2, \omega D_1 \Phi_2)\) - indefinite interval functions, \( D_2, T_2 \) - differential operators of the form (11), (12), (6) and (38).

Keywords: structure solutions, interval arithmetic, boundary value problems, R-function method.

Introduction

R-function method is used particularly for solving boundary value problems of mathematical physics, expressed differential, integral and integro-differential equations, which describes the mathematical models of various physical, technical, mechanical, etc. Process [1]. R-functions method allows us to construct coordinate sequences satisfying the boundary conditions exactly, without any approximations. However, when solving systems of differential (integro-differential) equations of high order due to bad of a matrix (full matrix of large orders, compiled as a result of sampling in the spatial variables using the method of R-functions) lost accuracy of the approximate solutions. Furthermore, such a loss of accuracy arise in cases where the initial data of the problem are not accurate, approximate values of the integrals are computed and error methods for solving governing equations, etc. These disadvantages can be eliminated by using interval method [2-4]. Hence the need to develop an algorithm combining R-functions method and interval method for solving practical problems, which we call the interval-valued R-function [5]. Therefore, we construct interval extensions for structural formulas constructed for the main types of boundary value problems transverse bending of thin plates, which is described by the differential equation in partial derivatives of the fourth order.

II. Constructing interval extensions of structural formula

For simplicity, we consider the problem of transverse bending of thin plates [9].

\[
D \Delta^2 u = q \\
D = \frac{E h^3}{12(1 - \nu^2)} - \text{flexural rigidity plate, } h - \text{thickness of the plate, } E - \text{young's modulus, } \nu - \text{poison's ratio, } q - \text{transverse load, } \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, u - \text{deflection plate.}
\]

In problems of transverse bending of thin plates on the boundary \( \Gamma \) (or portions \( \Gamma_i \)) is defined by the two boundary conditions.

1. Rigidly clamped plate. On the plate contour is zero deflection and rotation angles:

\[
\begin{align*}
\left. u \right|_{\Gamma} &= 0, \\
\left. \frac{\partial u}{\partial n} \right|_{\Gamma} &= 0
\end{align*}
\]

Here, the normal \( n \) to \( \Gamma \) in this case can be used the following structural formula [1]:

\[
u = \omega^2 \Phi
\]

It is assumed that \( |\nabla \omega| > 0 \) on \( \Gamma \). In practice there are options of rigid support with transverse displacements and rotations of the plate edge. For him, instead of the boundary conditions (2) are the conditions

\[
\begin{align*}
\left. u \right|_{\Gamma} &= \varphi_0, \\
\left. \frac{\partial u}{\partial n} \right|_{\Gamma} &= \psi_0
\end{align*}
\]
where, \( \varphi_0, \psi_0 \) - given functions on \( \Gamma \). Denote the extension to \( \Gamma \) as \( EC \varphi_0 = \varphi, EC \psi_0 = \psi \). (EC - operator bonding boundary values) will continue in \( \Gamma \) and also the structure of the solution of (1), (3) building type polynomial R-functions method has the form [1]:

\[
\mathbf{u} = \varphi + \psi \mathbf{w} - D_1 \varphi + \psi \mathbf{w}^2 \Phi
\]

(5)

Where, \( \Phi \) - undefined function, \( D_1 \) - differential operator with the following species,

\[
D_1 \mathbf{u} = \sum_{i=1}^{n} \frac{\partial \omega}{\partial x_i} \frac{\partial \mathbf{u}}{\partial x_i},
\]

(6)

and, \( \omega \) - equation of the boundary region, the following species

\[
\omega = \begin{cases}
0, & (x, y) \in \Omega \\
0, & (x, y) \in \Gamma \\
0, & (x, y) \in \Gamma \cup \Omega
\end{cases}
\]

(7)

Applying interval arithmetic operations, construct interval extension solution structure (5):

\[
\begin{align*}
[w_{\omega}] &= [\varphi + \psi \mathbf{w}] = [\omega \mathbf{w}] + [\mathbf{w}] \varphi \\
&= [\varphi + \min\{\omega, \max\} + \max\{\omega, \min\}] + [\min\{\omega, \max\} + \max\{\omega, \min\}]
\end{align*}
\]

and get

\[
[w_{\omega}] = [\varphi + \min\{\omega, \max\} + \max\{\omega, \min\}] + [\min\{\omega, \max\} + \max\{\omega, \min\}]
\]

(8)

Equation (7) is the interval structure of the solution of the problem (4).

Here

\[
L = \{w_{\omega}, \omega \mathbf{w}, \omega \mathbf{w}^2\}, L_1 = \{w_{D_1 \varphi}, w_{D_1 \psi}, w_{D_1 \mathbf{w}}\},
\]

(9)

\[
L_2 = [\varphi^2 \Phi, \omega^2 \Phi, \omega^2 \Phi, \omega^2 \Phi, \omega^2 \Phi, \omega^2 \Phi], \quad [\Phi, \Phi, \Phi]
\]

undefined interval function.

2. Simply supported plate. For her, the boundary conditions have the form

\[
\mathbf{u} |_{\Gamma} = 0,
\]

(10)

where, \( \mathbf{n}_0 \) and \( \mathbf{v} \) - direction of the normal and tangential to \( \Gamma \) respectively. As shown in [1], this condition can also be written as

\[
\mathbf{u} |_{\Gamma} = 0,
\]

(11)

where, \( \mathbf{v} \) - radius of curvature of \( \Gamma \).

Structure of the solution of (1), (9) the construction method of the form R-polynomial function has the form [1]:

\[
u = \omega \Phi_1 - \frac{\omega^3}{3}[D_2(\mathbf{w}) + \omega \mathbf{w}^2 \Phi_2]
\]

(12)

where, \( \Phi_1, \Phi_2 \) - undefined function and \( D_2, T_2 \) - differential operators, with the following form [1].

\[
D_2 = \sum_{i=1}^{2} \frac{\partial \mathbf{w}}{\partial x_i} \frac{\partial \mathbf{u}}{\partial x_i} = \sum_{i=1}^{2} \frac{\partial \mathbf{w}}{\partial x_i} \frac{\partial \mathbf{u}}{\partial x_i}
\]

(13)

Taking into account that

\[
[w_{\omega}] = [\omega \mathbf{w}] = \left[ \begin{array}{c}
\omega \mathbf{w}_1 \\
\omega \mathbf{w}_2
\end{array} \right],
\]

(14)

\[
X_1 = \min \{X_1\}, \quad X_1 = \max \{X_1\}
\]

Taking interval arithmetic operations, construct interval extension solution structure (9):

\[
[w_{\omega}] = \left[ \begin{array}{c}
\omega \mathbf{w}_1 \\
\omega \mathbf{w}_2
\end{array} \right], \quad \left[ \begin{array}{c}
\omega \mathbf{w}_1 \\
\omega \mathbf{w}_2
\end{array} \right] = \left[ \begin{array}{c}
\omega \mathbf{w}_1 \\
\omega \mathbf{w}_2
\end{array} \right] 
\]

(15)

where, \( \alpha_1, \alpha_2 \) - undefined function.

2. Simply supported plate. For her, the boundary conditions have the form

\[
\mathbf{u} |_{\Gamma} = 0,
\]

(16)

where, \( \mathbf{n}_0 \) and \( \mathbf{v} \) - direction of the normal and tangential to \( \Gamma \) respectively. As shown in [1], this condition can also be written as

\[
\mathbf{u} |_{\Gamma} = 0,
\]

(17)

where, \( \mathbf{v} \) - radius of curvature of \( \Gamma \).

Structure of the solution of (1), (9) the construction method of the form R-polynomial function has the form [1]:

\[
u = \omega \Phi_1 - \frac{\omega^3}{3}[D_2(\mathbf{w}) + \omega \mathbf{w}^2 \Phi_2]
\]

(18)

where, \( \Phi_1, \Phi_2 \) - undefined function and \( D_2, T_2 \) - differential operators, with the following form [1].
3. Elastically fixed plate. For them deflection function $u$ satisfies the boundary conditions on $\Gamma$

$$u|_{\Gamma} = 0,$$  \hspace{1cm} (18)

where $\rho_0$ - the radius of curvature of the boundary $\Gamma$, $k_0$ - flexural rigidity of termination.

Structure of the solution of (1), (18) and (19) the construction method of the form $R$-polynomial function has the form [1]:

$$u = \omega \Phi_1 + \frac{1}{2} \omega^3 \Phi_2 - \omega^3 \Phi_1 \big(2u_0 + q_0 \omega + k_1 \omega^2 + 2D_1 \phi_2 \big)$$  \hspace{1cm} (20)

where $\Phi_1, \Phi_2$ - undefined functions and $D_2, T_2, T_1$ - differential operators of the form (11) and (12), (6).

Applying interval arithmetic operations, construct interval arithmetic structure of the solution (20):

$$[u, \bar{u}] = \left[ \omega \Phi_1, \Phi_1 \big(2u_0 + q_0 \omega + k_1 \omega^2 + 2D_1 \phi_2 \big) \right]$$

or

$$[u, \bar{u}] = \left[ \omega \Phi_1, \Phi_1 \big(2u_0 + q_0 \omega + k_1 \omega^2 + 2D_1 \phi_2 \big) \right]$$

taking into account that

$$\left[ a_1, \bar{a}_1 \right] X_2 = \left[ a_1, \bar{a}_1 \right] X_3 = \left[ a_1, \bar{a}_1 \right] X_4 = \left[ a_1, \bar{a}_1 \right] X_5,$$

where $\overline{X_{11}} = \min \{X_{11}, \bar{X}_{11} \}$, $\overline{X_{12}} = \max \{X_{12}, \bar{X}_{12} \}$, $\overline{X_{13}} = \min \{X_{13}, \bar{X}_{13} \}$, $\overline{X_{14}} = \max \{X_{14}, \bar{X}_{14} \}$

and obtain

$$[u, \bar{u}] = \left[ \omega \Phi_1, \Phi_1 \big(2u_0 + q_0 \omega + k_1 \omega^2 + 2D_1 \phi_2 \big) \right]$$

In the interval extension structure (15), (17), $[\Phi_2, \bar{\Phi}_2] -$ undefined gap function, $D_2, T_2$ - differential operators of the form (11) and (12).
\[
\begin{bmatrix}
W_1, W_2 \\
W_3
\end{bmatrix}
\begin{bmatrix}
\nu_0, T_2, \omega, \nu, T_2, \bar{\omega}
\end{bmatrix} = \begin{bmatrix}
W_3, W_3
\end{bmatrix},
\]
where
\[
W_3 = \min\{W_3\}, \quad \bar{W}_3 = \max\{W_3\},
\]
and
\[
\frac{1}{\omega^2} \left( \int_{\Omega} \frac{\partial u}{\partial n} \right) \bigg|_{\Gamma_1} = 0;
\]
\[
\frac{\partial u}{\partial n} \bigg|_{\Gamma_1} = 0;
\]
\[
\frac{\partial u}{\partial t} \bigg|_{\Gamma_2} = 0;
\]
\[
\frac{\partial^2 u}{\partial t^2} + \left( \frac{1}{\rho_0} + k \right) \frac{\partial u}{\partial t} \bigg|_{\Gamma_1} = 0,
\]
where, \(\nu_0, \rho_0, k_0\) and Poisson's ratio are, respectively, the radius of curvature \(r_2\) and stiffness fixing plate. As before, we denote \(EC\rho_0^{-1} = \rho^{-1}\), \(ECk_0 = k\) (EC - extension operator boundary values).

Structure of the solution of (1), (24) and (25) the construction method of the form R-polynomial function has the form [1]:
\[
\begin{bmatrix}
\phi_1, \phi_2 \\
\bar{\phi}_1, \bar{\phi}_2
\end{bmatrix} = \begin{bmatrix}
\phi_1, \phi_2 \\
\bar{\phi}_1, \bar{\phi}_2
\end{bmatrix},
\]
where, \(\phi_1, \phi_2\) - undefined function and \(D_2, T_2, D_1\) - differential operators of the form (11) and (12), (6).

Applying interval arithmetic operations, construct interval arithmetic structure of the solution (27):
\[
\begin{bmatrix}
\phi_1, \phi_2 \\
\bar{\phi}_1, \bar{\phi}_2
\end{bmatrix} = \begin{bmatrix}
\phi_1, \phi_2 \\
\bar{\phi}_1, \bar{\phi}_2
\end{bmatrix},
\]
where
\[
\frac{1}{\omega^2} \left( \int_{\Omega} \frac{\partial u}{\partial n} \right) \bigg|_{\Gamma_1} = 0;
\]
\[
\frac{\partial u}{\partial n} \bigg|_{\Gamma_1} = 0;
\]
\[
\frac{\partial u}{\partial t} \bigg|_{\Gamma_2} = 0;
\]
\[
\frac{\partial^2 u}{\partial t^2} + \left( \frac{1}{\rho_0} + k \right) \frac{\partial u}{\partial t} \bigg|_{\Gamma_1} = 0,
\]
where, \(\nu_0, \rho_0, k_0\) and Poisson's ratio are, respectively, the radius of curvature \(r_2\) and stiffness fixing plate. As before, we denote \(EC\rho_0^{-1} = \rho^{-1}\), \(ECk_0 = k\) (EC - extension operator boundary values).

Structure of the solution of (1), (24) and (25) the construction method of the form R-polynomial function has the form [1]:
\[
\begin{bmatrix}
\phi_1, \phi_2 \\
\bar{\phi}_1, \bar{\phi}_2
\end{bmatrix} = \begin{bmatrix}
\phi_1, \phi_2 \\
\bar{\phi}_1, \bar{\phi}_2
\end{bmatrix},
\]
where, \(\phi_1, \phi_2\) - undefined function and \(D_2, T_2, D_1\) - differential operators of the form (11) and (12), (6).
where

\[ h_0 T_{2}^{(2)} \left[ \omega_{2} \omega_{2} \right] \phi \left[ \frac{B_5}{B_5} \right] = \left[ B_5 \right], \]

\[ B_5 = \min \{ B_5 \}, \]

and obtain (28) as follows:

\[ \left[ u, \bar{u} \right] = \left[ B_1, B_1 \right] \left[ \left( B_2 - B_2 \right) + \left( B_3 - B_3 \right) \right] - \left[ B_1, B_1 \right] \left[ \left( B_5 - B_5 \right) + \left( B_5 - B_5 \right) \right], \]

or

\[ \left[ u, \bar{u} \right] = \left[ B_1, B_1 \right] \left[ B_2 - B_2 \right] + \left[ B_1, B_1 \right] \left[ B_3 - B_3 \right] - \left[ B_1, B_1 \right] \left[ B_5 - B_5 \right] \]

or

\[ \left[ u, \bar{u} \right] = \left[ B_1, B_1 \right] \left[ B_2 - B_2 \right] + \left[ B_1, B_1 \right] \left[ B_3 - B_3 \right] - \left[ B_1, B_1 \right] \left[ B_5 - B_5 \right], \]

taking into account that

\[ \left[ B_1, B_1 \right] \left[ B_2 - B_2 \right] = \left[ C_4, C_4 \right], \]

where

\[ C_4 = \min \{ C_4 \}, \]

and obtain (30) as follows:

\[ \left[ u, \bar{u} \right] = \left[ B_1, B_1 \right] \left[ B_2 - B_2 \right] + \left[ B_1, B_1 \right] \left[ B_3 - B_3 \right] - \left[ B_1, B_1 \right] \left[ B_5 - B_5 \right], \]

or

\[ \left[ u, \bar{u} \right] = \left[ B_1, B_1 \right] \left[ B_2 - B_2 \right] + \left[ B_1, B_1 \right] \left[ B_3 - B_3 \right] - \left[ B_1, B_1 \right] \left[ B_5 - B_5 \right], \]

or

\[ \left[ u, \bar{u} \right] = \left[ B_1, B_1 \right] \left[ B_2 - B_2 \right] + \left[ B_1, B_1 \right] \left[ B_3 - B_3 \right] - \left[ B_1, B_1 \right] \left[ B_5 - B_5 \right], \]

5. Plate partially clamped rigidly and partly free.

Let \( \Gamma_1 \) - \( \Gamma_2 \) section of the border, in which the plate is rigidly clamped and the rest of the boundary \( \Gamma_2 = \Gamma \setminus \Gamma_1 \) free. The boundary conditions in this case are

\[ u \left|_{\Gamma_1} = 0; \right. \]

\[ \left. \frac{\partial u}{\partial n} \right|_{\Gamma_1} = 0; \]

\[ \left. \frac{\partial^2 u}{\partial n^2} + \frac{\partial^2 u}{\partial \tau^2} \right|_{\Gamma_2} = 0; \]

\[ \left. \frac{\partial^2 u}{\partial n^2} + \left( 2 - \mu_0 \right) \frac{\partial^2 u}{\partial \tau^2} \right|_{\Gamma_2} = 0. \]

Structure of the solution of (1), (34), (35) and (36) the construction method of the form R-polynomial function has the form [1]:

\[ u = \frac{w_0}{w_1 + w_2} \left( \omega_3 \Phi_1 + \Phi_0 \right) \]

\[ + \frac{w_0}{w_1 + w_2} \left( \omega_3 \Phi_2 + \Phi_0 \right) \]

\[ + \frac{w_0}{w_1 + w_2} \left( \omega_3 \Phi_3 + \Phi_0 \right) \]

\[ + \frac{w_0}{w_1 + w_2} \left( \omega_3 \Phi_4 + \Phi_0 \right) \]

\[ + \frac{w_0}{w_1 + w_2} \left( \omega_3 \Phi_5 + \Phi_0 \right) \]

\[ + \frac{w_0}{w_1 + w_2} \left( \omega_3 \Phi_6 + \Phi_0 \right). \]

\[ C_2 = \min \{ C_2 \}, \]
\[
\begin{align*}
\begin{bmatrix} u, \tilde{u} \end{bmatrix} &= \frac{\left[ \omega_1, \omega_1 \right]^2}{\left[ \omega_1, \omega_1 \right]^2 + \left[ \omega_2, \omega_2 \right]^4} \\
&\quad \left\{ \frac{\omega_1^2}{\omega_1^2 + \omega_2^2} \left[ \Phi_1, \Phi_1 \right] + \frac{\omega_2^2}{\omega_1^2 + \omega_2^2} \left[ \Phi_2, \Phi_2 \right] - \frac{\omega_1 \omega_2}{\omega_1^2 + \omega_2^2} \left[ \Phi_1, \Phi_2 \right] \right\} \\
d_{2}^{(2)} \left[ \left( \Phi_1, \Phi_1 \right) \right] - \frac{\omega_2}{2} \left[ \Phi_2, \Phi_2 \right] = \frac{\omega_2}{2} \left[ \Phi_2, \Phi_2 \right] = \left[ \Phi_2, \Phi_2 \right], \quad \text{(39)}
\end{align*}
\]

taking into account that

\[
\frac{\left[ \omega_1, \omega_1 \right]^2}{\left[ \omega_1, \omega_1 \right]^2 + \left[ \omega_2, \omega_2 \right]^4} = \left[ V_2, \bar{V}_2 \right],
\]

where

\[
V_2 = \min \{ V_2, \bar{V}_2 \}, \quad V_2 = \max \{ V_2, \bar{V}_2 \},
\]

\[
V_2 = \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4
\]

\[
D_2^{(2)} \left[ \left( \Phi_1, \Phi_1 \right) \right] = \left[ V_2, \bar{V}_2 \right],
\]

\[
\begin{align*}
\left( 2 - v_0 \right) \left( D_2^{(2)} T_2^{(2)} \right) \frac{\omega_2}{2} \left[ \Phi_1, \Phi_1 \right] &= \left( 2 - v_0 \right) \left( D_2^{(2)} T_2^{(2)} \right) \frac{\omega_2}{2} \left[ \Phi_1, \Phi_1 \right] = \left[ V_2, \bar{V}_2 \right],
\end{align*}
\]

\[
\begin{align*}
\omega_2^2 D_2^{(2)} \left[ \left( \Phi_1, \Phi_1 \right) \right] &= \left[ V_2, \bar{V}_2 \right],
\end{align*}
\]

where

\[
\begin{align*}
V_2 &= \min \{ V_2, \bar{V}_2 \}, \quad \bar{V}_2 = \max \{ V_2, \bar{V}_2 \},
\end{align*}
\]

\[
V_2 = \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4
\]

\[
\begin{align*}
\left[ V_2, \bar{V}_2 \right] &= \left[ V_2, \bar{V}_2 \right],
\end{align*}
\]

where

\[
\begin{align*}
V_2 &= \min \{ V_2, \bar{V}_2 \}, \quad \bar{V}_2 = \max \{ V_2, \bar{V}_2 \},
\end{align*}
\]

\[
V_2 = \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4
\]

\[
\begin{align*}
\left[ V_2, \bar{V}_2 \right] &= \left[ V_2, \bar{V}_2 \right],
\end{align*}
\]

where

\[
\begin{align*}
V_2 &= \min \{ V_2, \bar{V}_2 \}, \quad \bar{V}_2 = \max \{ V_2, \bar{V}_2 \},
\end{align*}
\]

\[
V_2 = \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4 \omega_2^4
\]
where

\[ G_4 = \min \{ G_4 \}, \overline{G_4} = \max \{ G_4 \}, G_4 = \left\{ V_1 M_1, V_2 M_1, V_3 M_1, V_4 M_1 \right\}, \]

\[ M_4 = \min \{ M_4 \}, \overline{M_4} = \max \{ M_4 \}, M_4 = \left\{ V_1 V_4, V_2 V_4, V_3 V_4, V_4 V_4 \right\}, \]

\[ \left[ V_1, V_1 \right] \left[ V_4, V_4 \right] \left[ V_5, V_5 \right] = \left[ G_5, \overline{G_5} \right], \]

where

\[ G_5 = \min \{ G_5 \}, \overline{G_5} = \max \{ G_5 \}, G_5 = \left\{ V_1 M_2, V_2 M_2, V_3 M_2, V_4 M_2 \right\}, \]

\[ M_5 = \min \{ M_5 \}, \overline{M_5} = \max \{ M_5 \}, M_5 = \left\{ V_1 V_4, V_2 V_4, V_3 V_4, V_4 V_4 \right\}, \]

\[ \left[ V_1, V_1 \right] \left[ V_4, V_4 \right] \left[ V_5, V_5 \right] = \left[ G_6, \overline{G_6} \right], \]

where

\[ G_6 = \min \{ G_6 \}, \overline{G_6} = \max \{ G_6 \}, G_6 = \left\{ V_1 M_3, V_2 M_3, V_3 M_3, V_4 M_3 \right\}, \]

\[ M_6 = \min \{ M_6 \}, \overline{M_6} = \max \{ M_6 \}, M_6 = \left\{ V_1 V_4, V_2 V_4, V_3 V_4, V_4 V_4 \right\}, \]

\[ \left[ V_1, V_1 \right] \left[ V_4, V_4 \right] \left[ V_5, V_5 \right] = \left[ G_7, \overline{G_7} \right], \]

where

\[ G_7 = \min \{ G_7 \}, \overline{G_7} = \max \{ G_7 \}, G_7 = \left\{ V_1 V_2, V_2 V_2, V_3 V_2, V_4 V_2 \right\}, \]

\[ \left[ V_1, V_1 \right] \left[ V_5, V_5 \right] = \left[ G_8, \overline{G_8} \right], \]

and obtain (41) as follows:

\[ \begin{align*}
  u & = \frac{g_1 + g_2 - g_3 - g_4 + g_5 + g_6 + g_7 + g_8}{\overline{u}}
  \end{align*} \]

In the interval extension structure (40), (41), (42),

\[ \left[ \varphi_1, \overline{\varphi_1} \right], \left[ \varphi_2, \overline{\varphi_2} \right] \] - undefined interval function,

\[ D_2, T_2, D_3, D_3 \] - differential operators of the form (11), (12), (6) and (38).

When constructing interval extensions of structural formulas, we consider problems (1) on the transverse bending of thin plates and 5 problems on a plate — A rigidly clamped plate, A freely supported plate, Elastically fixed plates, Partially rigidly clamped and partially elastically fixed plates, Plates partially rigidly clamped and partially free and created software library “Sol4Interval” based on the object orientation of the C++ programming language. This library makes it possible to construct an interval 3D graph for interval expansion of structures (Fig-1).
As a result of the calculations, the dependences (Fig. 1) of the maximum temperature in the zone of action of ionizing radiation on qiborite on the heat flux density at different depths in the body of the material and on the surface (x = 0) for different times of action were obtained.

There are three main areas for the successful use of interval analysis and interval methods on the basis of the Sol4Interval software library:

- Solving practical problems that have interval or, more generally, limited data uncertainty.
- Strict accounting of rounding errors in calculations with floating point numbers on digital computers.
- New approaches to solving traditional mathematical problems (such as, for example, the global optimization problem, global evidence-based solution of systems of nonlinear equations, etc.)

**Conclusion**

Interval extensions built structures making basic types of boundary value problems: rigidly clamped plate partially clamped rigidly and partially free plate for differential equations of the fourth order. The results can be used to construct interval value structure solutions for other boundary value problems of differential equations of different order. Resulting interval values the solution structure is easily implemented in computers.

**LITERATURE**


[5] Nazirov S. A. Multidimensional interval valued R-function. / / Problems of


[18]Application of Fuzzy Arithmetic to Quantifying the Effects of Uncertain Model Parameters, Michael Hans, University of Stuttgart.