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The first constraint problem for equation doesn’t have the fourth order curve

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Cover Page Footnote

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Erratum

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TO'RTINCHI TARTIBLI BUZILISHGA EGA BO'LMAGAN TENGLAMA UCHUN BIR CHEGARAVIY MASALALAR

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Annotatsiya: *Maqola ishida olingan natijalarning barchasi yangi bo'lib, ikkinchi tartibli turli tartibda buzilishga ega bo'lgan aralash tipdagi tenglamalar uchun aralash masalalar va yuqori tartibli buzilishga ega bo'lmagan chegaraviy masalalar birinchi marotaba o'rganilmoqda. O'rganilayotgan masalalarning o'zaro bir qiymatli yechilishi isbotlangan.*

Kalit so'zlar: *Hosila, tenglamalar, giperbolik tip, parabolik tip, Fur'ye usuli, soha, yechim yagonaligi, masala qo'yilishi, regulyar yechim, boshlang'ich shartlar, chegaraviy shartlar, differensial, integral.*

THE FIRST CONSTRAINT PROBLEM FOR EQUATION DOESN'T HAVE THE FOURTH ORDER CURVE

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Abstract: *All results of the dissertation work are new. Mixed type equations with one and two line degeneration for mixed type equations of second and higher kind are investigated. Sufficient conditions are found for uniqueness and existence of solutions of the formulated problems.*

Key words:*derivate, the equations, hyperbolic type, parabolic type, method Fure, space, uniqueness of the solution, formulation of the problem, regular solution, initial conditions, border conditions, differensial, integral.*

ГРАНИЧНАЯ ЗАДАЧА ДЛЯ УРАВНЕНИЙ ЧЕТВЕРТОГО ПОРЯДКА БЕЗ РАЗРУШЕНИЙ

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Аннотация: *В данной статье исследуется смешанная краевая задачи для уравнения второго порядка смешанного типов с вырождением разного порядка. Получены новые результаты. Доказана единственность решения изучаемой задачи .*

Клучовые слова: *производная, уравнения, гиперболический тип, параболический тип, метод Фурье, область, единственность решения, постановка задачи, регулярное решение, начальные условия, граничные условия, дифференциал,интеграл.*

Amaliyotda shunday masalalar uchraydiki, ularning matematik modeli sifatida hosil qilingan differensial tenglamalarning umumiy yechimidan xususiy yechimini ajratib olish uchun Koshi masalasidan emas, balki izlanayotgan funksiya va uning hosilasini integrallash oralig'ining boshlang'ich va oxirgi nuqtasidagi qiymatlaridan foydalaniladi. Odatda bunday masalalar chegaraviy masalalar nomi bilan yuritiladi.

So'ngi o'nyillikda Rossiya fanlar akademiyasi Kabardin-Balkar ilmiy markazi tadqiqotchilari tomonidan matematik modellarning biologik jarayonlarga tadbiq qilish natijasida aniqlandiki, ko'plab biologik jarayonlar xususiy hosilali elliptik, giperbolik, parabolik va aralash tipdagi tenglamalar bilan tavsiflash mumkin ekan.

Hozirgi kunga kelib turli xil biologik jarayonlar va hodisalarning matematik modellashtirishga bag'ishlangan ko'plab maqolalar va kitoblar mavjud bo'lib, yangi davrni ya'ni "Matematik biologiya tenglamalari" nomi bilan yuritilmoqda.

O.A.Ladijenskaya [1] va V.A.Il'inning [2] mashhur ishlaridan so'ng Fur'ye usulidan foydalanib buzilishga ega bo'lmagan giperbolik, parabolik va aralash tipdagi tenglamalar uchun chegaraviy masalalar o'rganilgan. Buni quyidagicha tushuntirish mumkin, ya'ni Fur'ye usuli (o'zgaruvchilarni ajratish usuli) giperbolik tipdagi tenglamalar uchun qaralayotgan masalaning sohasiga va tenglamadagi koeffitsientga ortiqcha shartlarsiz o'rganish imkonini beradi.

O'tgan asrning 60 yillariga kelib buziladigan aralash tipdagi chiziqli va kvazichizikli giperbolik va parabolik tipdagi tenglamalar uchun Fur'ye usulining tatbiqi sezilarli ravishda kengaytirildi, keyinchalik esa yuqori tartibli tenglamalar uchun ham Farg'onalik olimlar D.X.Karimov [3-4], K.B.Baykuziev [5], M.Kasimov [6], B.S.Kalanov [7] tomonidan tadqiq etilgan. A.Yu.Sazonovning [8] ishida singulyar koeffitsientli giperbolik va parabolik tipdagi tenglamalarni tadqiq etishda Fur'ye usulini qo'llagan.

Aytaylik $\Omega=\{(x,t):0<x<p,0<t<T\}$ bo'lsin. U holda Ω sohada

$$Lu\equiv u_{xxxx}+u_{tt}=f(x,t) \tag{1}$$

tenglamani qaraymiz, bu yerda $f(x,t)$ - berilgan funksiya**.**

Masalaning qo'yilishi va yechimning yagonaligi.

A-**Masala.** (1) tenglamaning Ω sohada quyidagi shartlarni qanoatlantiruvchi $u(x,t)$ yechimi topilsin:

$$u(x,0)=\psi_1(x), \qquad 0\le x\le p. \tag{2}$$

$$u_t(x,0)=\psi_2(x), \qquad 0\le x\le p. \tag{3}$$

$$u_x(0,t)=\varphi_1(t), \qquad 0\le t\le T. \tag{4}$$

$$u_x(p,t)=\varphi_2(t), \qquad 0\le t\le T. \tag{5}$$

$$u_{xxx}(0,t)=\varphi_3(t), \qquad 0\le t\le T. \tag{6}$$

$$u_{xxx}(p,t)=\varphi_4(t), \qquad 0\le t\le T. \tag{7}$$

1-Ta'rif. $u(x,t)\in C^{3,1}_{x,t}(\overline{\Omega})\cap C^{4,2}_{x,t}(\Omega)$ funksiya Ω sohada $f(x,t)\in C(\overline{\Omega})$ bo'lganda A masalaning ***regulyar yechimi*** deyiladi, agar u Ω sohada (2)-(7) shartlarni va (1) tenglamani qanoatlantirsa.

1-*Teorema.* Agar A masalaning regulyar yechimi mavjud bo'lsa, u holda u yagonadir.

Isbot. Aytaylik ikkita $u_1(x,t)$ va $u_2(x,t)$ yechimlar mavjud bo'lsin. Bu yechimlar ayirmasi bir jinsli (1) tenglamani va (2)-(7) shartlarga mos bir jinsli chegaraviy va boshlang'ich shartlarni qanoatlantiradi. Bu ayirmani $u(x,t)$ bilan belgilaymiz, yani

$$u(x,t)=u_1(x,t)-u_2(x,t). \tag{8}$$

Ma'lumki

$$\mathrm{X}_0(x)=\frac{1}{\sqrt{p}},\quad \mathrm{X}_n(x)=\sqrt{\frac{2}{p}}\cos\lambda_n x,\quad \lambda_n=\frac{n\pi}{p},\quad n=1,2... \tag{9}$$

funksiya $L_2(0,p)$ da to'liq ortonormallangan sistemani tashkil etadi.

$[9]$ ga asosan

$$d_n(t)=\int_0^p u(x,t)\mathrm{X}_n(x)dx,\quad n=0,1,... \tag{10}$$

funksiyani ko'rib chiqamiz. Bu funksiyaga asosan quyidagi funksiyani kiritamiz:

$$d_{n,\varepsilon}(t)=\int_{\varepsilon}^{p-\varepsilon} u(x,t)\mathrm{X}_n(x)dx,\quad 0<\varepsilon<p, \tag{11}$$

bunda $(\varepsilon, p-\varepsilon)\neq\varnothing$.

(11) tenglikni t bo'yicha ikki marotaba differensiallasak, quyidagi

$$d''_{n,\varepsilon}(t)=-\int_{\varepsilon}^{p-\varepsilon} u_{xxxx}(x,t)\mathrm{X}_n(x)dx \tag{12}$$

funksiyani to'rt marotaba bo'laklab integrallab, $\varepsilon\to 0$ da limitga o'tamiz, u holda

$$\lim_{\varepsilon\to 0} d''_{n,\varepsilon}(t)=-\lim_{\varepsilon\to 0}\left[\mathrm{X}_n(p-\varepsilon)u_{xxx}(p-\varepsilon,t)-\mathrm{X}_n(\varepsilon)u_{xxx}(\varepsilon,t)\right]+$$
$$+\lim_{\varepsilon\to 0}\left[\mathrm{X}'_n(p-\varepsilon)u_{xx}(p-\varepsilon,t)-\mathrm{X}'_n(\varepsilon)u_{xx}(\varepsilon,t)\right]-$$
$$-\lim_{\varepsilon\to 0}\left[\mathrm{X}''_n(p-\varepsilon)u_{x}(p-\varepsilon,t)-\mathrm{X}''_n(\varepsilon)u_{x}(\varepsilon,t)\right]+$$
$$+\lim_{\varepsilon\to 0}\left[\mathrm{X}'''_n(p-\varepsilon)u(p-\varepsilon,t)-\mathrm{X}'''_n(\varepsilon)u(\varepsilon,t)\right]-\lim_{\varepsilon\to 0}\lambda_n^4\int_{\varepsilon}^{p-\varepsilon}\mathrm{X}_n(x)u(x,t)dx,$$

(4)-(7) ga mos bir jinsli shartlarni hisobga olganimizda, (8) dan quyidagi natija kelib chiqadi:

$$d_n^0(t)=-\lambda_n^4\int_0^p u(x,t)\mathrm{X}_n(x)dx,\qquad n=0,1,...$$

ya'ni, oddiy differensial tenglamaga kelamiz. Oxirgi tenglikdan (10) ni hisobga olib quyidagicha yozishimiz mumkin bo'ladi:

$$d_n''(t)+\lambda_n^4 d_n(t)=0, \qquad n=0,1,... \tag{13}$$

ma'lumki uning yechimi quyidagi ko'rinishda bo'ladi:

$$d_n(t)=C_{1n}\cos\lambda_n^2 t+C_{2n}\sin\lambda_n^2 t.$$

Bu yerdagi C_{1n} va C_{2n} noma'lum koeffitsientlarni topish uchun (2) va (3) bir jinsli shartlarni ishlatamiz, ya'ni $d_n(0)=0,\ d_n'(0)=0$ bundan esa, $C_{1n}=0, C_{2n}=0$ ga ega bo'lamiz, ya'ni barcha $\forall t\in[0,T]$ uchun $d_n(t)=0$ ekanligi kelib chiqadi. U holda (10) tenglik quyidagi ko'rinishni oladi:

$$\int_0^p u(x,t)X_n(x)dx=0, \qquad n=0,1,... \tag{14}$$

bu tenglikdan esa $u(x,t)$ funksiya (9) ning to'liq sistemasi bilan ortogonalligi kelib chiqadi, ya'ni masalaning yechimi yagonadir.
Teorema isbotlandi.

A masala yechimining mavjudligi.

A-masalaning yechimini mavjudligini isbotlash uchun o'zgaruvchilarni ajraladigan usulini qo'llaymiz.

2-teorema. Agar $f\in C_{x,t}^{2,0}(\overline{\Omega}),\quad f_{xxx}\in L_2(\Omega),\quad f_x(0,t)=f_x(p,t)=0$ va
$\psi_3^{(4)}(x)\in C[0,p],\quad \psi_3^{(5)}\in L_2(0,p)$ va $\psi_3'(0)=\psi_3'(p)=0,$
$\psi_3'''(0)=\psi_3'''(p)=0,\quad \psi_4^{(2)}(x)\in C[0,p],\ \psi_4^{(3)}(x)\in L_2(0,p),$ va
$\psi_4'(0)=\psi_4'(p)=0,$

shartlarni qanoatlantirsa, u holda A-masalaning $u(x,t)$ regulyar yechimi mavjud bo'lib va u $u(x,t)\in C_{x,t}^{4,2}(\overline{\Omega})$ bo'ladi, bu yerda $\psi_3(x),\psi_4(x)$ - keyin aniqlanadi.

Isbot. (4)-(7) chegaraviy shartlar bir jinsli bo'lmaganligi sababli bu masalani yechish uchun, to'g'ridan to'g'ri o'zgaruvchilari ajralatish usulini qo'llab bo'lmaydi. Lekin xususiy hosilali differensial tenglamalar nazariyasidan ma'lumki, bu masalani bir jinsli chegaraviy masalaga olib kelishimiz mumkin.

Haqiqatdan ham, yordamchi funksiyani kiritamiz

$$w(x,t)=x\varphi_1(t)+\left[\varphi_2(t)-\varphi_1(t)\right]\frac{x^2}{2p}-\frac{p^2}{8\pi^2}\left[\frac{p}{2\pi}\sin\frac{2\pi}{p}x-x\right]\times$$
$$\times\left[\varphi_4(t)+\varphi_3(t)\right]+\frac{p^2}{2\pi^2}\left[\frac{p}{\pi}\sin\frac{\pi}{p}x-x+\frac{x^2}{p}\right]\left[\varphi_4(t)-\varphi_3(t)\right]. \tag{15}$$

Tekshirib ko'rish qiyin emaski

$$w_x(0,t)=\varphi_1(t); \qquad w_x(p,t)=\varphi_2(t),$$
$$w_{xxx}(0,t)=\varphi_3(t); \qquad w_{xxx}(p,t)=\varphi_4(t).$$

Masalani yechimini yig'indi ko'rinishda yozamiz

$$u(x,t)=v(x,t)+w(x,t). \tag{16}$$

Bu yerda $v(x,t)$ - yangi noma'lum funksiya. Shunday qilib (16) ga asoslanib, biz keyingi masalaga keldik:

*A*1 **masala**. Ω sohada

$$v_{xxxx}+v_{tt}=g(x,t) \tag{17}$$

tenglamaning $v(x,t)$ yechimini topingki u quyidagi chegaraviy shartlarni qanoatlantirsin:

$$v_x(0,t)=v_x(p,t)=0, \qquad 0\le t\le T, \tag{18}$$

$$v_{xxx}(0,t)=v_{xxx}(p,t)=0, \qquad 0\le t\le T, \tag{19}$$

$$v(x,0)=\psi_3(x), \qquad 0\le x\le p, \tag{20}$$

$$v_t(x,0)=\psi_4(x), \qquad 0\le x\le p. \tag{21}$$

(16) ga asosan

$$v(x,0)=u(x,0)-w(x,0)\equiv\psi_3(x), \qquad 0\le x\le p,$$

$$v_t(x,0)=u_t(x,0)-w_t(x,0)\equiv\psi_4(x), \qquad 0\le x\le p.$$

Bir jinsli (17) tenglamaning yechimini quyidagi ko'rinishda izlaymiz:

$$v(x,t)=\mathrm{X}(x)\cdot T(t) \tag{22}$$

(17) tenglamaga (22) ni qo'yib, o'zgaruvchilarga ajratganimizda

$$\frac{\mathrm{X}''(x)}{\mathrm{X}(x)}=\frac{T''(t)}{T(t)}$$

tenglikning chap tomoni x ga, o'ng tomoni t ga bog'liq bo'ladi. Agar o'zgarmas bo'lsa ya'ni ikki tomoni na x ga na t ga bog'liq bo'lmasa, bu o'zgarmasni λ^4 bilan belgilaymiz. Unda oxirgi tenglikdan, ikkita $\mathrm{X}(x)$ va $T(t)$ ga nisbatan oddiy differensial tenglama kelib chiqadi. Ya'ni

$$\mathrm{X}^{IV}(x)-\lambda^4\mathrm{X}(x)=0, \tag{23}$$

tenglama va (18) va (19) shartlarga asosan

$$\mathrm{X}'(0)=\mathrm{X}'(p)=\mathrm{X}'''(0)=\mathrm{X}'''(p)=0$$

shartlarni qanoatlantiruvchi yechimini topish masalasini hosil qilamiz. Tekshirib ko'rish qiyin emaski (23) tenglamaning umumiy yechimi

$$\mathrm{X}(x)=C_1e^{-\lambda x}+C_2e^{\lambda x}-C_3\cos\lambda x+C_4\sin\lambda x. \tag{24}$$

bo'lib, chegaraviy shartlarga bo'ysundiradigan bo'lsak

$$\mathrm{X}_n(x)=C_{3n}\cos\frac{n\pi}{p}x, \qquad n=0,1,..., \tag{25}$$

natijani olishimiz mumlin.

$\mathrm{X}_n(x)$ ni normallaganimizda, $C_{3n}=\sqrt{\dfrac{2}{p}}$ ni $n=0$ da esa $C_n=\dfrac{1}{\sqrt{p}}$ larni aniqlashimiz mumkin. Haqiqatdan ham $n=0$ da

$$X_0(x)=C_{30} \quad \Rightarrow \quad X_0^2(x)=C_{30}^2 \quad \Rightarrow \quad \int_0^p X_0^2(x)dx=\int_0^p C_{30}^2 dx \quad \Rightarrow$$

$$C_{30}^2\int_0^p dx=1 \quad \Rightarrow \quad C_{30}^2 p=1 \quad \Rightarrow \quad C_{30}=\frac{1}{p}$$

$n=1,2,...$ bo'lganda

$$X_n^2(x)=C_{3n}^2\cos^2\frac{\pi n}{p}x \Rightarrow \int_0^p X_n^2(x)dx=\int_0^p C_{3n}^2\cos^2\frac{\pi n}{p}xdx \Rightarrow \frac{C_{3n}^2}{2}\int_0^p\left(1+\cos\frac{2\pi n}{p}x\right)dx=1$$

$$\Rightarrow \frac{C_{3n}^2}{2}\left(x+\frac{p}{2\pi n}\sin\frac{2\pi n}{p}x\right)\Bigg|_0^p=1 \Rightarrow C_{3n}=\sqrt{\frac{2}{p}}$$

Demak (25), (9) ko'rinishida yoziladi.

Endi (17) tenglamaning yechimini quyidagi qator ko'rinishida yozamiz

$$v(x,t)=\sum_{n=0}^{\infty}v_n(t)X_n(x), \tag{26}$$

(26) qatorni (17) tenglamaga qo'yganimizda,

$$\sum_{n=0}^{\infty}\left[v_n^{IV}(t)+\lambda_n^4 v(t)\right]\cdot X_n(x)=g(x,t). \tag{27}$$

(17) tenglamaning o'ng tomonidagi $g(x,t)$ funksiyani $(0,p)$ oraliqda $X_n(x)$ bo'yicha Fur'ye qatoriga yoyamiz

$$g(x,t)=\sum_{n=0}^{\infty}g_n(t)\cdot X_n(x), \tag{28}$$

bu yerda

$$g_n(t)=\int_0^p g(x,t)\cdot X_n(x)dx. \tag{29}$$

(28) ni (27) ga qo'yganimizda $v_n(t)$ funksiyaga nisbatan quyidagi oddiy differensial tenglamaga kelamiz.

$$v_n^{IV}(t)+\lambda_n^4 v(t)=g_n(t), \qquad n=0,1,... \tag{30}$$

Ushbu hosil qilingan tenglamani o'zgarmasni variatsiylash usuli bilan yechib quyidagini aniqlashimiz mumkin

$$v_n(t)=a_n(0)\cos\lambda_n^2 t+b_n(0)\sin\lambda_n^2 t+\frac{1}{\lambda_n^2}\int_o^t g_n(\tau)\cdot\sin\lambda_n^2(t-\tau)d\tau, \quad n=1,2,... \tag{31}$$

Agar $n=0$ bo'lsa, (30) tenglama

$$v_0^{IV}(t)=g_0(t)$$

ko'rinishda bo'lib uning yechimi quyidagicha bo'ladi

$$v_0(t)=a_0(0)t+b_0(0)+\int_0^t(t-\tau)g_0(\tau)d\tau \tag{32}$$

(31), (32) va (9) yechimlarni (26) qatoriga qo'ysak,

$$v(x,t)=\left[a_0(0)t+b_0(0)+\int_0^t(t-\tau)g_0(\tau)d\tau\right]\frac{1}{\sqrt{p}}+ \tag{33}$$

$$+\sqrt{\frac{2}{p}}\sum_{n=1}^{\infty}\left[a_n(0)\cos\lambda_n^2t+b_n(0)\sin\lambda_n^2t+\frac{1}{\lambda_n^2}\int_o^t g_n(\tau)\cdot\sin\lambda_n^2(t-\tau)d\tau\right]\cdot\cos\lambda_n x.$$

yechimni hosil qilamiz. Noma'lum $a_n(0)$ va $b_n(0)$ koeffitsentlarni topish uchun (20), (21) shartlarni qo'llaymiz va quyidagi koeffitsentlarni topamiz

$$a_0(0)=\frac{1}{\sqrt{p}}\int_0^p\psi_4(x)dx,\qquad b_0(0)=\frac{1}{\sqrt{p}}\int_0^p\psi_3(x)dx, \tag{34}$$

$$a_n(0)=\sqrt{\frac{2}{p}}\int_0^p\psi_3(x)\cos\lambda_n xdx,\qquad n=1,2,... \tag{35}$$

$$b_n(0)=\sqrt{\frac{2}{p}}\frac{1}{\lambda_n^2}\int_0^p\psi_1(x)\cos\lambda_n xdx,\qquad n=1,2,... \tag{36}$$

Haqiqatdan ham

$$v(x,0)=b_0(0)\frac{1}{\sqrt{p}}+\sqrt{\frac{2}{p}}\sum_{n=1}^{\infty}\left[a_n(0)\cos\lambda_n x\right]=\psi_3(x)$$

$$\frac{1}{\sqrt{p}}\int_0^p b_0(0)dx+\sqrt{\frac{2}{p}}\int_0^p\sum_{n=1}^{\infty}\left[a_n(0)\cos\lambda_n x\right]dx=\int_0^p\psi_3(x)dx$$

$$b_0(0)\sqrt{p}+\sqrt{\frac{2}{p}}\sum_{n=1}^{\infty}\left[a_n(0)\int_0^p\cos\lambda_n xdx\right]=\int_0^p\psi_3(x)dx$$

$$b_0(0)\sqrt{p}=\int_0^p\psi_3(x)dx-\sqrt{\frac{2}{p}}\sum_{n=1}^{\infty}\left[a_n(0)\frac{1}{\lambda_n}\sin\lambda_n x\Big|_0^p\right]=\int_0^p\psi_3(x)dx$$

$$\Rightarrow\qquad b_0(0)=\frac{1}{\sqrt{p}}\int_0^p\psi_3(x)dx$$

Xuddi shunigdek

$$\frac{\partial}{\partial t}v(x,t)=\left[a_0(0)+\int_0^t g_0(\tau)d\tau\right]\frac{1}{\sqrt{p}}+$$

$$+\sqrt{\frac{2}{p}}\sum_{n=1}^{\infty}\left[-a_n(0)\lambda_n^2\sin\lambda_n^2t+b_n(0)\lambda_n^2\cos\lambda_n^2t+\frac{\lambda_n^2}{\lambda_n^2}\int_o^t g_n(\tau)\cdot\cos\lambda_n^2(t-\tau)d\tau\right]\cdot\cos\lambda_n x.$$

$$\frac{\partial}{\partial t}v(x,0)=\frac{1}{\sqrt{p}}a_0(0)+\sqrt{\frac{2}{p}}\sum_{n=1}^{\infty}\left[\lambda_n^2b_n(0)\cos\lambda_n x\right]=\psi_4(x).$$

$$\frac{1}{\sqrt{p}}\int_0^p a_0(0)dx=\int_0^p \psi_4(x)dx-\sqrt{\frac{2}{p}}\sum_{n=1}^{\infty}\left[\lambda_n^2 b_n(0)\int_0^p \cos\lambda_n x dx\right]=$$

$$=\int_0^p \psi_4(x)dx-\sqrt{\frac{2}{p}}\sum_{n=1}^{\infty}\left[b_n(0)\sin\lambda_n x\Big|_0^p\right]=\int_0^p \psi_4(x)dx.$$

$$\Rightarrow \qquad a_0(0)=\frac{1}{\sqrt{p}}\int_0^p \psi_3(x)dx$$

Endi

$$v(x,0)=b_0(0)\frac{1}{\sqrt{p}}+\sqrt{\frac{2}{p}}\sum_{n=1}^{\infty}\left[a_n(0)\cos\lambda_n x\right]=\psi_3(x) \qquad / * \ \cos\lambda_k x \qquad k=1,2,3,...$$

$$\frac{1}{\sqrt{p}}b_0(0)\cos\lambda_k x+\sqrt{\frac{2}{p}}\sum_{n=1}^{\infty}\left[a_n(0)\cos\lambda_n x\cos\lambda_k x\right]=\psi_3(x)\cos\lambda_k x$$

$$\frac{1}{\sqrt{p}}b_0(0)\int_0^p \cos\lambda_k x dx+\sqrt{\frac{2}{p}}\sum_{n=1}^{\infty}\left[a_n(0)\int_0^p \cos\lambda_n x\cos\lambda_k x dx\right]=\int_0^p \psi_3(x)\cos\lambda_k x dx$$

$n\neq k$ bo'lganda $\int_0^p \cos\lambda_n x\cos\lambda_k x dx=0$ bo'lganligi uchun, $n=k$ bo'lgandagina qaraladi.

$$\frac{1}{\lambda_n\sqrt{p}}b_0(0)\sin\lambda_n x\Big|_0^p+\sqrt{\frac{2}{p}}a_n(0)\int_0^p \cos^2\lambda_n x dx=\int_0^p \psi_3(x)\cos\lambda_n x dx$$

$$\sqrt{\frac{2}{p}}a_n(0)\int_0^p \frac{1}{2}dx+\sqrt{\frac{2}{p}}a_n(0)\int_0^p \frac{\cos 2\lambda_n x}{2}dx=\int_0^p \psi_3(x)\cos\lambda_n x dx$$

$$\sqrt{\frac{2}{p}}a_n(0)\frac{p}{2}+\sqrt{\frac{2}{p}}\frac{a_n(0)}{2}\frac{1}{2\lambda_n}\sin\lambda_n x\Big|_0^p=\int_0^p \psi_3(x)\cos\lambda_n x dx$$

$$\Rightarrow \qquad a_n(0)=\sqrt{\frac{2}{p}}\int_0^p \psi_3(x)\cos\lambda_n x dx$$

$$\frac{\partial}{\partial t}v(x,0)=\frac{1}{\sqrt{p}}a_0(0)+\sqrt{\frac{2}{p}}\sum_{n=1}^{\infty}\left[\lambda_n^2 b_n(0)\cos\lambda_n x\right]=\psi_4(x). \qquad / * \ \cos\lambda_k x \qquad k=1,2,...$$

$$\frac{1}{\sqrt{p}}a_0(0)\cos\lambda_k x+\sqrt{\frac{2}{p}}\sum_{n=1}^{\infty}\left[\lambda_n^2 b_n(0)\cos\lambda_n x\cos\lambda_k x\right]=\psi_4(x)\cos\lambda_k x.$$

$$\frac{1}{\sqrt{p}}a_0(0)\int_0^p \cos\lambda_k x dx+\sqrt{\frac{2}{p}}\sum_{n=1}^{\infty}\left[\lambda_n^2 b_n(0)\int_0^p \cos\lambda_n x\cos\lambda_k x dx\right]=\int_0^p \psi_4(x)\cos\lambda_k x dx.$$

Yuqoridagi kabi mulohaza yuritadigan bo'lsak,

$$\frac{1}{\lambda_n\sqrt{p}}a_0(0)\sin\lambda_n x\Big|_0^p+\sqrt{\frac{2}{p}}\lambda_n^2 b_n(0)\int_0^p \frac{1+\cos 2\lambda_n x}{2}dx=\int_0^p \psi_4(x)\cos\lambda_n x dx.$$

$$\sqrt{\frac{2}{p}}\lambda_n^2 b_n(0)\frac{p}{2}+\sqrt{\frac{2}{p}}\lambda_n^2\frac{b_n(0)}{2}\frac{b_n(0)}{2\lambda_n}\sin 2\lambda_n x\Big|_0^p=\int_0^p\psi_4(x)\cos\lambda_n xdx.$$

$$\Rightarrow \qquad b_n(0)=\frac{1}{\lambda_n^2}\sqrt{\frac{2}{p}}\int_0^p\psi_4(x)\cos\lambda_n xdx.$$

Shunday qilib, masalaning yechimi (33) ko'rinishda bo'ladi. Undagi $a_0(0)$, $b_0(0)$, $a_n(0)$ va $b_n(0)$ koeffisientlar (34)-(36) formulalari bilan aniqlanadi. Natijada biz (33) formula bilan berilgan masalaning rasman yechimini Ω sohada o'rgandik.

1-lemma. $\forall t\in[0,T]$ va $n=1,2,...$ bo'lganda quyidagi baholashlar o'rinli bo'ladi

$$|v_0(t)|\le C_1;\quad |v_n(t)|\le\left[|a_n(0)|+|b_n(0)|\right]+\frac{\sqrt{T}}{\lambda_n^2}\|g_n\|_{L_2(0,T)} \tag{37}$$

$$|v_0'(t)|\le C_2;\quad |v_n'(t)|\le\lambda_n^2\left[|a_n(0)|+|b_n(0)|\right]+\sqrt{T}\,\|g_n\|_{L_2(0,T)}$$

$$|v_0''(t)|\le C_3;\quad |v_n''(t)|\le\lambda_n^4\left[|a_n(0)|+|b_n(0)|\right]+C\|g_n\|_{L_2(\bar{\Omega})}+\lambda_n^2\sqrt{T}\,\|g_n\|_{L_2(0,T)}$$

bu yerda C_1, C_2, C_3 - musbat o'zgarmas sonlar.

Isbot. (31) ni baholaymiz. Avvalam bor (31) ni uchinchi xadini baholaymiz va Koshi-Bunyakov tengsizligini qo'llaymiz. Bunda

$$\frac{1}{\lambda_n^2}\left|\int_0^t g_n(\tau)\sin\lambda_n^2(t-\tau)d\tau\right|\le\frac{1}{\lambda_n^2}\left|\int_0^t g_n(\tau)d\tau\right|\le\frac{1}{\lambda_n^2}\sqrt{T\int_0^T g_n^2(\tau)d\tau}=\frac{\sqrt{T}}{\lambda_n^2}\|g_n\|_{L_2(0,\tau)}.$$

U holda (31) uchun

$$|v_n(t)|\le\left[|a_n(0)|+|b_n(0)|\right]+\frac{\sqrt{T}}{\lambda_n^2}\|g_n\|_{L_2(0,T)}$$

(34) dan ko'rish qiyin emaski $a_0(0)$ va $b_0(0)$ lar chegaralangan. Shunigdek $|t-T|\le T$ ekanligini hisobga olib va Koshi-Bunyakov tengsizligini qo'llab, (32) dan quyidagi kelib chiqadi

$$|v_0(t)|=C_1+\left|\int_0^t(t-\tau)g_0(\tau)d\tau\right|\le C_1+\sqrt{\int_0^T T^2d\tau\int_0^\tau g_0^2(\tau)d\tau}=C_1+T\sqrt{T}\cdot\|g_0\|_{L_2(0,\tau)}\le C_2,$$

bu yerda $C_1,C_2=const>0.$

Koshi-Bunyakovskiy tengsizligiga asoslanib, $v_0'(t),v_0''(t)$ va $v_n'(t),v_n''(t)$ hosilalarni ham yuqoridagidek baholashimiz mumkin bo'ladi.

Lemma isbotlandi.

Endi topgan (33) yechimimizni yaqinlashuvchi ekanligini va

$$\frac{\partial^4 v}{\partial x^4}=\sum_{n=1}^{\infty}\lambda_n^4 v_n(t)\cdot X_n(\tau) \tag{38}$$

$$\frac{\partial^2 v}{\partial t^2}=-\sum_{n=1}^{\infty}\lambda_n^4 v_n(t)\cdot X_n(x)+g_0(t)\cdot X_0(x)+\sum_{n=1}^{\infty}g_n(t)\cdot X_n(x), \tag{39}$$

qatorlarni tekshiramiz. Buning uchun (38) ni yaqinlashishligini tekshirish bizga yetarli bo'ladi. (38) qatorning yaqinlashishidan (39) qatorning yaqinlashishi kelib chiqadi, chunki (39) qator (38) va Fur'ye qatorlari $g(x,t)$ funksiyadan tarkib topgan. Fur'ye qatorlar nazariyasidan ma'lumki $g(x,t)$ funksiyaga qo'yilgan shartlar, $\overline{\Omega}$ sohada absalyut va tekis yaqinlashadi. Endi (38) qatorning yaqinlashuvchiligini ko'rib chiqamiz, uning uchun

$$\sum_{n=1}^{\infty} \lambda_n^4 |v_n(t)|. \tag{40}$$

mojarant qator bo'ladi. Topilganlarni lemmadagi (37) ni (40) ga qo'yib va o'zgarmas $\sqrt{T}$ ni qoldirib yozsak,

$$\sum_{n=1}^{\infty}\left[\lambda_n^4 |a_n(0)| + \lambda_n^2 |b_n(0)| + \lambda_n^2 \cdot \|g_n\|_{L_2(0,T)}\right]$$

hosil qilamiz.

Biz

$$\sum_{n=1}^{\infty}\left[\lambda_n^4 |a_n(0)| + \lambda_n^2 |b_n(0)|\right], \tag{41}$$

$$\sum_{n=1}^{\infty} \lambda_n^2 \cdot \|g_n\|_{L_2(0,T)}. \tag{42}$$

qatorlarning yaqinlashuvchiligini isbotlashimiz kerak. Buning uchun (35) ni besh marta, (36) ni esa uch marta bo'laklab integrallab

$$a_n(0) = \int_0^p \psi_3(x)\cos\frac{\pi n}{p}xdx = \frac{p}{\pi n}\psi_3(x)\sin\frac{\pi n}{p}x\Big|_0^p\Big|^{=0} - \frac{p}{\pi n}\int_0^p \psi_3'(x)\sin\frac{\pi n}{p}xdx =$$

$$= -\frac{p}{\pi n}\int_0^p \psi_3'(x)\sin\frac{\pi n}{p}xdx = \frac{p^2}{(\pi n)^2}\psi_3'(x)\cos\frac{\pi n}{p}x\Big|_0^p + \frac{p^2}{(\pi n)^2}\int_0^p \psi_3''(x)\cos\frac{\pi n}{p}xdx =$$

$\psi_3(x)$ funksiya uchun qo'shimcha shartlar qo'yib, ya'ni $\psi_3'(0)=0,\ \psi_3'(p)=0$ da

$$= \frac{p^2}{(\pi n)^2}\int_0^p \psi_3''(x)\cos\frac{\pi n}{p}xdx = \frac{p^3}{(\pi n)^3}\psi_3''(x)\sin\frac{\pi n}{p}x\Big|_0^p - \frac{p^3}{(\pi n)^3}\int_0^p \psi_3'''(x)\sin\frac{\pi n}{p}xdx =$$

$$= -\frac{p^3}{(\pi n)^3}\int_0^p \psi_3'''(x)\sin\frac{\pi n}{p}xdx = \frac{p^4}{(\pi n)^4}\psi_3'''(x)\cos\frac{\pi n}{p}x\Big|_0^p + \frac{p^4}{(\pi n)^4}\int_0^p \psi_3^{(4)}(x)\cos\frac{\pi n}{p}xdx =$$

qo'shimcha $\psi_3'''(x)$ ga shartlar qo'yamiz, ya'ni $\psi_3'''(0)=0, \psi_3'''(p)=0$. U holda

$$= \frac{p^4}{(\pi n)^4}\int_0^p \psi_3^{(4)}(x)\cos\frac{\pi n}{p}xdx = \frac{p^5}{(\pi n)^5}\psi_3^{(4)}(x)\sin\frac{\pi n}{p}x\Big|_0^p + \frac{p^5}{(\pi n)^5}\int_0^p \psi_3^{(5)}(x)\sin\frac{\pi n}{p}xdx$$

Demak,

$$a_n(0) = \sqrt{\frac{2}{p}}\frac{p^5}{(\pi n)^5}\int_0^p \psi_3^{(5)}(x)\sin\frac{\pi n}{p}xdx.$$

Xuddi shuningdek

$$b_n(0)=\sqrt{\frac{2}{p}}\frac{p^2}{(\pi n)^2}\int_0^p \psi_4(x)\cos\frac{\pi n}{p}xdx=\sqrt{\frac{2}{p}}\frac{p^3}{(\pi n)^3}\psi_4(x)\sin\frac{\pi n}{p}x\Big|_0^p-\sqrt{\frac{2}{p}}\frac{p^3}{(\pi n)^3}\int_0^p \psi_4'(x)\sin\frac{\pi n}{p}xdx=$$

$$=-\sqrt{\frac{2}{p}}\frac{p^3}{(\pi n)^3}\int_0^p \psi_4'(x)\sin\frac{\pi n}{p}xdx=\sqrt{\frac{2}{p}}\frac{p^4}{(\pi n)^4}\psi_4'(x)\cos\frac{\pi n}{p}x\Big|_0^p-\sqrt{\frac{2}{p}}\frac{p^4}{(\pi n)^4}\int_0^p \psi_4''(x)\cos\frac{\pi n}{p}xdx=$$

$\psi_4(x)$ funksiya uchun qo'shimcha shartlar qo'yib, ya'ni $\psi_4'(0)=0,\ \psi_4'(p)=0$ da

$$=-\sqrt{\frac{2}{p}}\frac{p^4}{(\pi n)^4}\int_0^p \psi_4''(x)\cos\frac{\pi n}{p}xdx=-\sqrt{\frac{2}{p}}\frac{p^5}{(\pi n)^5}\psi_4''(x)\sin\frac{\pi n}{p}x\Big|_0^p+\sqrt{\frac{2}{p}}\frac{p^5}{(\pi n)^5}\int_0^p \psi_4'''(x)\sin\frac{\pi n}{p}xdx$$

Demak,

$$b_n(0)=\sqrt{\frac{2}{p}}\frac{p^5}{(\pi n)^5}\int_0^p \psi_4'''(x)\sin\frac{\pi n}{p}xdx$$

va $\psi_3(x)$ va $\psi_4(x)$ funksiyalar shartlarini hisobga olib, quyidagiga erishamiz,

$$a_n(0)=-\left(\frac{p}{\pi}\right)^5\frac{b_n^{(5)}}{n^5},\qquad b_n(0)=\left(\frac{p}{\pi}\right)^3\frac{a_n^{(3)}}{n^3},\tag{43}$$

bu yerda

$$b_n^{(5)}=\sqrt{\frac{2}{p}}\int_0^p \psi_3^{(5)}(x)\sin\lambda_n xdx,\qquad a_n^{(3)}=\sqrt{\frac{2}{p}}\int_0^p \psi_4^{(3)}(x)\sin\lambda_n xdx.$$

(43) ni (41) ga qo'yib, quyidagi natijaga kelamiz

$$\sum_{n=1}^{\infty}\left[\frac{\left|a_n^{(5)}\right|}{n}+\frac{\left|b_n^{(3)}\right|}{n}\right]\tag{44}$$

Bu qatornig yaqinlashuvchanligi elementar tengsizlikdan kelib chiqadi, ya'ni

$$\frac{\left|a_n^{(5)}\right|}{n}\le\frac{1}{2}\left[\left(a_n^{(5)}\right)^2+\frac{1}{n^2}\right],\qquad \frac{\left|b_n^{(3)}\right|}{n}\le\frac{1}{2}\left[\left(b_n^{(3)}\right)^2+\frac{1}{n^2}\right],$$

va

$$\sum_{n=1}^{\infty}\left[\left(a_n^{(5)}\right)^2+\left(b_n^{(3)}\right)^2\right],\qquad \sum_{n=1}^{\infty}\frac{1}{n^2}.$$

Birinchi qatorning yaqinlashuvchiligi bo'lak-bo'lak uzluksiz bo'lgan $\psi_3^{(5)}(x)$, $\psi_4^{(3)}(x)$ funksiyalar uchun Bessel tengsizligidan kelib chiqadi. Ikkinchi qator umumlashgan garmonik $\sum_{n=1}^{\infty}\frac{1}{n^{\alpha}}$ qator, $\alpha>1$ bo'lganda yaqinlashuvchi bo'ladi. Demak, (41) qator yaqinlashadi. (29) ni uch marta bo'laklab integrallasak va $g(x,t)$ ga qo'yilgan shartlarni hisobga olsak, quyidagiga erishamiz

$$g_n(t)=\frac{1}{\lambda_n^3}\overline{g_{n3}}(t),\qquad n=1,2,...\tag{45}$$

bu yerda

$$\overline{g_{n3}}(t)=\sqrt{\frac{2}{p}}\int_0^p \frac{\partial^3 g}{\partial x^3}\sin\lambda_n x dx, \qquad n=1,2,...$$

(45) ni hisobga olib (42) qatorni quyidagi ko'rinishda yozamiz

$$\sum_{n=1}^{\infty}\frac{1}{\lambda_n}\left\|\overline{g_{n3}}\right\|_{L_2(0,T)} \tag{46}$$

Endi (46) qatorning yaqinlashuvchiligini ko'rsatamiz

$$\sum_{n=1}^{\infty}\frac{1}{\lambda_n}\left\|\overline{g_{n3}}\right\|_{L_2(0,T)} \le \sqrt{\frac{p^2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}}\cdot\sqrt{\sum_{n=1}^{\infty}\left\|\overline{g_{n3}}\right\|^2_{L_2(0,T)}}$$

Ohirgi tengsizlikning o'ng tomonidagi birinchi qator, yuqorida ko'rsatilgandek yaqinlashadi. Endi, ohirgi tengsizlikning o'ng tomonidagi ikkinchi qatorning yaqinlashuvchiligini ko'rsatamiz:

$$\left\|\frac{\partial^3 g}{\partial x^3}\right\|^2_{L_2(\Omega)}=\left(\frac{\partial^3 g}{\partial x^3},\frac{\partial^3 g}{\partial x^3}\right)_{L_2(\Omega)}=\left(\sqrt{\frac{2}{p}}\sum_{n=1}^{\infty}\overline{g_{n3}}(t)\cdot\sin\lambda_n x,\sqrt{\frac{2}{p}}\sum_{m=1}^{\infty}\overline{g_{m3}}(t)\cdot\sin\lambda_m x,\right)_{L_2(\Omega)}=$$

$$=\frac{2}{p}\int_0^T\int_0^p\sum_{n=1}^{\infty}\overline{g_{n3}}(t)\cdot\sin\lambda_n x\sum_{n=1}^{\infty}\overline{g_{m3}}(t)\cdot\sin\lambda_m x dx dt=\sum_{n=1}^{\infty}\int_0^T\overline{g_{n3}}^2(t)dt=\sum_{n=1}^{\infty}\left\|\overline{g_{n3}}\right\|^2_{L_2(0,T)^2},$$

ya'ni biz shunday tenglikka keldik:

$$\sum_{n=1}^{\infty}\left\|\overline{g_{n3}}\right\|^2_{L_2(0,T)^2}=\left\|\frac{\partial^3 g}{\partial x^3}\right\|^2_{L_2(\Omega)} \tag{47}$$

(47) tenglik bo'lak-bo'lak va barcha ortonormallangan funksiyalar uchu *Parseval* tengligiga teng kuchli. Demak, (46) qator yaqinlashadi. Shunday qilib biz, (40) mojorand qatorning yaqinlashuvchiligini ko'rsatdik. Demak, $\overline{\Omega}$ sohada (38) va (39) qatorlar absalyut va tekis yaqinlashadi. Olingan yechimning yaqinlashuvchiligi (38) va (39) qatorlarning yaqinlashuvchiligidan kelib chiqadi.

2. teoremaning shartlari bajarilsa, unda A-masalaning $u(x,t)$ yechimini ikki marta t bo'yicha va to'rt marta x bo'yicha hadlab differensiallanuvchi bo'ladi, bundan kelib chiqadiki topilgan qatorlar $\overline{\Omega}$ sohasida absolyut va tekis yaqinlashadi. Buni esa $u(x,t)\in C^{4,2}_{x3}\left(\overline{\Omega}\right)$ ko'rinishda yozishimiz mumkin. *Teorema isbotlandi.*

Aytib o'tish kerakki, agar (4), (5) shartlarida $u(x,t)$ funksiyani va (6), (7) shartlarida $u_{xx}(x,t)$ funksiya berilganda, (15) funksiyaning quyidagicha tanlash talab etiladi:

$$w(x,t)=\varphi_1(t)+\left[\varphi_2(t)-\varphi_1(t)\right]\cdot\frac{x}{p}+\frac{p^2}{8\pi^2}\left[\varphi_4(t)+\varphi_3(t)\right]\times$$

$$\times\left(1-\cos\frac{2\pi}{p}x\right)+\frac{p^2}{2\pi^2}\left[\varphi_4(t)-\varphi_3(t)\right]\cdot\left(\cos\frac{\pi}{p}x-1+\frac{2x}{p}\right).$$

Bu holda ham masalani o'zgaruvchilari ajraladigan usul bilan yechish mumkin.

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