MATHEMATICAL MODELING OF STRESS-STRAIN STATE OF LOADED RODS WITH ACCOUNT OF TRANSVERSE BENDING

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MATHEMATICAL MODELING OF STRESS-STRAIN STATE OF LOADED RODS WITH ACCOUNT OF TRANSVERSE BENDING

Anarova Sh.A., Olimov M., Ismoilov Sh.M.

Abstract. Mathematical simulation of static and dynamic processes of strain taking into account transverse bending under loading is presented in the paper in linear and geometrically nonlinear statements. An extensive analysis of research work carried out in universities and science centers all over the world is given, the relevance of the problem and the areas of solution application are emphasized. The mathematical correctness of the problem statement is shown. Variations of kinetic, potential energy, volume and surface forces are determined for mathematical models of static and dynamic processes of strain with taking into account transverse bending of loaded rods in linear and geometrically nonlinear statements. Based on the theory of elastic strains and the refined theory of Vlasov-Dzhanelidze-Kabulov, and using the Ostrogradsky-Hamilton variation principle, a mathematical model of the statics and dynamics of the process of rod points displacement is developed for transverse bending in linear and geometrically nonlinear statements. Equations of a mathematical model with natural initial and boundary conditions in a vector form are given. A computational algorithm is developed for calculating the statics and dynamics of rods under loading in linear and geometrically non-linear statements using the central finite differences of the second order of accuracy. The strain processes when the rod is rigidly fixed at two edges are considered in linear and geometrically nonlinear statements. The calculation results obtained are given in the form of graphs. The propagation of longitudinal, transverse vibrations and the angle of inclination along the length of the rod was studied at different points of times. Linear and geometrically non-linear vibration results are analyzed and compared.

Keywords: oscillations, rod, mathematical model, transverse bending, moving

Introduction

A special attention all over the world is paid to the development of a mathematical model and an automated system for calculating linear and geometrically nonlinear problems of building parts and structures under spatial loading. In this regard, the creation of an automated system for estimating the stress-strain state of rod-like structures when studying the strain processes of spatially loaded rods based on modern information technologies, as well as their improvement is one of the main tasks. In developed countries of the world, including the USA, France, Canada, Japan, China, the United Arab Emirates, Iran, Russia, Ukraine, Kazakhstan and others, the development of mathematical models, computational algorithms and the software for calculating rod-type structure material is very important.

A number of scientific results have been obtained during the studies conducted all over the world in mathematical simulation and improvement of automated systems for static and dynamic calculations, linear and nonlinear strain processes in rods under spatial loading. These results include the improved models of dynamic functional properties of rotating beams (Mecánicos Universidad Tecnológica Nacional, Argentina) [1-2], nonlinear problems of free oscillations of rotating composite beams developed by S.P. Tymoshenko S.P. Timoshenko’s nonlinear problem of a beam on the basis of a modified theory of paired stresses (Amirkabir University of Technology and Sharif University of Technology, Iran) [3-4,5-6], the study of dynamic behavior of the drill string, improvement and development of an experimental device (Institut National des Sciences Appliquees de Lyon, France) [7-8], a nonlinear solution for the S.P. Tymoshenko’s inclined beam oscillations under moving force has been obtained, mathematical models of oscillation of flexural-torsion, longitudinal-flexural and longitudinal-torsion waves are developed based on the A.P. Filipov’s theory of elastic strain in the rod system (Institute of Mechanical Engineering Problems of Russian Academy of Sciences, Russian Federation) [9-12].

All over the world, in order to solve the problems of strain processes of structure elements such as rods in construction, aviation, rocket production, shipbuilding, mechanical engineering and oil industry, the following promising areas are being studied: the development of mathematical models and computational algorithms, improvement and creation of software for linear and nonlinear strain processes in the S.P.Timoshenko beam, simulation of the stress-strain state of rod-like structure elements, that take into account strain processes, geometric and physical nonlinearity, a complete study of intense flexural-torsion, longitudinal-flexural and longitudinal-torsion oscillations propagating in rods[11-12].

Statement of the problem. This article proposes a mathematical simulation of geometrically non-linear rod problems with allowance for transverse bending. In this case, the spatial strain of the rods takes into account two components of the longitudinal displacements $u_1(x,t)$, $u_2(x,t)$, and one component of the transverse displacements $u_3(x,t)$. Then the displacements of the rod points takes the form [13]:

$$u_1(x, y, z, t) = u - z\alpha, \quad u_2 = 0,$$

$$u_3(x, y, z, t) = w,$$

where $u$, $w$ are the displacements of the rod midline; $\alpha$ is the angle of rotation of the section at a pure bending; $u_1$, $u_3$ are the components of the
displacement vector. Here, the \( u, w, \alpha \) are the functions to be found in the spatial variable \( x \) and time \( t \), and there are no restrictions on the external load.

**Mathematical model.** Using the Ostrogradsky-Hamilton variation principle [13-15]:

\[
\delta \int_{t_i}^{t_f} (K - \Pi + A) \, dt = 0
\]

(2)

where \( K, P \) are kinetic and potential energy; \( A \) is the work of external volume and surface forces.

We derive a mathematical model of nonlinear problems of rods under static and dynamic spatial loading.

When calculating the variation of kinetic energy, the following ratio is used [13-15]:

\[
\int_{t}^{t_f} \rho \frac{\partial u}{\partial t} \delta \frac{\partial u}{\partial t} \, dt + \rho \frac{\partial u_2}{\partial t} \delta \frac{\partial u_2}{\partial t} \, dt, \quad \int_{t}^{t_f} \delta \Pi \, dt = \int_{t}^{t_f} \sum_{i=1}^{3} \sigma_{ii} \delta \varepsilon_{ii} \, dt = \int_{t}^{t_f} \sum_{i=1}^{3} \left( \sigma_{11} \delta \varepsilon_{11} + \sigma_{12} \delta \varepsilon_{12} + \sigma_{13} \delta \varepsilon_{13} \right) \, dt
\]

where \( \sigma_{11}, \sigma_{12}, \sigma_{13} \) are the elements of the stress tensor; \( \varepsilon_{11}, \varepsilon_{12}, \varepsilon_{13} \) are the elements of the strain tensor.

Take the Cauchy relations [5, 10, 11-13]:

\[
\varepsilon_{11} = \frac{1}{2} \left( \frac{\partial u_1}{\partial x} \right)^2, \quad \varepsilon_{12} = \frac{1}{2} \left( \frac{\partial u_1}{\partial x} \right) \left( \frac{\partial u_2}{\partial x} \right), \quad \varepsilon_{13} = \frac{1}{2} \left( \frac{\partial u_1}{\partial x} \right) \left( \frac{\partial u_3}{\partial x} \right)
\]

According to (1), \( \frac{\partial u_3}{\partial z} = 0 \). So

\[
\varepsilon_{11} = \frac{1}{2} \left( \frac{\partial u_1}{\partial x} \right)^2, \quad \varepsilon_{12} = \frac{1}{2} \left( \frac{\partial u_2}{\partial x} \right)^2, \quad \varepsilon_{13} = \frac{1}{2} \left( \frac{\partial u_3}{\partial x} \right)^2.
\]

Stress components based on the Hooke law are written in the form:

\[
\sigma_{11} = E\varepsilon_{11} = E \frac{\partial u_1}{\partial x}, \quad \sigma_{13} = G\varepsilon_{13} = G \left( \frac{\partial u_1}{\partial x} + \frac{\partial u_3}{\partial x} \right).
\]

where \( E \) is the elasticity modulus, \( G = \frac{E}{2(1+\mu)} \) is the shear modulus, \( \mu \) is the Poisson ratio.

For the variation of the external forces work the following formula is applied [13-17]:

\[
\int_{t}^{t_f} \delta Adt = \int_{t}^{t_f} \sum_{i=1}^{3} F_i \delta u_i \, dv + \int_{s}^{s_f} \sum_{s=1}^{3} q_i \delta u_i \, ds + \int_{t}^{t_f} \sum_{s=1}^{3} f_i \delta u_i \, ds = \int_{t}^{t_f} \sum_{s=1}^{3} \left( F_i \delta u_i + F_2 \delta u_2 + F_3 \delta u_3 \right) dv \]

\[
= \int_{v}^{v_f} \left( F_1 \delta u_1 + F_2 \delta u_2 + F_3 \delta u_3 \right) dv + \int_{s}^{s_f} \left( q_1 \delta u_1 + q_2 \delta u_2 + q_3 \delta u_3 \right) ds + \int_{s}^{s_f} \left( f_1 \delta u_1 + f_2 \delta u_2 + f_3 \delta u_3 \right) ds,
\]

where \( F_i \) are the components of volume forces, referred to the unit volume, \( q_i \) is the surface forces, referred to the unit surface area of the rod; \( f_i \) is the end face forces, respectively.

Here, \( u, w, \alpha \) are the functions of the \( x \) coordinate and time \( t \), therefore, the derivatives with respect to \( z \) are zero. When considering a symmetric section, the static moments are zero too. Then the variation equation is simplified and takes the following form [18-22]:

The last integral is

\[\int \left[ \int \left( -\rho F \frac{\partial^2 u}{\partial t^2} + E \frac{\partial^2 u}{\partial x^2} + f_3 \right) + q_3 \right] \, dx \, \delta u \]

\[+ \left[ \int \left( \rho I_y \frac{\partial^2 \alpha}{\partial t^2} - E I_y \frac{\partial^2 \alpha}{\partial x^2} + M(f_i) \right) + M(q_i) \right] \, dx \, \delta \alpha \]

\[+ \left[ \int \left( -\rho F \frac{\partial^2 w}{\partial t^2} + G \left( F \frac{\partial^2 w}{\partial x^2} \right)^2 + \frac{\partial w}{\partial x} \left( F \frac{\partial^2 u}{\partial x^2} \right) + \frac{\partial^2 w}{\partial x^2} E \left( F \frac{\partial^2 u}{\partial x^2} \right) + \frac{\partial^2 w}{\partial x^2} \right) + \frac{\partial^2 w}{\partial x^2} E \left( F \frac{\partial^2 u}{\partial x^2} \right) + \frac{\partial w}{\partial x} \right] \, dx \, \delta w \]

\[+ \frac{\partial^2 w}{\partial x^2} E \left( F \frac{\partial^2 u}{\partial x^2} \right) + \frac{\partial w}{\partial x} \right] \, dx \, \delta w \]
\[ \Phi^{k-1}_2 = \frac{\partial^2 w^{k-1}}{\partial x^2} \left( \frac{l}{a} \frac{\partial u^{k-1}}{\partial x} \right) + \frac{l^2}{EFa} \frac{\partial w^{k-1}}{\partial x} \cdot \frac{l^2}{M(\varphi)} \left( f_i + \bar{q}_i \right) = 0. \]

Boundary conditions:
\[ \left[ -l \frac{\partial \bar{w}}{\partial x} + \frac{l^2}{EFa} \varphi_1 \right] \frac{\partial \bar{u}}{\partial x} = 0, \]
\[ \left[ -\frac{1}{2(1+\mu)} \left( l \frac{\partial \bar{w}}{\partial x} \right) - \Phi^{k-1}_2 + \frac{l^2}{EFa} \varphi_1 \right] \frac{\partial \bar{w}}{\partial x} = 0. \]

Initial conditions:
\[ \left[ \frac{\partial \bar{u}}{\partial t} \right]_{t=0} = 0, \]
\[ \left[ \frac{\partial \bar{w}}{\partial t} \right]_{t=0} = 0, \]
\[ \left[ \frac{l}{EFa} \frac{\partial \varphi_1}{\partial t} \right]_{t=0} = 0. \]

Thus, using the hypothesis (1), a system of equations for problems of geometrically non-linear rods under spatial loading with allowance for transverse bending is obtained. By specifying the boundary conditions at \( x = 0 \) and \( x = l \), one can solve a number of practical problems.

Computational algorithm. The solution of differential equations of motion (4) with the corresponding boundary (5) and initial (6) conditions obtained from the variation principle (2) in a scalar form is rather difficult. Therefore, the system of differential equations, the boundary and initial conditions will be represented in a vector form.

Introduce the vectors in the following form [24]:
\[ \bar{U} = \left[ \bar{u}, \bar{w}, \varphi_1 \right]^T, \]
\[ \bar{F} = \left[ \left( f_i + \bar{q}_i \right), \left( f_i + \bar{q}_i \right), \left( M(f_i) + M(q_i) \right) \right]^T. \]

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\[ F(\varphi) = \left[ \varphi_1, \varphi_2, M(\varphi_1) \right]^T, \]
\[ \Phi^{k-1} = \left[ 0, \Phi_2^{k-1}, 0 \right]^T, \]
\[ \tilde{\Phi}^{k-1} = \left[ 0, \tilde{\Phi}_2^{k-1}, 0 \right]^T. \] (7)

The system of equations (4), boundary conditions (5) and initial conditions (6), with introduced matrix elements, are written in the vector form:

\[ M \frac{\partial^2 \tilde{U}^k}{\partial t^2} + A \frac{\partial^2 \tilde{U}^k}{\partial x^2} + E \Phi^{k-1} + \tilde{D} \tilde{F} = 0, \] (8)

\[ -M \frac{\partial \tilde{U}^k}{\partial t} \biggr|_r = 0, \] (9)

\[ \left[ -A \frac{\partial \tilde{U}^k}{\partial x} + E \Phi^{k-1} + \tilde{D} \tilde{F}(\varphi) \right] \biggr|_r = 0. \] (10)

From the system of equations (4) \( M \) is:

\[ M = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & 0 \\ 0 & 0 & m_{33} \end{bmatrix}. \]

Matrix elements have the values:

\[ m_{11} = -1, \quad m_{22} = -1, \quad m_{33} = -\frac{I_y}{Fa^2}. \]

Matrix \( A \) in the system of equations (4) has the following form:

\[ A = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}. \]

Elements of matrix \( A \) from the system of equations (4) have the form:

\[ a_{11} = 1, \quad a_{22} = \frac{G}{E}, \quad a_{33} = \frac{I_y}{Fa^2}. \]

Matrix \( D \) in the system of equations (4) has the following form:

\[ D = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix}. \]

Elements of matrix \( D \) have the values

\[ d_{11} = d_{22} = \frac{l^2}{E Fa}, \quad d_{33} = -\frac{l^2}{E Fa^2}. \]

Matrix \( \tilde{A} \) from boundary conditions (6) has the form:

\[ \tilde{A} = \begin{bmatrix} \tilde{a}_{11} & 0 & 0 \\ 0 & \tilde{a}_{22} & 0 \\ 0 & 0 & \tilde{a}_{33} \end{bmatrix}. \]

Elements of matrix \( \tilde{A} \) from boundary conditions (6) have the form:

\[ \tilde{a}_{11} = -1, \quad \tilde{a}_{22} = -\frac{G}{E}, \quad \tilde{a}_{33} = -\frac{I_y}{Fa^2}. \]

Matrix \( \tilde{D} \) from boundary conditions (6) has the following form:

\[ \tilde{D} = \begin{bmatrix} \tilde{d}_{11} & 0 & 0 \\ 0 & \tilde{d}_{22} & 0 \\ 0 & 0 & \tilde{d}_{33} \end{bmatrix}. \]

where

\[ \tilde{d}_{11} = \tilde{d}_{22} = \frac{l^2}{E Fa}, \quad \tilde{d}_{33} = \frac{l^2}{E Fa^2}. \]

At initial conditions, the elements of matrices have opposite signs relative to the elements of the matrix of systems of equations \( M_{n,y} = -M_{c,y} \).

The linear problem is solved, which is the first approximation of the solution.

\[ M \frac{\partial^2 \tilde{U}^k}{\partial t^2} + A \frac{\partial^2 \tilde{U}^k}{\partial x^2} + \tilde{D} \tilde{F} = 0, \] (11)

\[ -M \frac{\partial \tilde{U}^k}{\partial t} \biggr|_r = 0, \] (12)

\[ \left[ A \frac{\partial \tilde{U}^k}{\partial x} + \tilde{D} \tilde{F}(\varphi) \right] \biggr|_r = 0. \] (13)

Further, an iteration process is organized until the following condition is satisfied:

\[ \text{MAX} \left| \tilde{U}_{i,j}^k - U_{i,j}^{k-1} \right| \leq \varepsilon, \] here \( k \) - is the number of iterations.

When constructing a computational algorithm for a system of differential equations (11) with initial (12) and boundary (13) conditions, the central finite-difference relations of second-order accuracy are applied [23-24].

After some mathematical calculations, a system of algebraic equations is built in the form:

\[ \tilde{U}_{i,j}^k = \tilde{A} \tilde{U}_{i-1,j}^k + \tilde{B} \tilde{U}_{i,j}^k - \tilde{C} \tilde{U}_{i+1,j}^k - \tilde{F}_{i,j}. \]

Coefficients of all resolving equations have the form:
\[
\begin{align*}
\bar{A} &= \frac{\tau^2 AM^{-1}}{h^2}, \quad \bar{B} = 2 + \frac{\tau^2 AM^{-1}}{h^2}, \\
\bar{C} &= \frac{\tau^2 AM^{-1}}{h^2}, \\
\bar{F}_{i,j} &= D\tau^2 M^{-1}\bar{F}_{i,j},
\end{align*}
\]

The solution order of the problem is:
1) \(k = 1\);
2) at \(i = 1, j = 0\) the following equation is solved
\[
\bar{U}_{1,1} = \frac{1}{2} \left[ A\bar{U}_{1,0} + B\bar{U}^0_{1,0} + 2\tau M^{-1}\bar{U}^0_{1,0} + \bar{P}_{0,0} - \bar{F}_{1,0} \right],
\]
3) \(i = i + 1\);
4) if the condition \(i \geq N - 2\) is not met, to proceed to point 2, otherwise – to point 3;
5) at \(i = i, j = 1\) the following equation is solved
\[
\bar{U}_{i,2} = -\bar{A}\bar{U}_{i-1,1} + \bar{B}\bar{U}_{i,1} - \bar{C}\bar{U}_{i+1,1} - \bar{U}^0_{i,0} - \bar{F}_{i,0},
\]
6) \(i = i + 1\);
7) if the condition \(i \geq N - 2\) is not met, to proceed to point 5, otherwise – to point 6;
8) at \(i = N - 1, j = 1\) the following equation is solved
\[
\bar{U}_{N-1,2} = \bar{A}\bar{U}_{N-2,1} + \bar{B}\bar{U}^0_{N-1,1} - \bar{U}^0_{N-1,0} + \bar{P}_{N,1} - \bar{F}_{N-1,1},
\]
9) at \(i = 1, j = j\) the following equation is solved
\[
\bar{U}_{i,j+1} = -\bar{A}\bar{U}_{0,j} + \bar{B}\bar{U}_{1,j} - \bar{C}\bar{U}_{2,j} - \bar{U}_{1,j-1} - \bar{F}_{1,j},
\]
10) \(i = i + 1\);
11) if the condition \(i \geq N - 2\) is not met, to proceed to point 9, otherwise – to point 10;
12) at \(i = N - 1, j = j\) the following equation is solved
\[
\bar{U}_{N-1,j+1} = \bar{A}\bar{U}_{N-2,j} + \bar{B}\bar{U}_{N-1,j} - \bar{U}_{N-1,j-1} + \bar{P}_{N,j} - \bar{F}_{N-1,j},
\]
13) \(j = j + 1\);
14) if the condition \(j > M\) is not met, to proceed to point 12, otherwise – to point 13;
15) \(k = k + 1\);
16) computation of nonlinear terms;
17) to proceed to point 1;
18) if the condition \(|\bar{U}^k_{i,j} - \bar{U}^{k-1}_{i,j}| \leq \varepsilon\) is met, to proceed to point 19, otherwise – to point 12;
19) end.

In a static statement equation (8) and boundary conditions (10) are rewritten in the form
\[
A \frac{\partial^2 \bar{U}^k}{\partial x^2} + E\bar{F}^{k-1} + D\bar{F} = 0,
\]
\[
\begin{bmatrix} A \frac{\partial \bar{U}^k}{\partial x} + B\bar{U}^k + E\bar{F}^{k-1} + D\bar{F}(\varphi) \end{bmatrix} \delta \bar{U} = 0.
\]

The linear problem is solved and thus the zero approximation of the solution is made:
\[
A \frac{\partial^2 \bar{U}^k}{\partial x^2} + D\bar{F} = 0,
\]
\[
\begin{bmatrix} A \frac{\partial \bar{U}^k}{\partial x} + B\bar{U}^k + D\bar{F}(\varphi) \end{bmatrix} \delta \bar{U} = 0.
\]

When constructing a computational algorithm for the system of differential equations (16) with boundary conditions (17), the central finite-difference relations of the second-order accuracy are applied [24].

Consider the boundary conditions of the case when the two ends of the rod are rigidly fixed.
In this case, the boundary conditions are
\[
\bar{U} \Big|_{x=0} = 0, \quad \frac{\partial \bar{U}}{\partial x} \Big|_{x=0} = 0,
\]
\[
\bar{U} \Big|_{x=l} = 0, \quad \frac{\partial \bar{U}}{\partial x} \Big|_{x=l} = 0.
\]

Solution order of the problem is:
1. At \(i = 0\) the following equation is solved
\[
\bar{U}_1 = -\left(1 + \bar{C}^{-1}\bar{A}\right)^{-1}\tilde{C}^{-1}\tilde{F}_0.
\]
2. At \(i = 1\) the following equation is solved
\[
\bar{U}_2 = \tilde{C}^{-1}(\bar{C}^{-1}\bar{A} + B\bar{U}_1 - \tilde{F}_1).
\]
3. At \(i \geq 2\) the following equation is solved
\[
\bar{U}_{i+1} = \tilde{C}^{-1}\left(-\bar{A}\bar{U}_{i-1} + B\bar{U}_i - \tilde{F}_i\right).
\]
4. At \(i = N - 2\) the following equation is solved

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\[ \tilde{U}_{N-1} = \hat{C}^{-1} \left( -\tilde{A}\tilde{U}_{N-3} + \tilde{B}\tilde{U}_{N-2} - \hat{F}_{N-2} \right). \]

As a result the first approximation of the problem is obtained:
\[ A\frac{\partial^2 \tilde{U}^k}{\partial x^2} = -DF - \hat{E}\tilde{U}^{k-1}. \]

Further, an iteration process is going on until the following condition is met:
\[ \text{MAX} \left| \tilde{U}^k - \tilde{U}^{k-1} \right| \leq \epsilon. \]

Applying the central finite-difference relations of the second-order accuracy [3, 11-13, 17] for \( \Phi_2^{k-1} \) and \( \bar{\Phi}_2^{k-1} \), we have
\[ \Phi_2^{k-1} = \left[ \frac{1}{h^2} (w_{r+1} - 2w_r + w_{r-1}) \right] \left[ \frac{l}{2ah} (u_{r+1} - u_r) \right] + \left[ \frac{1}{2h} (w_{r+1} - w_{r-1}) \right] \left[ \frac{l}{ah^2} (u_{r+1} - 2u_r + u_{r-1}) \right] \]
and
\[ \bar{\Phi}_2^{k-1} = \frac{1}{2h} (w_{r+1} - w_{r-1}) \left( \frac{a}{2h} (u_{r+1} - u_{r-1}) \right). \]

**Computational experiment.** Consider the numerical solution of the problem. The following parameters are used to calculate the rod: the Young’s modulus \( E = 2 \times 10^5 \text{ Pa} \), the Poisson’s ratio \( \nu_1 = 0.3 \) (for steel), the length \( l = 10 \text{ m} \), cross-sections \( a = 0.02 \text{ m} \), \( b = 0.02 \text{ m} \), surface loads \( q_1 = 0.015 \text{ H} \), \( q_3 = 0.02 \text{ H} \), \( M = 0.012 \text{ H} \cdot \text{m} \).

The results obtained correspond to the specified boundary value problems (18) [28].

Figures 1 – 4 show the results of rod oscillations in the form of graphs for the longitudinal displacement \( u \) transverse displacements \( w \) the angle of inclination \( \alpha \) of the sections for boundary value problems; the rod is rigidly fixed at two ends.

Figure 1 shows the graphs of the longitudinal displacement \( u \) varying over time \( t \). The longitudinal displacement \( u \) at each point symmetrically increases from the boundary to the center along the rod length. The sinusoid of longitudinal displacement is formed over time.

Figure 2 presents a graph of the time change in the 20th point when dividing the rod length into 40 parts. In the remaining nodes, the sinusoids are obtained, but with a smaller amplitude.

**Figure 1. Change in the longitudinal displacement \( u \times 10^7 \) along the rod length**
Figure 2. Change in the longitudinal displacement $u$ in one point of the rod over time $t$.

Figure 3. Change in the transverse displacement $w \times 10^4$ along the rod length.

Figure 3 shows the graphs of the change in the transverse displacement $W$ over time $t$. The transverse displacement $W$ at each point increases symmetrically from the boundary to the center along the rod length.
Figure 4. Change in the lateral displacement $w$ at one point of the rod over $t$

Figure 5. Change of the angle of inclination $\alpha \times 10^4$ along the rod length

Figure 5 shows the graphs of the angle of inclination $\alpha$ varying over time $t$. The greatest amplitude is reached in the middle of the rod. First it decreases and then increases, almost reaching a state of rest. These changes take 120 time steps.

The amplitude of the sinusoidal change in the angle of inclination is approximately 2, and the interval of change is (-3.8; 0.0).
Figure 6. Change in longitudinal $10^4 \cdot u$ and transverse $10^4 \cdot w$ displacements, in the angle of inclination at pure bending $10^4 \cdot \alpha$ at one point of the rod over time $t$.

Figure 6 shows the change in longitudinal $u$, transverse $w$ displacements and in the angle of inclination $\alpha$ along the rod length. Each parameter has its own sinusoid over time.

Figure 7. Change in longitudinal $10^4 \cdot u$ and transverse $10^4 \cdot w$ displacements, in the angle of inclination at pure bending $10^4 \cdot \alpha$ at one point of the rod over time $t$.
Figures 8 - 10 show the comparison of linear and non-linear results of the rods displacements. It has been established that an account of nonlinear terms leads to a more accurate definition of the dynamic model of rods, i.e. to a decrease in the amplitudes of transverse oscillations and an increase in the frequencies as compared to linear cases.

Figure 8. Comparison of linear and non-linear results of longitudinal displacement $U$

Figure 9. Comparison of linear and non-linear results of transverse displacement $W$
ences with the nonlinear-

\[ \alpha \]

\( \cdot \)

\( \cdot \)

\( \cdot \)

1

0

-1

-2

-3

-4

-5

lin. non-lin.

\[ \begin{array}{c}
0 & 200 & 300 & 400 & 500 & 600 & 700 & 800 \\
\hline
1 & 7 & 13 & 19 & 25 & 31 & 37 & 43
\end{array} \]

**Figure 10. Comparison of linear and non-linear results in the angle of inclination \( \alpha \).**

**Conclusion**

The results obtained are consistent with the results of studies on the nonlinear dynamics of rods; a comparison of elastic elements displacements in the linear statement gives differences with the nonlinear statement of the problem. This suggests that the rod structure of geometric and physical parameters studied in this paper is considered as essentially non-linear one. Therefore, the linear simulation of this problem will lead to the first (overestimated) approximation of the solution. Besides, it will reflect the variety of complex oscillatory processes that occur at multiple frequencies, which will remain outside the framework of linear dynamic models.

**REFERENCES**


